

Equilibrium with Benchmarking Institutions*

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Abstract

In this paper we develop an asset pricing model with heterogeneous institutional investors, and we provide a comprehensive analysis of the effects of benchmark heterogeneity on equilibrium prices and portfolio allocations. We find that an institution's holdings are higher for those assets that are exclusively part of its benchmark, and that are financed by taking short positions in the assets that are only in other institution's benchmark, while keeping a zero cash balances at all times. This result implies that correlation across benchmarks is negative. We define a measure of asymmetry between benchmarks and show how it effects asset prices and portfolio allocations. Institutions revert their holdings to their benchmarks when fundamental volatility is high – *flight to benchmark* – thus creating a demand pressure on the overlapping part of benchmarks, which in turn pushes prices up and Sharpe ratios down even further. Our model also addresses the twin stocks discrepancy, the low volatility puzzle and the asset class effect. We conclude our analysis by characterizing an endogenous choice of benchmark, and show that institutions optimally select the same fully diversified index.

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1 Introduction

Heterogeneity in the funds' industry is substantial, Kacperczyk, Sialm, and Zheng (2005) show that funds differ substantially in their industry concentration and the more concentrated funds tend to follow distinct investment styles. Furthermore, institutional ownership is gradually increasing: by the end of 2010 it reached 67% in the equity market. Salient aspect of the funds' industry is that funds' performance is measured relative to an index. This became quite explicit when in 1998 the Security and Exchange Commission (SEC) required mutual funds to report their benchmark indices, thus making their relative performance readily available to all investors. Understanding how this affects institutions' incentives and ultimately asset prices is paramount and is the focus of this study.

In this paper we develop an asset pricing equilibrium model with multiple institutions in order to study the effects of benchmark heterogeneity on equilibrium prices and asset allocations. An institutional investor is a fund manager whose performance is measured relative to an index, which is comprised of some given combination of assets. Heterogeneity arises since each institution has potentially a different benchmark. We first derive asset pricing quantities and portfolio allocations in analytical forms and we analyze their equilibrium behavior for different index configurations. We then show how our model is able to rationalize some of the most pervasive asset pricing irregularities that have received much attention in the empirical literature. We conclude by deriving an endogenous index configuration as part of the institutions' choice set.

Our main results are that institution's holdings are higher for assets that are exclusively in their benchmarks than assets that are also benchmarked by the other institution, and are financed by taking short positions in the assets that are exclusively in the other institution's benchmark, to the extent allow by their other holdings. In addition, it is never optimal for institutions to hold cash when they both have benchmark motives, since the Sharpe ratios of the other institutions' benchmarked assets are negative, taking a short position yields an above zero Sharpe ratio. Furthermore, as the index configuration becomes less symmetric, adding an asset to one benchmark and deleting it from the other increase prices and volatilities, and decrease Sharpe ratios. When the fundamental volatility is high, institutions revert their holdings to their benchmarks, thereby creating a demand pressure on assets that are common to both benchmarks, which further increase prices and decrease Sharpe ratios. Our model can explain the twin-stock price discrepancy the low-volatility puzzle and the asset class effect. We conclude our results by endogenizing the

In our economic setting, we consider multiple risky assets with the same payoff distribution. Unlike Basak and Pavlova (2013) we do not restrict the aggregate dividend's distribution to be a Geometric Brownian Motion, but instead we allow it to naturally arise as a sum of individual payoffs. The economy is populated by multiple institutions that can invest in a cash account and in the risky assets. Their preferences are characterized by a CRRA utility function defined over their terminal wealth relative to a benchmark. We refer to an institutional investor with no benchmark as retail investor. In reduced form, this utility structure captures the salient feature of relative performance concerns and the incentives of institutional investors to outperform their benchmarks. Institutions optimal terminal wealth and portfolio allocations are determined in equilibrium such that dividends are consumed and asset markets clear. These assets are claims to dividends, and their prices, volatilities and Sharpe ratios are determined endogenously. We present the equilibrium asset prices and portfolio allocations for general specification of indices and in Section 4 we investigate a special case of geometric averages of the risky assets with equal weights. This specification is commonly used in practice (i.e., *S&P500 Commodity Index (SPCI)*) and it preserves tractability without sacrificing the economic intuition.

The equilibrium is characterized by an optimal risk sharing rule, which determines how much each institution consumes of the dividends. We show that the optimal shares are given by weighted averages of the institution's benchmarks. In states of the world where one benchmark is higher than the others, the related institution's share will be higher. The weight of each institution is determined by its *initial weight* divided by its *index weight*. The initial weight reflects the initial share of institutions and the index weight reflects the co-movement of the index with the pricing kernel. When the co-movement is high the index weight decreases, and it reduces the share of the corresponding institution.

Importantly, equilibrium asset prices and portfolios depend on two hedging demands, *aggregate hedging* and *index hedging*. With the first, institutions hedge against future fluctuations in the aggregate dividend volatility. With the second, instead, they hedge against future fluctuations in each institution's index holdings. The two hedging demands have counter effects on asset prices and on portfolio allocations. Risk aversion greater than one is paramount, since risk aversion equals one removes the index hedging and the heterogeneity in asset prices and portfolio allocations vanishes. Asset pricing implications strongly depend on the benchmark heterogeneity and configurations; however, we can analytically compare two assets if a subset of institutions that benchmark one asset also benchmark the other. In this case, the asset benchmarked by the subset of institutions has a lower price and a higher Sharpe ratio. Intuitively, when a bigger set of institutions follow an asset, the demand pressure on that asset causes an increase in price and a decrease in the Sharpe ratio, so that it becomes a less desirable investment for institutions with no benchmarking motives.

To study the role of benchmark heterogeneity, we focus our analysis on the case of two in-

stitutions and ten assets. This setting allows us to characterize, without loss of generality, the equilibrium implications for: (i) assets that are benchmarked by both institutions; (ii) assets that are benchmarked by only one institution; and (iii) assets that are not benchmarked by either of the them. The number of assets of each these *types* is a key determinant of the equilibrium quantities. As shown in previous literature, institutions' portfolio allocations are tilted towards their benchmarks. However, the heterogeneity of benchmarks plays a key role in determining the magnitude of the allocation distortion and its distribution within the benchmark. Four economic considerations determine the institutions' holdings in equilibrium. First, because of diversification purposes, institutions' share is equally distributed within assets of the same type. Second, an institution acts as a retail investor with respect to those assets that are not in its benchmark but that are, instead, in the benchmark of the other. This implies that one institution provides supply to the other by taking a short position on these assets. Third, when benchmarks overlap, the institutions provide supply to one another by decreasing their demand for the overlapping assets. As last, it is suboptimal for institutions to hold cash when they both have benchmarks, as it would be more profitable taking a short position in the assets that are benchmarked only by the other institution.

In the sensitivity analysis we define a measure of asymmetry in the index configuration and show that when the index configuration is more asymmetric the distortion in asset prices and portfolio allocations is exacerbated. When an asset is deleted from a narrow benchmark and is included in a wide benchmark: assets that are common to both benchmarks increase in prices and decrease in Sharpe ratios. In such cases, the deletion from the narrow benchmark has stronger effects than the addition to the wide benchmark. Wealth levels incur an upward jump when the second benchmark is introduced, since a new demand for benchmarked assets is introduced and prices immediately increase.

In the analysis of how asset prices and portfolio allocations change when the fundamental volatility increases we find the *flight to benchmark* behavior. Institutions become more concentrated in their benchmarks in episodes of high fundamental volatility. The distortion is exacerbated the narrower the index, since institutions' holdings outside their benchmarks are higher, implying that there are more funds to reallocate. When the benchmarks overlap, this creates a further demand pressure on the overlapping part of the benchmark which subsequently pushes their prices up and Sharpe ratios down even further.

There are several implications our model produces that are ubiquitous in the empirical data. The first implication is coined *idiosyncratic risk pricing*, since some of the idiosyncratic shocks effect prices much more strongly than others. All shocks appear in the aggregate hedging component but only a few appear in the index hedging component. It appears that shocks that are propagated through the index hedging component have profound effect on asset prices. Indeed, when dividends are significantly correlated the price difference between benchmarked and non benchmarked

assets is substantial. It is the idiosyncratic part of the dividend stream that drives the difference. The economic intuition for this effect arises from the fact that institutions fail to diversify in the usual sense, their portfolios are tilted towards their benchmarks, therefore they are more exposed to shocks in these assets. This result is aligned with the empirical evidence. Campbell, Lettau, Malkiel, and Xu (2001) and Goyal and Santa-Clara (2003) and Boyer, Mitton, and Vorkink (2010), claim that due to investors' failure to diversify, idiosyncratic risk matters for pricing. Idiosyncratic pricing provides a rational explanation to the twin stock price discrepancy, documented by Rosenthal and Young (1990) and Dabora and Froot (1999), and the parent and subsidiary stock discrepancy, documented by Mitchell, Pulvino, and Stafford (2002) and Lamont and Thaler (2003). The idiosyncratic differences between twin companies and parent and subsidiary companies have major effects on stocks prices. Barberis and Thaler (2003) describe it as "most certainly a mispricing".

The second implication is the *low volatility puzzle*, "the greatest anomaly in finance" as coined by Baker, Bradley, and Wurgler (2010). They observe in the data that low volatility portfolios outperform high volatility portfolios. Put differently, low volatility portfolios' Sharpe ratios are higher than high volatility portfolios' Sharpe ratios. This result was first explored by Ang, Hodrick, Xing, and Zhang (2006) and Ang, Hodrick, Xing, and Zhang (2009), where they show that stocks with high idiosyncratic volatility have extremely low returns, contradicting what a traditional asset pricing theory implies. As was described earlier, benchmarking institutions can be responsible for this anomaly.

The third implication, explored by Harris and Gurel (1986) and Shleifer (1986), is the stock inclusion effect: a stock price increases with inclusion in- and decreases with exclusion out- of an index. In their empirical tests they control both for fundamental changes to the underlying firm and for changes to the firm's quality that inclusion implies. Chen, Noronha, and Singal (2004) show that this phenomenon is more pronounced in recent years. During 1976-1989 stock inclusion increased stock prices on the day of announcement by 3.1% while during 1989-2000 stock inclusion has increased stock prices by 5.45%. This empirical fact aligns with the increased size of the institutional sector over these years, as was reported by Blume and Keim (2012), but can also be attributed to the growth of passive index funds, as Hortasu and Syverson (2004) report. This is in agreement with our model, predicting that institutions tilt their portfolios towards the indexed stocks, effectively increasing their prices. Furthermore, the magnitude of the change depends on the size of the institutional sector, as the empirical findings suggest.

The last implication, explored by Vijh (1994) and Barberis, Shliefer, and Wurgler (2005), is the comovement of assets within the same class. In the traditional asset pricing theory, since the comovement of prices reflects the comovement of fundamental values, index inclusion shouldn't affect assets' comovement. The evidence however, suggests otherwise: stock inclusion leads to a

shift in the correlation structure of returns, as our model predicts.

In the last section we find the endogenous choices of benchmarks. The discussion about optimal benchmark was previously in a contractual context. In a principal-agent framework, the investor offers to the manager a compensation contract that optimally yields the manager to closely follow the benchmark. However, most of the models in the literature consider a partial equilibrium setting, where prices and benchmarks are exogenously predetermined. In contrast, we derive the optimal benchmarks when prices and index choices are endogenously and jointly determined. We abstract from the principal-agent problem and assume that each reported benchmark arises from an optimal contract, given prices and benchmarks. When all the different institutions report their benchmarks and prices are revealed we can identify if the benchmarks are optimal for the given prices. In the case where institutions are obliged to report benchmarks, the optimal index configuration arises when all institutions are fully diversified and follow the exact same benchmark, i.e., herd. This result implies that there is a substantial additional cost for the principal to offer the manager an optimal benchmark contract that is not the fully diversified index. For example, fund managers require higher fees for benchmarking the Russell 1000 Growth index as opposed to the S&P500 index.

Brennan (1993) was the first to introduce financial institutions to an asset pricing equilibrium model. He proposed a static model with two types of agents, retail investors, who are mean-variance return optimizers, and institutions, who are mean-variance return optimizers relative to the benchmark index. Both types have constant absolute risk aversion preferences. In this model, equilibrium prices are higher and the expected return is characterized by two factors: the market portfolio and the benchmark portfolio.

Currently, the literature on benchmarking is divided into two main branches. The first branch emphasizes the agency friction between the fund managers and the fund investors as the main source of asset pricing anomalies. Allen (2001), in his presidential address, expresses the importance of implementing agency frictions into asset pricing equilibrium models with institutions. Cornell and Roll (2005) explore the asset pricing implications in a static setting and Cuoco and Kaniel (2011) in a dynamic continuous time setting. The second branch, abstracts away from the agency friction problem. Gomez and Zapatero (2003) and Brennan and Li (2008), among others, explore the asset pricing implications in a static setting, while Basak and Pavlova (2013) in a dynamic continuous time setting. The main tradeoff between the two approaches is that in the first, the interaction mechanism is rich but the economic structure is simple, whereas in the second it is the other way around. Nonetheless, in both cases, due to modeling challenges most of the benchmarking literature is in static settings. A key shortcoming of the static models as well as single risky asset models is that they are unable to explore changes in asset allocations due to shifts in risk exposure or in model parameters. Therefore, models of these types, apart from the inclusion effect, are unable to address the phenomena described above.

Cuoco and Kaniel (2011) were the first to explore institutions' effects on asset prices in a dynamic equilibrium setting. They model the fund manager explicitly and introduce an agency friction between the fund investor and the fund manager. The fund manager cares about the index due to a performance based fees structure in his utility preference. They are the first to introduce a constant relative risk aversion preference to a dynamic equilibrium model with an institutional investor. Their model exhibits both the asset price inclusion effect and the decline in Sharpe ratio relative to the no benchmark case. However, some of their results are in contrast with empirical findings. First, in their model asset volatilities are lower relative to the no benchmark case. Second, in their model the size of the institutional sector does not affect the asset pricing quantities. Empirical evidence suggests that the effects discussed above are more pronounced in recent years, suggesting that the size of the institutional sector does matter. See for example, Barberis et al. (2005) in the asset class effect, Chen et al. (2004) in asset price inclusion effect.

Basak and Pavlova (2013) also considered a dynamic setting with one institutional investor and one retail investor. In their model, the institution has a log utility in wealth and is affine in the index, while the retail investor has log utility in wealth. In contrast to Cuoco and Kaniel (2011), they abstract away from the agency friction but are able to introduce a richer economic setting. Therefore, they are the first to show that benchmarking institutions' effects on asset prices can also address the asset class effect and the counter cyclicity of Sharpe ratios. In addition, instead of letting the aggregate dividends and the index to naturally arise as sums of the independent dividends, they restrict them to be a geometric Brownian Motion. By doing this they are able to present the asset pricing quantities in closed form. However, they are unable to explore within and across benchmark heterogeneities.

Lastly, there is a large body of work on delegated portfolio management. In this literature, models introduce an agency friction between fund managers and fund investors and usually incorporate a very rich interaction mechanism with a very simple security market structure. Recent papers that explore the equilibrium effects in a dynamic setting include Kapur and Timmermann (2005), Arora and Ou-Yang (2006), He and Krishnamurthy (2012), He and Krishnamurthy (2013), Vayanos and Woolley (2013), Buffa, Vayanos, and Woolley (2014) and Brunnermeier and Sannikov (2014). In addition, our paper also relates to the discussion about optimal benchmarks and how the compensation contracts should be structured. This literature includes Bhattacharya and Pfleiderer (1985), Starks (1987), Stoughton (1993), Heinkel and Stoughton (1994), Das and Sundaram (2002), Palomino and Prat (2003), Larsen (2005), Liu (2005), Cadenillas, Cvitanic, and Zapatero (2007), Li and Tiwari (2009) and Lioui and Poncet (2013). Moreover, our paper relates to the literature on relative wealth concerns. For example, DeMarzo, Kaniel, and Kremer (2004) and DeMarzo, Kaniel, and Kremer (2008) show that competition on local resources leads investors to care about their relative wealth in the community. As a result, investors are over-investing and herding.

We contribute to the literature on benchmarking institutions in two main aspects. In the first aspect, we are the first to incorporate heterogeneity in benchmarks in an asset pricing equilibrium model. We provide a complete analysis of the heterogeneity effects on asset prices and portfolio allocations. We find that institutions tilt towards the benchmark is exacerbated in the independent part of their benchmark and it is financed by taking a short position on the other institutions' benchmark. In addition, adding an asset to a benchmark might have reversed effects on the assets within it: it increases prices and volatilities and decreases Sharpe ratios of the common component of the benchmark. In addition, we observe that institutions revert their holdings to the benchmark in episodes of high fundamental volatility, which creates a further demand pressure on the common component of the benchmark that further increases prices.

The rest of the paper is organized as follows. In section 2 we set up the economy. In section 3 we derive the main general equilibrium result, asset prices, portfolios and key results of the model. In section 4 we analyze the heterogeneity implications of institutions. In Section 4.5 we derive the optimal index configuration, and in Section 5 we summarize our key results and conclude.

2 The Economic Setup

In this section we develop a model to capture the heterogeneous effects of benchmarks on asset prices and portfolios. It is a complete market economy, where the primitives of the model are as simple as possible without losing the effects of benchmarks.

Dividends: There are N independent risky dividends paid at time T . We denote the j 'th risky dividend by D_{jT} . Over time information about D_{jT} is revealed by the dynamics

$$dD_{jt} = D_{jt}(\mu dt + \sigma d\omega_{jt}), \quad D_{j0} > 0 \quad (1)$$

where μ and σ are positive constants, ω_{jt} is a dividend specific Brownian Motion and D_{j0} is the given initial level. We denote by D_T the sum of the individual dividends, $D_T = \sum_{j=1}^N D_{jT}$. Its dynamics are characterized by

$$\frac{dD_t}{D_t} = \mu dt + \sigma_t^{D'} d\omega_t,$$

where σ_t^D is a $N \times 1$ vector, where its j 'th entry is equal to

$$\frac{D_{jt}\sigma}{D_t}, \quad (2)$$

ω_t is a $N \times 1$ vector of the individual Brownian Motions and the $'$ signifies vector transpose.

Assets: There are $N + 1$ traded assets in the economy, N risky assets and a cash account. For the

sake of simplicity the cash account interest rate r , which is exogenous, is set to zero. The risky assets are claims to the dividends paid at time T , where we denote the price of asset j by S_{jt} and its dynamic by

$$dS_{jt} = S_{jt} (\mu_{jt}^S dt + \sigma_{jt}^{S'} d\omega_t). \quad (3)$$

The asset price S_{jt} , its drift, μ_{jt}^S , and its vector of volatilities, σ_{jt}^S , are endogenous and determined in equilibrium.

Institutions: There are M heterogeneous institutions, where institution's i preference over terminal wealth is characterized by

$$E \left[\frac{1}{1-R} \left(\frac{W_{iT}}{S_{iT}^I} \right)^{1-R} \right], \quad (4)$$

where R is the risk aversion parameter assumed to be greater than one, $R > 1$, and S_{iT}^I is the index. The i 'th institution is endowed with λ_i shares of the total assets market, such that $\sum_{i=1}^M \lambda_i = 1$. Its wealth dynamics are characterized by

$$dW_{it} = W_{it} \pi_{it}' (\mu_t^S dt + \sigma_t^{S'} d\omega_t), \quad (5)$$

where π_t is a vector of portfolio allocations invested in each asset, μ_t^S is a vector where the j 'th instance is characterized by μ_{jt}^S and σ_t^S is a matrix where the j 'th row is asset specific vector of sensitivities characterized by σ_{jt}^S ¹. The terminal wealth $\{W_{iT}\}_{i=1}^M$ and portfolio allocations $\{\pi_{iT}\}_{i=1}^M$ are jointly determined in equilibrium.

Benchmarks: χ is a binary, $M \times N$, matrix representing the index configuration, where

$$\chi_{ij} = \begin{cases} \alpha_{ij} & \text{if the } j\text{'th asset is included in the benchmark of the } i\text{'th institution,} \\ 0 & \text{else} \end{cases} \quad (6)$$

where α_{ij} is the weight of each asset in the i th benchmark and n_i is the number of assets in the i 'th institution's benchmark. No arbitrage condition implies that asset prices equal dividend payoffs when they materialize, $S_{jT} = D_{jT}$. Therefore instead of working with the endogenous function of assets: S_{iT}^I , we can work with the exogenous function of dividends: I_{iT} . Institution i chooses a benchmark I_{iT} from the set \mathcal{J}_i characterized by

$$\mathcal{J}_i \equiv \left\{ I_{iT} \mid \frac{dI_{it}}{I_{it}} \equiv \mu_t^{I_i} dt + \sigma_t^{I_i'} d\omega_t, \text{ where } \mu_t^{I_i} \text{ and } \sigma_t^{I_i'} \text{ are measurable, bounded and continuously differentiable functions of the underlying shocks} \right\}. \quad (7)$$

\mathcal{J}_i incorporates many possible indices we observe in the data, such as arithmetic and geometric averages with general finite weights. For example, a weighted arithmetic average of stock returns

¹The wealth dynamics in (5) takes into consideration that $r = 0$

is characterized by $I_{iT} = \sum_{j=1}^N \chi_{ij} D_{jT}$, where $\alpha_{ij} = 1$ for all assets within the i 'th benchmark, and an equally weighted geometric average of stock prices by $\prod_{j=1}^N (D_{jT})^{\chi_{ij}}$ where $\alpha_{ij} = \frac{1}{n_i}$ for all assets within the i 'th benchmark. The only viable restriction on the benchmark is that its evolution has bounded drift and volatility coefficients. Institutions' objective functions (4), incorporate the relative to the benchmark wealth concerns of institutions; they strive to post higher returns when the benchmark is high, which translates into a positive marginal utility of wealth with respect to the index, $U_{WS} > 0$.

Definition 1 (Equilibrium). *In an economy with dividends given by (1), benchmark configuration given by (6) and benchmarks from (7). For initial endowments of $\{\lambda_i\}_{i=1}^M$ shares of the total asset market, such that $\sum_{i=1}^M \lambda_i = 1$, an equilibrium is described by the stochastic processes $\{S_{jt}\}_{j=1}^N$ and $\{\pi_{it}\}_{i=1}^M$, and the random variables $\{W_{iT}\}_{i=1}^M$ on the filtered probability space defined by the vector of Brownian Motions, $\{\omega_{it}, t \in [0, T]\}$. Such that, given prices, $\{S_{jt}\}_{j=1}^N$, each institution i optimally chooses π_{it} and W_{iT} given its budget constraint (5) to maximize its utility (4), and markets clear:*

$$\begin{aligned} \sum_{i=1}^M W_{iT} &= D_T, \\ \sum_{i=1}^M W_{it} \pi_{it} &= S_t, \\ \sum_{i=1}^M W_{it} (1 - \pi'_{it} \mathbf{1}) &= 0, \end{aligned}$$

where $t \in [0, T]$ and $\mathbf{1}$ is a $N \times 1$ vector of ones.

3 General Equilibrium

We obtain the asset pricing quantities and portfolio allocations by first postulating a risk sharing rule, P_{iT} . This quantity determines the share of the aggregate wealth that each investor consumes at maturity. It is state dependent and related to the agent's benchmark index. By using P_T we then obtain the pricing kernel M_T , which also depends on the index configuration.

Theorem 1 (Equilibrium).

The optimal sharing rule, $W_{iT} = P_{iT} D_T$, is characterized by

$$P_{iT} = \frac{A_i (I_{iT})^{1-\frac{1}{R}}}{\sum_{i=1}^M A_i (I_{iT})^{1-\frac{1}{R}}} \quad (8)$$

and the pricing kernel, M_T , is characterized by

$$M_T = D_T^{-R} \left(\sum_{i=1}^M A_i I_{iT}^{1-\frac{1}{R}} \right)^R. \quad (9)$$

The weight adjustment, A_i , is given by

$$A_i = \frac{\lambda_i}{\lambda_1} \frac{E \left[\left(D_T^{-R} \left(A_1 I_{1T}^{1-\frac{1}{R}} + \dots + A_i I_{iT}^{1-\frac{1}{R}} + \dots + A_M I_{MT}^{1-\frac{1}{R}} \right)^R I_{1T} \right)^{1-\frac{1}{R}} \right]}{E \left[\left(D_T^{-R} \left(A_1 I_{1T}^{1-\frac{1}{R}} + \dots + A_i I_{iT}^{1-\frac{1}{R}} + \dots + A_M I_{MT}^{1-\frac{1}{R}} \right)^R I_{iT} \right)^{1-\frac{1}{R}} \right]} \quad (10)$$

and for a fixed set of weight adjustments, $A_1 \equiv 1, A_2, \dots, A_{i-1}, A_{i+1}, \dots, A_M$, with finite values there exist a unique fixed point that satisfies (10).

The results in Theorem 1 do not depend on the structure of the index. Institutions may differ both in assets within the benchmark and in the benchmark structure.

An important observation is that an economy with institutional herding is equivalent to an economy with just one major institution with the same initial share as the sum of the herding institutions. Usually, with CRRA preference, when risk aversions are identical, it is possible to aggregate investors into a single representative agent. However, due to heterogeneity in the indices, the aggregation is possible only if their indices are identical.

Proposition 1 (Equivalence).

If we replace a subset of institutions with the same benchmark index, \hat{I} , with a single aggregate institution with initial share equals to $\sum_{i \in \hat{I}} \lambda_i$, then equilibrium prices and portfolios for the rest of the participants are unchanged.

The optimal share is a weighted average of the indices to the power of one minus the risk tolerance. The weight of institution i is determined by the *weight adjustment*, A_i , which is fixed overtime. In addition, the optimal share moves between zero and one with values closer to one when the i 'th index is doing relatively better than the rest of the indices and closer to zero otherwise. Therefore, consumption of institution i is determined by how well its index performs relative to others.

In the representation above the weight adjustments are determined relative to the first investor, i.e., $A_1 \equiv 1$. Furthermore, they are comprised of two components: the *initial adjustment* divided by the *risk adjustment*. The initial adjustment implies that institutions with large initial shares have bigger effects on institutions' shares. The risk adjustment, which measures the relative performance of the indices, implies that institutions with indices that are on average performing relatively better, subsequently reduce their share of the aggregate by increasing their risk adjustment. For example, suppose that high realizations of the i 'th index are pretty common and that high realizations of the k 'th index are pretty rare. Then, the optimal share of the i 'th investor will be adjusted downward and the optimal share of the k 'th investor will be adjusted upward through their respective adjustment costs. In contrast, in the baseline case with no benchmarks, the weight adjustments

are reduced to the initial adjustment and the optimal share of each market participant is solely determined by its initial share.

The state price density in this economy is also a multiplication of two factors. The first is the *aggregate dividend* factor, from the baseline Lucas economy case. The second factor is the *index factor* and it arises due to the benchmarking concerns of institutions. It is a weighted average of the different benchmarks according to their weight adjustments. The index factor is what drives the differences in between equilibrium quantities of indexed and non indexed assets.

Theorem 2 (Equilibrium Asset Prices and Portfolio Allocations).

Sharpe ratios are given by

$$\theta_t = RE_t \left[\frac{M_T}{E_t[M_T]} \sigma_T^D \right] - (R-1) \sum_{i=1}^M E_t \left[\frac{M_T}{E_t[M_T]} P_{iT} \frac{\mathcal{H}_{t,T}^{I_i}}{I_{iT}} \right] \chi_i. \quad (11)$$

Asset volatilities are given by

$$\begin{aligned} \sigma_{jt}^S &= \sigma e_j + RE_t \left[\left(\frac{M_T}{E_t[M_T]} - \frac{M_T D_{jT}}{E_t[M_T D_{jT}]} \right) \sigma_T^D \right] \\ &\quad - (R-1) \sum_{i=1}^M E_t \left[\left(\frac{M_T}{E_t[M_T]} - \frac{M_T D_{jT}}{E_t[M_T D_{jT}]} \right) P_{iT} \frac{\mathcal{H}_{t,T}^{I_i}}{I_{iT}} \right] \chi_i. \end{aligned} \quad (12)$$

Optimal portfolios (wealth volatilities) are given by

$$\begin{aligned} \sigma_t^S \pi_{it} &= \theta_t + \frac{R-1}{R} E_t \left[\frac{I_{iT}^{1-\frac{1}{R}} M_T^{1-\frac{1}{R}}}{E_t \left[I_{iT}^{1-\frac{1}{R}} M_T^{1-\frac{1}{R}} \right]} \frac{\mathcal{H}_{t,T}^{I_i}}{I_{iT}} \right] \bar{\chi}_i \\ &\quad - (R-1) E_t \left[\frac{I_{iT}^{1-\frac{1}{R}} M_T^{1-\frac{1}{R}}}{E_t \left[I_{iT}^{1-\frac{1}{R}} M_T^{1-\frac{1}{R}} \right]} \sigma_T^D \right] + \frac{(R-1)^2}{R} \sum_{k=1}^M E_t \left[\frac{I_{iT}^{1-\frac{1}{R}} M_T^{1-\frac{1}{R}}}{E_t \left[I_{iT}^{1-\frac{1}{R}} M_T^{1-\frac{1}{R}} \right]} P_T^k \frac{\mathcal{H}_{t,T}^{I_k}}{I_{kT}} \right] \chi_k. \end{aligned} \quad (13)$$

The i 'th institution's wealth is given by $W_{it} = \frac{E_t[M_T P_{iT} D_T]}{E_t[M_T]}$, the j 'th asset price is given by $S_{jt} = \frac{E_t[M_T D_{jT}]}{E_t[M_T]}$, where χ_i is the i 'th row of matrix χ , signifying the i 'th institution benchmarked assets and e_j is a unit vector and $\mathcal{H}_{t,T}^{I_i}$ is a general hedging demand against fluctuations in I_{it} for $t \in [0, T]$.

There are two hedging demands in this framework: *aggregate hedging* and *index hedging*. In the first, participants hedge against future fluctuations in the aggregate dividend volatility. It is not unique to our framework as it stems from any dynamic model with stochastic opportunity set. Whereas in the second, market participants hedge against future fluctuations in the index holdings of all market participants, which is captured by their optimal share multiplied by their

corresponding indices. It is unique to our model and stems due to institutional benchmarking. $R > 1$ is an important ingredient of the results, since when $R = 1$, log utility preference, the index hedging component vanishes and assets' characteristics become identical.

4 Implications of Benchmarks' Heterogeneity

For the sake of simplicity for the rest of the paper we assume that institutions' index structure is geometric average with equal weights; they may differ only in which assets are included in their benchmarks ².

$$I_{iT} \equiv \prod_{j=1}^N (D_{jT})^{\chi_{ij}},$$

χ_{ij} is defined in (6) with $\alpha_{ij} = \frac{1}{n_i}$. The asset pricing quantities and portfolio allocations are then characterized by plugging $\mathcal{H}_{t,T}^{I_i} = \sigma I_{iT}$.

Empirical evidence suggests that funds differ in their industry concentration and choose their benchmarking indices accordingly. In this section we study the heterogeneity effects of benchmarks. In a series of comparisons between different index configurations we draw conclusions on how the index configuration effects asset prices. This comparative statics exercise provides guidelines on how market characteristics change once we change the index configuration.

In general, asset prices and Sharpe ratios are influenced by the number of institutions that benchmark them, their initial share and their benchmark indices. However, Proposition 2 describes a way that analytical comparisons can be made.

Proposition 2 (Asset Prices Effects).

Suppose that any institution i who benchmarks asset l also benchmarks asset k : $\chi_{\cdot,k} \geq \chi_{\cdot,l}$, and that the initial levels of dividends are the same then

1. $S_{lt} \leq S_{kt}$
2. $\theta_{lt} \geq \theta_{kt}$

where $t \in [0, T]$.

Proposition 2 provides theoretical foundation for comparing asset prices while keeping the index configuration fixed. We use these results to support the illustrations in this section.

²Results for arithmetic averages of assets' returns are described in Appendix B. Geometric averages are common in practice, our index specification exactly match the *S&P500 Commodity Index (SPCI)*. It is a good proxy for institutional benchmarking concerns in stock markets. In addition, the choice of an equally weighted geometric average buys us simpler analytical expressions as well as tractability. By setting $I_{iT} \equiv 1$, we are removing the benchmarking concerns of institutions, therefore the case of one institutional investor and a retail investor is nested with this choice of benchmark. Furthermore, the weight of each asset in the benchmark is self determined and equals to $1/n_i$.

For simplicity purposes let us assume that there are two institutions with the same initial share and ten assets that each institution can decide whether to include or exclude in its benchmark before the markets start to unfold. Benchmarks then remain fixed and cannot be changed. The benchmarking heterogeneity can be characterized by four types of assets.

Definition 2 (Types of Assets).

- *type10*: assets that are included only in the first benchmark
- *type01*: assets that are included only in the second benchmark
- *type00*: assets that are excluded from both benchmarks
- *type11*: assets that are included in both benchmarks

n_{10} , n_{01} , n_{00} and n_{11} denote the number of type specific assets, respectively.

For example, the index configuration given by $\frac{\chi_1}{\chi_2} = \frac{1100000111}{0011000111}$ is decomposed to two assets of *type10*, two assets of *type01*, three assets of *type00* and three assets of *type11*. In this characterization types' locations do no matter for type specific asset prices and portfolio allocations. In our example, $\frac{\chi_1}{\chi_2} = \frac{0001111100}{0001110011}$ generates the same type specific asset pricing quantities. Therefore, the number of assets from each type is what determine types' equilibrium prices and portfolio allocations³. In all examples we use the following order of types from left to right, *type10*, *type01*, *type00* and *type11*.

The relative benchmark sizes in the index configuration is one of the main driving forces effecting asset prices and portfolio allocations. In cases where one benchmark is wide and the other is narrow prices and portfolio allocations across benchmarks diverge, whereas when both benchmarks are either narrow or wide prices and portfolio allocations across benchmarks are equivalent: when benchmark sizes are identical asset prices and portfolio allocations are symmetric.

Definition 3 (Symmetry).

Let μ be the measure of asymmetry characterized by

$$\mu = \frac{|n_1 - n_2|}{N}. \tag{14}$$

We say that the index configuration χ is symmetric when $\mu = 0$.

$\mu \in [0, 1]$, it equals 0 when benchmarks are symmetric and it equals 1 when one institution is fully diversified and the other has no benchmarking motives. When the index configuration χ is symmetric asset prices and portfolio allocations assets are identical.

³For the simplicity of representation and without losing the economic intuition we assume that at the time of comparison, all dividend payoffs are identical, $D_{it} = D_{jt}$, for $\forall i, j = 1, 2, \dots, N$. However, it is possible to look at conditional distributions of asset prices and portfolio allocations in general.

Proposition 3 (Symmetry). *If $\mu = 0$, the initial share is identical and dividends' initial levels are the same then*

1. $S_t^{10} = S_t^{01}$
2. $\theta_t^{10} = \theta_t^{01}$
3. $\sigma_t^{10} = \sigma_t^{01}$
4. $\pi_{1t}^{10} = \pi_{2t}^{01}$
5. $\pi_{1t}^{01} = \pi_{2t}^{10}$
6. $\pi_{1t}^{11} = \pi_{2t}^{11}$

where $t \in [0, T]$ and the supper scripts 10 and 01 correspond to type10 and type01 assets, respectively.

We start the analysis by examining how portfolio allocations differ for the different types of assets. It has been established that institutions' benchmarking motives imply that they tilt their portfolios towards the benchmark. However, the strength of the tilt and the amount invested in the overlapping and independent parts of the benchmarks very much depend on the index configuration. First, due to diversification within type motive institutions allocate their funds equally among assets of the same type. Second, institutions play a retail investor role and provides supply for the other institution in assets exclusively in the other institutions' benchmark. Third, interestingly institutions invest lower fractions of wealth in assets that are included in both benchmarks than assets that are only included in their benchmarks. In some sense, each institution plays a retail investor role and provides some supply for the other institution by essentially decreasing its own demand for these assets. We continue by showing the benchmarking motives addresses some of the widely known puzzles in asset pricing.

4.1 Effects of Asymmetry

In this section we show how the asymmetry in the index configuration controlled by μ effects asset prices and portfolio allocations, while controlling for the common component. When the index configuration becomes asymmetric one benchmark is increasing in size and the other benchmark is decreasing in size. The *increasing benchmark* and its institution are denoted by I while the *decreasing benchmark* and its institution are denoted by D .

Figure 1 shows the sensitivity analyses of the portfolio allocations to changes in asymmetry, μ . In the top panel we observe the common components of the index configuration and in the bottom panel the independent components of the index configuration. Due to diversification motives: 1) the I (D) institution decreases (increases) its holding in each of the overlapping assets as μ increases. 2) the I (D) institution also decreases (increases) its holding in assets exclusively in its benchmark and increases (decreases) its holding in assets exclusively benchmarked by the D (I) institution. Lastly, we observe that the cash account remains zero for any level of asymmetry, since

there are always better Sharpe ratio investments than investment in the cash account, yielding a zero Sharpe ratio.

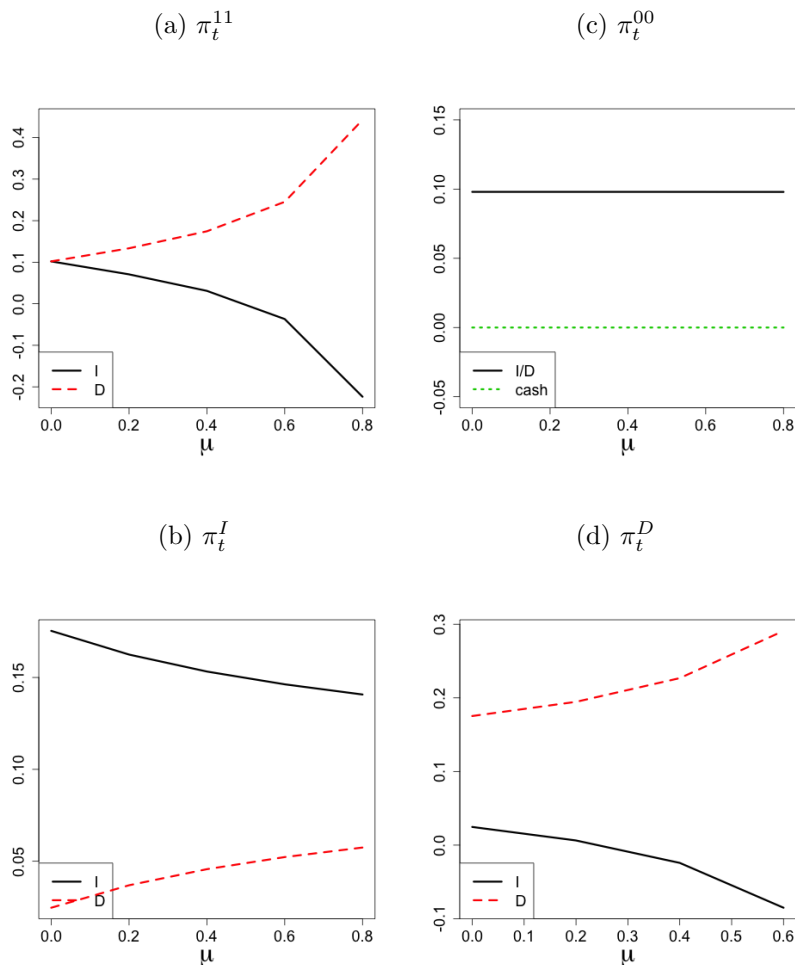


Figure 1. In these graphs we fix the common component, $n_{11} = 1$ and $n_{00} = 1$ and show how portfolio allocations depend on the relative size measure μ . The top panel reflects the portfolio allocations in the common components and the lower panel in the individual components. 11 represents an asset common to both benchmarks and 00 represents an asset excluded from both benchmarks. π^I reflects the holdings in the individual components of the increasing benchmark and π^D reflects the holdings in the individual components of the decreasing benchmark. Equivalently, I represents an asset exclusively within the increasing size benchmark and D represents an asset exclusively within the decreasing size benchmark. Parameters are $\mu = 0.03$, $\sigma = 0.15$, $\lambda_1 = \lambda_2 = 0.5$, $D_{j0} = 1$ for $j = 1, \dots, 6$, $T = 3$ and $R = 4$.

Exclusively benchmarked assets and commonly benchmarked assets behave differently to increase in asymmetry. Intuitively, prices of the I (D) benchmark decrease (increase), their Sharpe ratios increase (decrease) and their volatilities decrease (increase) as the asymmetry increases. The I benchmarked assets' prices decrease since now the benchmarking demand pressure is dispersed

among more assets. Tilting the portfolio implies that the benchmarked assets are more sensitive to cash flow shocks within the benchmark than to cash flow shocks outside the benchmark; a cash flow shock within the benchmark has a greater effect on portfolio holdings than a cash flow shock outside the benchmark, implying that their volatility is higher. As asymmetry increases the I benchmark sensitivity to shocks within it decreases, since now a smaller fraction of wealth is invested in each asset, thereby effects of a cash flow shock on portfolio adjustments are mitigated. Sharpe ratios of benchmarked assets are lower than Sharpe ratios of non benchmarked assets. Equilibrium implies that assets within a benchmark are undesirable investments to market participants without benchmarking motives in these assets. This result is obtained by decreasing the Sharpe ratios of benchmarked assets. As asymmetry increases the I benchmarked assets' Sharpe ratios increase. The benchmarking institution is more diversified within the benchmark, therefore maintaining equilibrium implies that the decline in Sharpe ratios is mitigated.

However commonly benchmarked assets prices increase, their Sharpe ratios decrease and their volatilities increase as asymmetry increases. There are two opposing forces at play. In the first force, the addition to the I benchmark implies that prices and volatilities decrease, and Sharpe ratios increase for assets within it, while in the second force the deletion from the D benchmark implies that prices and volatilities increase, and Sharpe ratios decrease for assets within it. The driver of this result is the fact that addition or deletion from a narrow benchmark effect asset prices more strongly than deletion or addition from a wide benchmark. A consequence of this result is that assets within each institution's benchmark might have very different pricing quantities depending on the index configuration, as can be seen in Figure 2 and Table 1.

Assets' volatilities can be thought of as sensitivities to cash flow shocks. We find that benchmarked assets' sensitivity to cash flow shocks within the benchmark is higher than the sensitivity to cash flow shocks excluded from the benchmark. Meaning that when a positive cash flow shock hits a benchmarked asset the corresponding gains are distributed among the benchmarked assets more than among the non benchmarked assets. In addition, the institutions may take a short position on the assets exclusively in the other institution's benchmark to the extent allowed by their holdings, implying that the sensitivity across benchmarks might be negative. The index configuration determines the magnitude of the sensitivity within and across benchmarks. In Figure 3, the left panel we observe that the correlation across benchmarks decreases as asymmetry increases due to a sharp increase in the volatilities of D benchmarked assets. In the right panel we observe that the correlation within the benchmark is different for the I and the D benchmarks. As the I benchmark increases, the fraction of wealth invested in each asset within it decrease and so are the sensitivities to cash-flow shocks, therefore correlation among assets within it decreases.

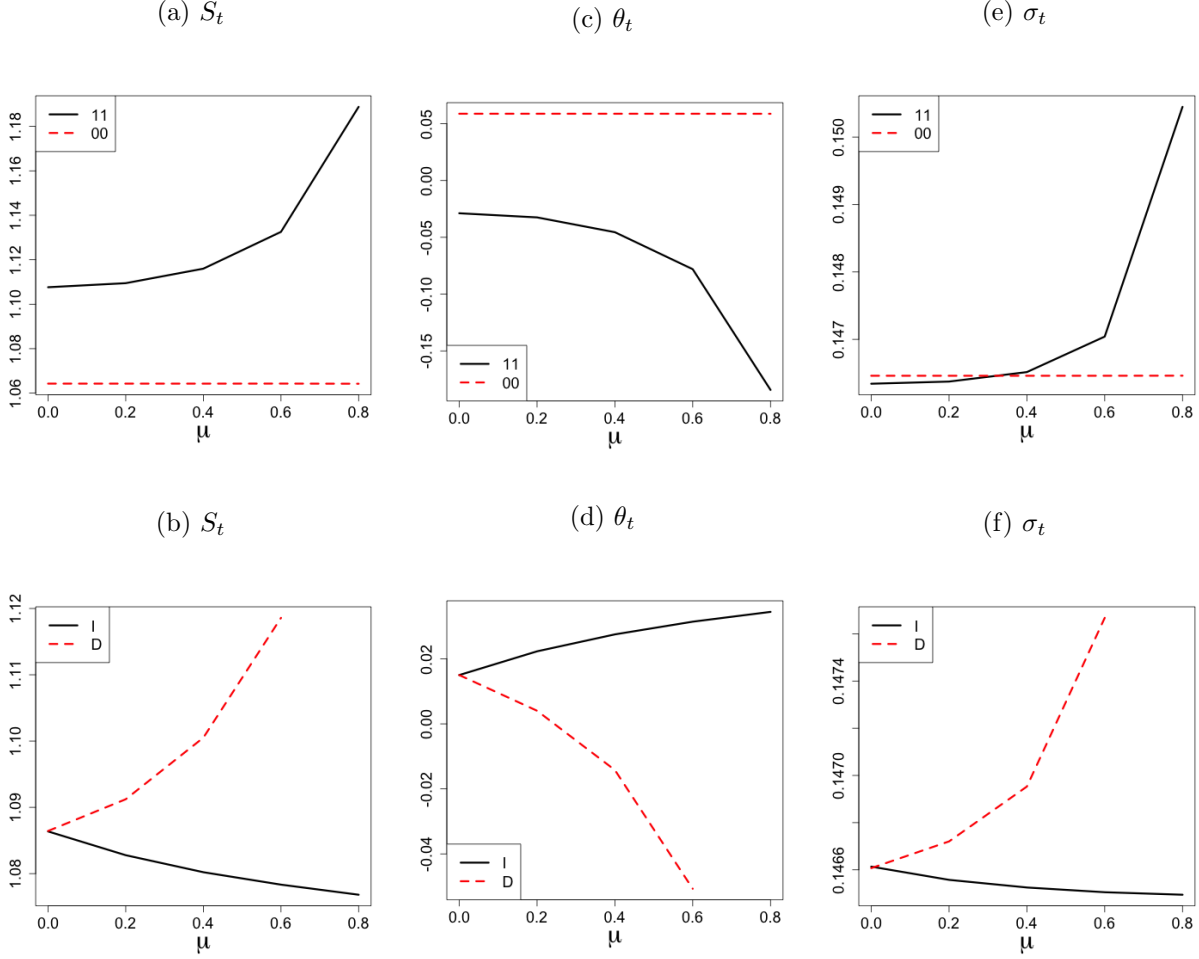


Figure 2. In these graphs we fix the common component, $n_{11} = 1$ and $n_{00} = 1$ and show how asset prices depend on the relative size measure μ . The top panel reflects asset prices of the common components and the lower panel asset prices of the individual components. 11 represents an asset common to both benchmarks and 00 represents an asset excluded from both benchmarks. I represents an asset exclusively within the increasing size benchmark and D represents an asset exclusively within the decreasing size benchmark. Parameters are the same as in Figure 1.

4.2 Effect of the Common Components

In this section, we fix the independent component controlled by the asymmetry μ and observe how changes in the common components effect asset prices and portfolio allocations. We find that the narrower the benchmarks the higher the benchmarking effects on asset prices and portfolio allocations. This is captured by looking on the difference between commonly non benchmarked and commonly benchmarked assets, $n_{00} - n_{11}$.

First, non benchmarked assets' prices and portfolio allocations are only mildly affected by the different index configurations, $\pi_t^{00} \approx 0.10$, $S_t^{00} \approx 1.06$ and $\theta_t^{00} \approx 0.06$ for any index configuration.

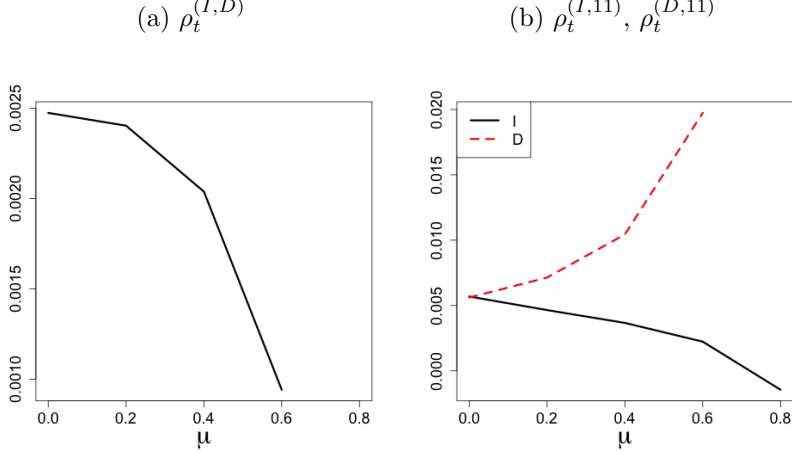


Figure 3. In these graphs we show how the correlation within and across benchmarks depend on the relative size measure μ , while keeping the common components fixed $n_{11} = 1$ and $n_{00} = 1$. The correlations are characterized by $\rho_t^{(j_1, j_2)} = \sigma_{j_1 t}^s \sigma_{j_2 t}^s / \|\sigma_{j_1 t}^s\| \|\sigma_{j_2 t}^s\|$, where $\sigma_{j_1 t}^s$ and $\sigma_{j_2 t}^s$ are type specific. Parameters are the same as in Figure 1. Figure 3a reflect the correlation across benchmarks and Figure 3b reflect the correlation within benchmarks. 11 represents an asset common to both benchmarks. I represents an asset exclusively within the increasing size benchmark and D represents an asset exclusively within the decreasing size benchmark. Parameters are the same as in Figure 1.

These assets are not exposed to the index hedging and only affected by the aggregate hedging. The aggregate hedging component is only mildly affected by the changes in the index configuration and therefore these asset prices' and portfolio allocations are only mildly affected.

Second, institutions may take short positions on the other institution's benchmarked assets to the extent allowed by their other holdings serving as collateral, as shown in the last row, the portfolio allocation π_{1t}^{01} in Table 1. More surprisingly they may also take short position on their own benchmark as shown in the first row, the portfolio allocation π_{1t}^{11} in Table 1. Furthermore, when the size of the benchmarks is identical $S_t^{10} = S_t^{01}$ and $\theta_t^{10} = \theta_t^{01}$ as Proposition 3 implies, and, $S_t^{11} \geq S_t^{10}, S_t^{01}$ and $\theta_t^{11} \geq \theta_t^{10}, \theta_t^{01}$, as Proposition 2 implies.

Third, the total wealth of institutions incur an upward jump when a retail investor becomes an institutional investor with benchmarking motives, as observed in Table 1. Introducing the second benchmark has two effects. In the first effect, the second institution tilts its portfolio towards its benchmarked assets, thereby increasing demand for these assets, pushing up their prices. In the second effect, the first institution increases its holdings on its benchmark financed by taking a short position on the second institution's benchmark which further increases assets prices within the first benchmark. In total, the two effects increase prices implying that the total economic wealth jumps.

Lastly, when both institutions are with benchmarking motives it is never optimal to hold the

Effects of the Common Component on Asset Prices									
	$\frac{\chi_1}{\chi_2}$	μ	$n_{00} - n_{11}$	S_t^{10}	S_t^{01}	S_t^{11}	θ_t^{10}	θ_t^{01}	θ_t^{11}
1	$\frac{1000000001}{0100000001}$	0	6	1.12	1.12	1.17	-0.05	-0.05	-0.16
2	$\frac{1000000111}{0100000111}$	0	2	1.09	1.09	1.12	0.00	0.00	-0.05
3	$\frac{1000011111}{0100011111}$	0	-2	1.08	1.08	1.11	0.02	0.02	-0.01
4	$\frac{1001111111}{0101111111}$	0	-6	1.08	1.08	1.09	0.03	0.03	0.00
5	$\frac{1110000001}{0001000001}$	0.2	4	1.09	1.12	1.15	0.00	-0.05	-0.11
6	$\frac{1110000111}{0001000111}$	0.2	0	1.08	1.09	1.11	0.02	0.00	-0.03
7	$\frac{1110011111}{0001011111}$	0.2	-4	1.08	1.08	1.10	0.03	0.02	-0.01

Effects of the Common Component on Portfolio Allocations												
	$\frac{\chi_1}{\chi_2}$	μ	$n_{00} - n_{11}$	π_{1t}^{10}	π_{1t}^{01}	π_{1t}^{11}	$cash_1$	π_{2t}^{10}	π_{2t}^{01}	π_{2t}^{11}	$cash_2$	$W_1 + W_2$
1	$\frac{1100000001}{0000000001}$	0.2	7	0.23	-	-0.14	0.00	-0.02	-	0.36	0.00	10.87
2	$\frac{1100000000}{0000000000}$	0.2	8	0.29	-	-	-0.37	0.01	-	0.39	0.37	10.75
3	$\frac{1110000001}{0000000001}$	0.3	5	0.19	-	-0.17	0.00	-0.02	-	0.36	0.00	10.87
4	$\frac{1110000000}{0000000000}$	0.3	6	0.23	-	-	-0.37	-0.02	-	-	0.37	10.75
5	$\frac{1111100000}{0000010000}$	0.4	4	0.17	-0.27	-	0.00	0.03	0.48	-	0.00	10.87

Table 1. $S_t^{00} \approx 1.06$, $\theta_t^{00} \approx 0.06$ and $\pi_t^{00} \approx 0.10$ for any index configuration. Parameters are the same as in Figure 1.

cash account since institutions always have a strategy that invoke a greater than zero Sharpe ratio investment for both. In contrast, this is not the case when only one institution has benchmarking motives. In such cases, the institution takes a short position on the cash account, to the extent allowed its other holdings, while the retail investor a long position, as shown in Table 1 in $cash_1$ and $cash_2$ columns.

4.3 Effects of Fundamental Volatility

We now turn our focus to how changes in dividends' volatilities effects on asset prices and portfolio allocations. Increasing dividends' volatilities imply that current information about the future cash flows is more noisy and reveals less about the materialized payoffs. In cases where either both institutions benchmark a fully diversified benchmark or both institutions have no benchmarks, increasing cash flow volatilities do not affect portfolio holdings, since institutions cannot mitigate the effects of the noise by reallocating funds. Subsequently asset prices fall, since volatilities increase and Sharpe ratio falls when dividends are more volatile. However, in other benchmark configurations institutions can mitigate the effects of the increase in cash flow volatility by reallocating their funds. In such cases assets' prices and Sharpe ratios might rise and assets' volatilities might fall as the cash flow volatilities increase. As before, the type of asset and the index configuration determines how the change in cash flow volatilities affect asset prices and portfolio allocations.

Institutions' benchmarking concerns imply that they revert funds to their benchmarks in episodes of high cash flow volatility: the *flight to benchmark* property. As the fundamental volatility increases institutions shift funds from the non benchmark assets to the benchmarked assets: they become more concentrated in their benchmarks. As was previously stressed, institutions' holdings in the non benchmarked assets are rigid to changes in the index configuration, therefore, the narrower the benchmark the more funds are allocated to other assets, implying that institutions can allocate more funds towards the benchmark when fundamental volatility increases, which results in bigger price distortions.

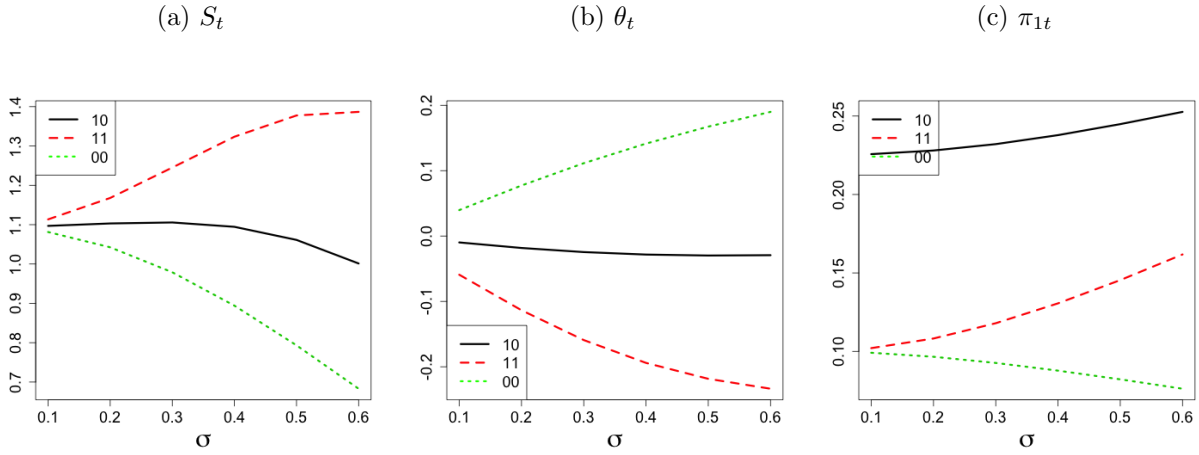


Figure 4. In these graphs we present the asset pricing quantities of the different types. The full, dashed and dotted lines characterize *type00*, *type11* and *type10* assets respectively, and the index configuration is characterized by $\frac{\chi_1}{\chi_2} = \frac{1100000001}{0011000001}$. Parameters are the same as in Figure 1.

Figure 4 illustrates the effects of asset prices and portfolios as the fundamental volatility increases. The demand pressure on the commonly benchmarked assets is stronger since both institutions revert their funds to the same assets, implying that the distortion in asset prices is stronger. Furthermore, when the fundamental volatility reaches a certain level, institutions' reaction to the increase in the fundamental volatility decreases, since the negative effects on payoffs becomes stronger than the positive effect of the increase in demand.

4.4 Asset Pricing Phenomena

There are several phenomena that are prevalent in the empirical analysis and can be explained by a model with institutions.

Idiosyncratic Pricing: In our model different idiosyncratic shocks have different prices. There are some idiosyncratic shocks that are much more pronounced in prices than others. Since institutions are not diversified in the usual sense, cash flow shocks within their benchmarks effect prices

much more than cash flow shocks outside their benchmarks. This outcome can address the twin stock and, parent and subsidiary price discrepancies, as shown in Table 1. As long as the institutions benchmark different assets their prices diverge, even if their dividend streams are highly correlated.

The Low Volatility Puzzle: Traditional asset pricing models imply that bearing more risk should be compensated by a higher expected return. However, this result is counterfactual, as Baker et al. (2010) shows. Low volatility investments outperform high volatility investments, which entails that the Sharpe ratio of low volatility stocks is higher than the Sharpe ratio of high volatility stocks. Institutions' demand for benchmarked assets pushes down their Sharpe ratio and increases their volatilities. Institutions' holdings are concentrated in the benchmark, implying that the sensitivity of benchmarked assets to cash flow shocks within the index is higher than to cash flow shock outside the index. In order to make the benchmarked asset less desirable to hold, in equilibrium their Sharpe ratios are reduced, while their volatilities remain high. Examples are shown in Table 1.

Asset Class Effect: This model, as observed in previous theoretical literature on institutions also shows that the presence of institutions generates the asset class effect: the increase in the correlation of stocks within the same benchmark index. Empirical evidence suggests that there are numerous patterns of comovement in asset returns. There are strong common factors in the returns of value stocks, stocks in the same industry, S&P500 stocks, small cap stocks and others, as was reported in Barberis et al. (2005).

There are several guidelines that a policy maker can extract from this model. For example, suppose that the question a policy maker tries to answer is whether funds' benchmarks overlap. He could then compare the difference in asset prices between the two different settings, while controlling for heterogeneities in the data. As shown above, changing the index configuration has profound implications on asset prices. If institutions' benchmarks do overlap it should result in different asset pricing quantities. In that sense, the model is very flexible, and can easily address economies with large opportunity sets with multiple institutions. Heterogeneity in the model's parameters can be introduced to accommodate the heterogeneities in the data, without tempering the analytical expressions.

4.5 Endogenous Benchmarks

The study of optimal benchmarks is of main interest in the literature on benchmarking institutions. These studies are mainly concentrated on cases of one institution in a partial equilibrium setting and a principal-agent framework. We provide insights on the optimal index configuration when multiple institutions jointly choose their optimal indices in an equilibrium setting, while we suppress the principal-agent problem. In this section we add an additional layer to the model and allow institutions to choose their benchmarks. In that sense, market participants report a benchmark

and prices are formed accordingly. By taking prices as given, institutions then decide if they would like to deviate from their reported benchmarks. For a given reported index configuration, if institutions unanimously decide not to deviate, then, the reported benchmarks represent the optimal index configuration. More formally, the value function of institution i is given by

$$v_0(I_{iT}) = \hat{\kappa}_i \left(E \left[\left(\hat{M}_T I_{iT} \right)^{1 - \frac{1}{R}} \right] \right)^R, \quad (15)$$

where $\hat{\kappa}_i$ is a constant and \hat{M}_T is the pricing kernel, both are determined by the reported benchmarks. Each institution then maximize its value function over the set of all possible indices

$$\max_{\chi_i} \{v_0(I_{iT})\}. \quad (16)$$

If the reported index configuration equals the index configuration that maximizes institutions' value functions then the reported benchmarks are in fact optimal. In this simplified maximization problem, institution i controls only which assets are included in its index. The weights of the benchmarked assets are self determined and distributed equally, conforming with the economic setup in Section 2. Furthermore, we assume that institutions are fully committed to their time zero benchmark. Meaning institutions commit to their time zero reported benchmark and afterwards deviation is not allowed. Eventhough it is reasonable to assume that institutions affect prices, in order to be consistent with the economic framework in Section 2 we model institutions as price takers. Interestingly, the strategic case does not change the optimal benchmark choice, institutions optimally benchmark the same fully diversified index even when considering the index choices effects on prices as well.

Proposition 4 (Optimal Index Configuration).

The optimal index configuration is unique and is obtained when institutions benchmark the same fully diversified index, subject to the constraint that institutions self report a benchmark.

The choices of indices have two effects on the optimal benchmark. The first is an aggregate effect, since other institutions' benchmarks affect each individual optimal share, they also affect the individual institution choice of benchmark. The second effect is an individual effect, each institution choice of benchmark implies a different discounting of the optimal share, a different distortion to the objective beliefs according to the choice of benchmark. These two effects together determine the optimal index configuration.

Previous literature argues that the fund investor's preference determines what is the optimal benchmark. For example, if the investor cares about growth stocks, then the manager should benchmark the Russell 1000 growth index. However, previous theoretical results do not consider the endogenous effects of prices and indices. In contrast, this paper shows that when other institutions' benchmarks are taken into account and prices emerge, the optimal benchmarks are fully diversified. It means that the optimal index configuration in this broad sense may not be optimal in

the individual sense, when prices and index choices are exogenous. In our example, a fund investor who cares about growth stocks will have to substantially increase the manager's compensation to make him deviate from the S&P500 index.

Table 2 represents the gains by deviating from the reported benchmark represented by $\bar{\chi}_1 = \bar{\chi}_2 = 1111$. When institutions do not deviate their benefits are highest, meaning $\chi_1 = \chi_2 = 1111$ is the optimal benchmark. It is obtained when both institutions are fully diversified and herd; they both hold the same fully diversified index. These outcomes are robust to changes in participants' initial share and to the addition of a retail investor with no benchmark.

Deviations from the reported benchmarks, $\bar{\chi}_1 = \bar{\chi}_2 = 1111$						
	0000	0001	0011	0111	1111	
0000	-9.99,-9.99	-12.26,-9.99	-11.82,-9.99	-11.68,-9.99	-11.61,-9.99	
0001	-9.99,-12.26	-12.26,-12.26	-11.82,-12.26	-11.68,-12.26	-11.61,-12.26	
0011	-9.99,-11.82	-12.26,-11.82	-11.82,-11.82	-11.68,-11.82	-11.61,-11.82	
0111	-9.99, -11.68	-12.26,-11.68	-11.82,-11.68	-11.68,-11.68	-11.61,-11.68	
1111	-9.99, -11.61	-12.26,-11.61	-11.82,-11.61	-11.68,-11.61	-11.61,-11.61	
0010		-11.82,-12.25				
0110		-11.82,-11.82	-11.68, -11.82			
1110		-11.82,-11.68	-11.68, -11.68			
0100			-11.68, -12.26			
1100			-11.68, -11.82			
1000				-11.68, -12.25		
1001				-11.68,-11.82		
1011				-11.68,-11.68		

Table 2. In the case where institutions are required to report a benchmark, $\chi_i \neq 0000$, $i = 1, 2$, the optimal index configuration is obtained when $\chi_1 = \chi_2 = 1111$, meaning institutions do not deviate from the reported benchmarks, as the red marking shows. The payoffs are in utils and the parameters are as in Figure 1.

In contrast, Table 3, represents the gains by deviating from the reported benchmark represented by $\bar{\chi}_1 = 0011$, $\bar{\chi}_2 = 0001$. No deviation is not optimal, meaning $\chi_1 = 0011$, $\chi_2 = 0001$ is not an optimal benchmark. This outcome as well is robust to changes in participants' initial share λ_i , and to the addition of a retail investor with no benchmark.

Deviations from the reported benchmarks, $\bar{\chi}_1 = 0011$ and $\bar{\chi}_2 = 0001$					
	0000	0001	0011	0111	1111
0000	-9.21,-9.21	-14.04,-9.21	-12.15,-9.21	-11.17,-9.21	-10.70,-9.21
0001	-9.21,-14.04	-14.04,-14.04	-12.15,-14.04	-11.17,-14.04	-10.70,-14.04
0011	-9.21,-12.15	-14.04,-12.15	-12.15,-12.15	-11.17,-12.15	-10.70,-12.15
0111	-9.21,-11.17	-14.04,-11.17	-12.15,-11.17	-11.17,-11.17	-10.70,-11.17
1111	-9.21,-10.70	-14.04,-10.70	-12.15,-10.70	-11.17,-10.70	-10.70,-10.70
0010		-14.04,-11.31			
0110		-14.04,-10.33	-12.15,-10.33		
1110		-14.04,-10.02	-12.15,-10.02		
0100			-12.15,-10.14		
1100			-12.15,-9.77		
1000				-11.17,-10.13	
1001				-11.17,-11.51	
1011				-11.17,-11.16	

Table 3. In the case where institutions are required to report a benchmark, $\chi_i \neq 0000$, $i = 1, 2$, the optimal index configuration is obtained when $\chi_1 = \chi_2 = 1111$, meaning institutions *deviate* from the reported benchmarks $\bar{\chi}_1 = 0011$, $\bar{\chi}_2 = 0001$, as the red marking shows. Parameters are as in Table 2.

5 Conclusions

In this paper we have incorporated heterogeneous institutions into an asset pricing equilibrium model by adding their different benchmarking concerns in preferences. We have shown that a relative subsistence benchmark with constant relative risk aversion preference has major implications on asset prices, within and across benchmarks. We show that institutions' tilt towards their benchmark is exacerbated on the independent part of their benchmark and is financed by taking short positions on the other institutions' benchmarked assets. In addition, adding an asset to the benchmark causes the overlapping assets to increase in prices and volatilities and decrease in Sharpe ratios while the opposite occurs in the independent part of the benchmark. Furthermore, we show that when institutions revert holdings to their benchmarks the fundamental volatility is high, which creates a further demand pressure on overlapping part of the benchmark that causes prices to increase and Sharpe ratios to decrease. Our model explains much of what was previously a puzzle. Such as, the institutional demand for low Sharpe ratio assets, asset price appreciation with inclusion, asset class effect and the twin stocks discrepancy are among the phenomena that we explain using institutional investors. Lastly, by endogenizing the benchmark choices, we find that the optimal index configuration is obtained when all institutions benchmark the same fully diversified index.

In addition, we have shown that institutions further tilt their portfolios towards their benchmarks as the cash flows become riskier, meaning institutions are even less diversified as the cash flows become more volatile. This flight to benchmark behavior of institutions amplifies bad eco-

conomic news and has the potential to explain systematic defaults and market crashes. In addition, when institutions' benchmarks overlap their behavior creates a price pressure on the overlapping assets, resulting in a price appreciation. Moreover, we have shown that the highest wealth effect is obtained when institutions benchmark the same narrow index. Lastly, by endogenizing the benchmark choices, we have shown that the optimal index configuration is obtained when all institutions benchmark the same fully diversified index.

Concluding, the results presented in this paper are qualitative results. In our analysis we aimed to keep the model as simple as possible by excluding heterogeneities in primitives. The set of parameters in the illustrations are the same with only minor deviations. However, the model is very flexible and can potentially be calibrated to match empirical data. For example by introducing heterogeneity in cash flow volatilities, reflecting empirical differences in asset classes. In addition, the flight to benchmark behavior of institutions suggests that by modeling default, this behavior can amplify the effects of high cash flow volatility. An increase in cash flow volatility increases the chances of a market crash and, at the same time, institutions become less diversified, which further increases the chances of a market crash. This amplification effect may play a key role in explaining systematic disasters.

A The Baseline Case

This section's main purpose is to illustrate the changes between economies with no institutions, the baseline case, and economies with institutions. The equilibrium, asset prices and portfolios are simply obtained by plugging $I_{iT} \equiv 1$ in Theorem 1 and Theorem 2. The results for this case were first introduced by Lucas (1978), and have been presented in many different settings in numerous papers. We outline it here to emphasize what features of the model change with the introduction of institutions.

By plugging $I_{iT} \equiv 1$, for $i = 1, 2, \dots, M$ in (8) we get that the optimal sharing rule of each agent is constant and equals to its initial share. That is, $P_{iT} \equiv \bar{P}^i = \lambda_i$, investor i holds a fixed λ_i share of the wealth.

Summary of Results for the Baseline Case:

The optimal sharing rule implies, $\bar{W}_{iT} = \bar{P}_i D_T$, is characterized by $\bar{P}_i = \frac{\lambda_i}{\sum_{i=1}^M \lambda_i} = \lambda_i$. Sharpe ratios are given by

$$\bar{\theta}_t = R \frac{E_t \left[D_T^{-R} \sigma_T^D \right]}{E_t \left[D_T^{-R} \right]}. \quad (17)$$

Asset volatilities are given by

$$\bar{\sigma}_{jt}^{S_j} = \sigma e_j + R E_t \left[\left(\frac{D_T^{-R}}{E_t \left[D_T^{-R} \right]} - \frac{D_T^{-R} D_{jT}}{E_t \left[D_T^{-R} D_{jT} \right]} \right) \sigma_T^D \right]. \quad (18)$$

Optimal portfolios are given by

$$\bar{\pi}_{1t} = (\bar{\sigma}_t^S)^{-1} \left(\bar{\theta}_t + (1 - R) E_t \left[\frac{D_T^{-R} D_T}{E_t \left[D_T^{-R} D_T \right]} \sigma_T^D \right] \right), \quad (19)$$

where the optimal wealth of agent i is given by $\bar{W}_{it} = \lambda_i \frac{E_t \left[D_T^{1-R} \right]}{E_t \left[D_T^{-R} \right]}$ and the j 's asset price is given by $\bar{S}_{jt} = \frac{E_t \left[D_T^{-R} D_{jT} \right]}{E_t \left[D_T^{-R} \right]}$.

B Linear Index

The index configuration was defined earlier in (6). In this special case we have

$$\bar{\chi}_{ij} = \begin{cases} 1 & \text{if the } j\text{'th asset is included in the benchmark of the } i\text{'th institution} \\ 0 & \text{else} \end{cases}$$

Avoiding confusing from the main body of work we denote $\bar{\chi}$ as the index configuration matrix, where the benchmark of investor i is a weighted arithmetic average of assets' returns characterized by

$$S_T^I = \sum_{j=1}^N \bar{\chi}_{ij} S_{jT}$$

$$I_{iT} = \sum_{j=1}^N \bar{\chi}_{ij} D_{jT}$$

Theorem 3 (Equilibrium Asset Prices and Portfolio Allocations).

Sharpe ratios are given by

$$\theta_t = RE_t \left[\frac{M_T}{E_t[M_T]} \sigma_T^D \right] - (R-1) \sigma \sum_{i=1}^M E_t \left[\frac{M_T}{E_t[M_T]} P_{iT} \frac{D_{iT}}{I_{iT}} \right] \bar{\chi}_i. \quad (20)$$

Asset volatilities are given by

$$\sigma_{jt}^S = \sigma e_j + RE_t \left[\left(\frac{M_T}{E_t[M_T]} - \frac{M_T D_{jT}}{E_t[M_T D_{jT}]} \right) \sigma_T^D \right]$$

$$- (R-1) \sigma \sum_{i=1}^M E_t \left[\left(\frac{M_T}{E_t[M_T]} - \frac{M_T D_{jT}}{E_t[M_T D_{jT}]} \right) P_{iT} \frac{D_{iT}}{I_{iT}} \right] \bar{\chi}_i. \quad (21)$$

Optimal portfolios (wealth volatilities) are given by

$$\sigma_t^S \pi_{it} = \theta_t + \frac{R-1}{R} E_t \left[\frac{I_{iT}^{1-\frac{1}{R}} M_T^{1-\frac{1}{R}}}{E_t \left[I_{iT}^{1-\frac{1}{R}} M_T^{1-\frac{1}{R}} \right]} \frac{D_{iT}}{I_{iT}} \right] \sigma \bar{\chi}_i \quad (22)$$

$$- (R-1) E_t \left[\frac{I_{iT}^{1-\frac{1}{R}} M_T^{1-\frac{1}{R}}}{E_t \left[I_{iT}^{1-\frac{1}{R}} M_T^{1-\frac{1}{R}} \right]} \sigma_T^D \right] + (R-1)^2 \frac{\sigma}{R} \sum_{k=1}^M E_t \left[\frac{I_{iT}^{1-\frac{1}{R}} M_T^{1-\frac{1}{R}}}{E_t \left[I_{iT}^{1-\frac{1}{R}} M_T^{1-\frac{1}{R}} \right]} P_T^k \frac{D_{iT}}{I_{iT}} \right] \bar{\chi}_k.$$

The wealth of the i 'th institution is given by $W_{it} = \frac{E_t[M_T P_{iT} D_T]}{E_t[M_T]}$ and the j 'th asset price by $S_{jt} = \frac{E_t[M_T D_{jT}]}{E_t[M_T]}$, where $\bar{\chi}_i$ is the i 'th row of the matrix $\bar{\chi}$, signifying the i 'th institution benchmarked

assets.

C Proofs

The following Lemma is a step in proving Theorem 1. It states sufficient conditions under which the expected values in the main theorem are finite.

Lemma 1. *Processes f_T and g_T with the evolution*

$$\begin{aligned}\frac{df_t}{f_t} &= \mu_t^f dt + \sigma_t^{f'} d\omega_t, & f_0 &\in (0, \infty), \\ \frac{dg_t}{g_t} &= \mu_t^g dt + \sigma_t^{g'} d\omega_g, & g_0 &\in (0, \infty),\end{aligned}$$

where $T < \infty$, μ_t^f , $\|\sigma_t^f\|^2$ and μ_t^g , $\|\sigma_t^g\|^2$ are bounded processes and away from zero, has a finite expected value and so are their powers and multiplication.

Proof. The bounded drift implies that $\mu_t^f \leq \bar{\mu}$, from some constant $\bar{\mu}$, and the bounded $\|\sigma_t^f\|^2$ implies that Novikov condition is satisfied. Therefore, we conclude that

$$\begin{aligned}E[f_T] &= f_0 E \left[\exp \left\{ \int_0^T \left(\mu_s^f - \frac{1}{2} \|\sigma_s^f\|^2 \right) ds + \int_0^T \sigma_s^{f'} d\omega_s \right\} \right] \\ &\leq f_0 \exp \{ \bar{\mu} T \} E \left[\exp \left\{ - \int_0^T \frac{1}{2} \|\sigma_s^f\|^2 ds + \int_0^T \sigma_s^{f'} d\omega_s \right\} \right] = f_0 \exp \{ \bar{\mu} T \} < \infty.\end{aligned}$$

Showing that the power and multiplication are also finite is done by using Ito's Lemma and observing that the process is also an exponential form with bounded coefficients, thereby has finite expected value. \square

Proof of Theorem 1 (Equilibrium) . We conjecture that markets are complete and adopt the equivalent martingale method approach pioneered by Karatzas, Lehoczky, and Shreve (1987), Cox and Huang (1989) and Cox and Huang (1991).

The first order condition of investor i is given by

$$W_{iT} = I_{iT} (y_i \xi_T I_{iT})^{-\frac{1}{R}} \geq 0,$$

where y_i is the Lagrange multiplier that can be obtained by plugging W_{iT} into the static budget constraint, $E[\xi_T W_{iT}] = \lambda_i S_{m0}$,

$$y_i^{-\frac{1}{R}} = \frac{\lambda_i S_{m0}}{E \left[\xi_T^{1-\frac{1}{R}} I_{iT}^{1-\frac{1}{R}} \right]} > 0.$$

By plugging the Lagrange multiplier back into the first order condition we get that

$$\xi_T W_{iT} = \lambda_i S_{m0} \frac{\xi_T^{1-\frac{1}{R}} I_{iT}^{1-\frac{1}{R}}}{E \left[\xi_T^{1-\frac{1}{R}} I_{iT}^{1-\frac{1}{R}} \right]}. \quad (23)$$

The sharing rule is optimal if it satisfies the market clearing condition in the consumption good and agents' first order conditions. By plugging the conjectured optimal share, (8), we get that

$$\sum_{i=1}^M W_{iT} = D_T \sum_{i=1}^M P_{iT} = D_T,$$

the sum of all shares equals one. In order to check that agents behave optimally we plug the conjectured optimal shares to the corresponding first order conditions,

$$\xi_T P_{iT} D_T = \xi_T \frac{A_i (I_{iT})^{1-\frac{1}{R}}}{\sum_{n=1}^M A_n (I_{nT})^{1-\frac{1}{R}}} D_T,$$

plugging the definition of A_i , dividing and multiplying by S_{m0} and $\xi_T^{1-\frac{1}{R}}$ and rearranging we get that

$$\xi_T P_{iT} D_T = \xi_T \left(\lambda_i S_{m0} \frac{(\xi_T I_{iT})^{1-\frac{1}{R}}}{E \left[(\xi_T I_T^1)^{1-\frac{1}{R}} \right]} \right) \left(\sum_{n=1}^M \lambda_n S_{m0} \frac{(\xi_T I_{nT})^{1-\frac{1}{R}}}{E \left[(\xi_T I_T^n)^{1-\frac{1}{R}} \right]} \right)^{-1} D_T.$$

By using (23) and recalling the market clearing condition in the consumption good we get the desired result,

$$\xi_T P_{iT} D_T = \xi_T (\xi_T W_{iT}) \frac{1}{\xi_T \sum_{n=1}^M W_{nT}} D_T = \xi_T W_{iT} \frac{1}{D_T} D_T = \xi_T W_{iT}.$$

Once we have shown that P_{iT} is the optimal sharing rule, finding ξ_T is done by plugging the optimal share to investor i first order condition,

$$P_{iT} D_T = \lambda_i S_{m0} \frac{\xi_T^{-\frac{1}{R}} I_{iT}^{1-\frac{1}{R}}}{E \left[\xi_T^{1-\frac{1}{R}} I_{iT}^{1-\frac{1}{R}} \right]},$$

dividing and multiplying by A_i and rearranging we get that

$$\xi_T = \left(\frac{E \left[(\xi_T I_{iT})^{1-\frac{1}{R}} \right] A_i}{\lambda_i S_{m0}} \frac{D_T P_{iT}}{A_i I_{iT}^{1-\frac{1}{R}}} \right)^{-R} = \left(\frac{E \left[(\xi_T I_{iT})^{1-\frac{1}{R}} \right] A_i}{\lambda_i S_{m0}} \right)^{-R} M_T \quad (24)$$

and plugging the definition of A_i from (10) we finally get that

$$\xi_T = \left(\frac{E \left[(\xi_T I_{1T})^{1-\frac{1}{R}} \right]}{\lambda_1 S_{m0}} \right)^{-R} M_T. \quad (25)$$

Both quantities are proportional with proportionality factor known at the onset. By taking conditional expectation at time t , when $t \in [0, T]$, and using the fact that the state price density is a martingale lead to the desired result.

In order to get the fixed point equation for A_i we use the result for ξ_T from (24) and write

$$E \left[(\xi_T I_{iT})^{1-\frac{1}{R}} \right] = E \left[\left(\frac{E \left[(\xi_T I_{iT})^{1-\frac{1}{R}} \right] A_i}{\lambda_i S_{m0}} \right)^{1-R} (M_T I_{iT})^{1-\frac{1}{R}} \right],$$

by rearranging this equation we get that

$$E \left[(\xi_T I_{iT})^{1-\frac{1}{R}} \right]^R = E \left[\left(\frac{A_i}{\lambda_i S_{m0}} \right)^{1-R} (M_T I_{iT})^{1-\frac{1}{R}} \right]$$

and by dividing the equation of the first investor by the equation of the i 'th investor we get the fixed point equation for A_i ,

$$A_i = \frac{\lambda_i}{\lambda_1} \frac{E \left[(M_{1T} I_{1T})^{1-\frac{1}{R}} \right]}{E \left[(M_T I_{iT})^{1-\frac{1}{R}} \right]}. \quad (26)$$

In order to derive the uniqueness of the solution, let us define an investor specific map $T_i(A_0) : \mathbb{R}^+ \rightarrow \mathbb{R}^+$,

$$T_i(A_0) \equiv \frac{\lambda_i}{\lambda_1} \frac{E \left[D_T^{1-R} \left(\sum_{n=1, \neq i}^M A_n I_{nT}^{1-\frac{1}{R}} + A_0 I_{iT}^{1-\frac{1}{R}} \right)^{R-1} (I_{1T})^{1-\frac{1}{R}} \right]}{E \left[D_T^{1-R} \left(\sum_{n=1, \neq i}^M A_n I_{nT}^{1-\frac{1}{R}} + A_0 I_{iT}^{1-\frac{1}{R}} \right)^{R-1} (I_{iT})^{1-\frac{1}{R}} \right]}, \quad i = 1, 2, \dots, M. \quad (27)$$

Let $A_h = A_0 + \epsilon$, for $\epsilon > 0$. By looking at the difference we get that

$$T_i(A_h) - T_i(A_0) \leq \frac{\lambda_i}{\lambda_1} \frac{E \left[D_T^{1-R} (I_{1T})^{1-\frac{1}{R}} \left\{ \left(\sum_{n=1, \neq i}^M A_n I_{nT}^{1-\frac{1}{R}} + A_h I_{iT}^{1-\frac{1}{R}} \right)^{R-1} - \left(\sum_{n=1, \neq i}^M A_n I_{nT}^{1-\frac{1}{R}} + A_0 I_{iT}^{1-\frac{1}{R}} \right)^{R-1} \right\} \right]}{E \left[D_T^{1-R} \left(\sum_{n=1, \neq i}^M A_n I_{nT}^{1-\frac{1}{R}} + A_0 I_{iT}^{1-\frac{1}{R}} \right)^{R-1} (I_{iT})^{1-\frac{1}{R}} \right]},$$

by defining

$$G_i(\epsilon) \equiv \left(\sum_{n=1, \neq i}^M A_n I_{nT}^{1-\frac{1}{R}} + A_h I_{iT}^{1-\frac{1}{R}} \right)^{R-1} - \left(\sum_{n=1, \neq i}^M A_n I_{nT}^{1-\frac{1}{R}} + A_0 I_{iT}^{1-\frac{1}{R}} \right)^{R-1},$$

noticing that $G_i(\epsilon)$ is twice differentiable function and deriving its Taylor expansion around zero we get that

$$\begin{aligned} G_i(\epsilon) &= 0 + (R-1) \left(\sum_{n=1, \neq i}^M A_n I_{nT}^{1-\frac{1}{R}} + A_0 I_{iT}^{1-\frac{1}{R}} \right)^{R-2} I_{iT}^{1-\frac{1}{R}} \epsilon \\ &\quad + \frac{(R-1)(R-2)}{2} \left(\sum_{n=1, \neq i}^M A_n I_{nT}^{1-\frac{1}{R}} + A_0 I_{iT}^{1-\frac{1}{R}} \right)^{R-3} \left(I_{iT}^{1-\frac{1}{R}} \right)^2 \epsilon^2 + \mathcal{O}(\epsilon^2). \end{aligned}$$

We conclude that

$$G_i(\epsilon) \leq \frac{(R-1)(R-2)}{2} \left(\sum_{n=1, \neq i}^M A_n I_{nT}^{1-\frac{1}{R}} + A_0 I_{iT}^{1-\frac{1}{R}} \right)^{R-3} \left(I_{iT}^{1-\frac{1}{R}} \right)^2 \epsilon^2 + \mathcal{O}(\epsilon^2),$$

since $R > 1$ implies that the first element of $G_i(\epsilon)$ is greater than zero. By plugging this back into the main inequality we get that

$$\begin{aligned} T_i(A_h) - T_i(A_0) &\leq \\ \frac{\lambda_i}{\lambda_1} &\frac{E \left[D_T^{1-R} (I_{1T})^{1-\frac{1}{R}} \frac{(R-1)(R-2)}{2} \left(\sum_{n=1, \neq i}^M A_n I_{nT}^{1-\frac{1}{R}} + A_0 I_{iT}^{1-\frac{1}{R}} \right)^{R-3} \left(I_{iT}^{1-\frac{1}{R}} \right)^2 \epsilon^2 + \mathcal{O}(\epsilon^2) \right]}{E \left[D_T^{1-R} \left(\sum_{n=1, \neq i}^M A_n I_{nT}^{1-\frac{1}{R}} + A_0 I_{iT}^{1-\frac{1}{R}} \right)^{R-1} (I_{iT})^{1-\frac{1}{R}} \right]} \end{aligned}$$

and by dividing both sides of the equation by ϵ and noticing that $\lim_{\epsilon \rightarrow 0} \frac{\mathcal{O}(\epsilon^2)}{\epsilon} = 0$ we conclude that

$$\lim_{\epsilon \rightarrow 0} \frac{T_i(A_h) - T_i(A_0)}{\epsilon} \leq \lim_{\epsilon \rightarrow 0} \left\{ q_1 \epsilon + q_2 \frac{\mathcal{O}(\epsilon^2)}{\epsilon} \right\} = 0.$$

q_1 and q_2 are finite as is shown below.

In addition, if we set $A_0 = 0$ we get that

$$T_i(0) = \frac{\lambda_i}{\lambda_1} \frac{E \left[D_T^{1-R} \left(\sum_{n=1, \neq i}^M A_n I_{nT}^{1-\frac{1}{R}} \right)^{R-1} (I_{1T})^{1-\frac{1}{R}} \right]}{E \left[D_T^{1-R} \left(\sum_{n=1, \neq i}^M A_n I_{nT}^{1-\frac{1}{R}} \right)^{R-1} (I_{iT})^{1-\frac{1}{R}} \right]} > 0.$$

Since we have established that for any A_0 the derivative of $T_i(0)$ equals 0 as well as $T_i(0) > 0$, $T_i(A_0)$ has a unique fixed point. Let us set the fixed point to be A_i and conclude that

$$A_i = T_i(A_i) \equiv \frac{\lambda_i}{\lambda_1} \frac{E \left[(M_T I_{1T})^{1-\frac{1}{R}} \right]}{E \left[(M_T I_{iT})^{1-\frac{1}{R}} \right]}.$$

Showing that the expected values are finite we use Ito's Lemma on M_t and get that

$$\begin{aligned} \frac{dM_t}{M_t} &= -R (\mu_t^D dt + \sigma_t^{D'} d\omega_t) + R \sum_{i=1}^M P_{it} \left(\mu_t^{I_i^R} dt + \sigma_t^{I_i^{R'}} d\omega_t \right) \\ &+ \frac{1}{2} \left(R(1+R) \|\sigma_t^D\|^2 - 2R^2 \sum_{i=1}^M P_{it} \sigma_t^{D'} \cdot \sigma_t^{I_i^R} + R(R-1) \sum_{i=1}^M \sum_{j=1}^M P_{it} P_{jt} \sigma_t^{I_i^{R'}} \cdot \sigma_t^{I_j^R} \right) dt \\ &\equiv \mu_t^M dt + \sigma_t^{M'} d\omega_t, \end{aligned} \tag{28}$$

where $\mu_t^{I_i^R}$ and $\sigma_t^{I_i^{R'}}$ are the drift and volatility of $I_{it}^{1-\frac{1}{R}}$ characterized by

$$\begin{aligned} \mu_t^{I_i^R} &\equiv \left(1 - \frac{1}{R} \right) \left(\mu_t^{I_i} - \frac{1}{R} \|\sigma_t^{I_i}\|^2 \right), \\ \sigma_t^{I_i^{R'}} &\equiv \left(1 - \frac{1}{R} \right) \sigma_t^{I_i}. \end{aligned}$$

Note that both μ_t^M , $\|\sigma_t^M\|$ and $\mu_t^{I_i}$, $\|\sigma_t^{I_i}\|$ are bounded, implying that so are the drift and the norm of the volatility of $M_t^{1-\frac{1}{R}} I_{it}^{1-\frac{1}{R}}$. The expected values in q_1 and q_2 can also be written as multiplications and powers of M_T , I_{iT} and D_T , which implies that they are finite as well.

By Lemma 1 we conclude that for a fixed set of adjustment weights the expected values in (10) are finite. □

Proof of Proposition 1 (Equivalence). The proof is done in two steps. In the first step, we show that the pricing kernels are identical. If every institution $i \in \hat{I}$ have the same benchmark

index then the risk adjustment costs of every institution $i \in \hat{I}$ are the same as well, implying that

$$\sum_{i \in \hat{I}} A_i = \sum_{i \in \hat{I}} \frac{\lambda_i}{\lambda_1} \frac{E \left[(M_T I_{1T})^{1-\frac{1}{R}} \right]}{E \left[(M_T I_{iT})^{1-\frac{1}{R}} \right]} = \left(\frac{\sum_{i \in \hat{I}} \lambda_i}{\lambda_1} \right) \frac{E \left[(M_T I_{1T})^{1-\frac{1}{R}} \right]}{E \left[(M_T I_{iT})^{1-\frac{1}{R}} \right]} \equiv \bar{A}_i.$$

If we plug this to the pricing kernel we get that

$$\begin{aligned} M_T^{\mathcal{M}_1} &= D_T^{-R} \left(A_1 I_{1T}^{1-\frac{1}{R}} + \dots + \sum_{i \in \hat{I}} A_i I_{iT}^{1-\frac{1}{R}} + \dots + A_M I_{MT}^{1-\frac{1}{R}} \right)^R \\ &= D_T^{-R} \left(A_1 I_{1T}^{1-\frac{1}{R}} + \dots + \bar{A}_i I_{iT}^{1-\frac{1}{R}} + \dots + A_M I_{MT}^{1-\frac{1}{R}} \right)^R = M_T^{\mathcal{M}_2}, \end{aligned}$$

where \mathcal{M}_1 denotes the original economy and \mathcal{M}_2 the new one. In the second step, we outline that if both economies have identical cash flows and identical pricing kernels then stock prices, stock volatilities and Sharpe ratios are the same. Moreover, since the optimal share, P_{iT} , is the same for the rest of the market participants, their portfolio holdings does not change as well. \square

Proof of Theorem 2 (Asset Prices and Portfolio Allocations). **Optimal wealth** is obtained by using the no arbitrage argument, $\xi_t W_{it} = E_t [\xi_T W_{iT}]$, by plugging the optimal share and the state price density, (25), we get that

$$W_{it} = \frac{E_t [M_T P_{iT} D_T]}{E_t [M_T]}. \quad (29)$$

Stocks prices are obtained the same way, by using the no arbitrage argument and using the state price density, (25), we get that

$$S_{jt} = E_t [\xi_{t,T} D_{jT}] = \frac{E_t [M_T D_{jT}]}{E_t [M_T]}. \quad (30)$$

Sharpe ratios are obtained by taking the Malliavin Derivative of the state price density, (25), doing so lead to

$$-\theta_t = \frac{E_t [\mathcal{D}_t M_T]}{E_t [M_T]}. \quad (31)$$

The next step is obtained by unfolding M_T according to (9),

$$-\theta_t = -R \frac{E_t \left[M_T \left(\mathcal{D}_t \log D_T - \mathcal{D}_t \log \sum_{i=1}^M A_i I_{iT}^{1-\frac{1}{R}} \right) \right]}{E_t [M_T]}. \quad (32)$$

The Malliavin derivative of the index is characterized by

$$\mathcal{D}_t \log \sum_{i=1}^M A_i I_{iT}^{1-\frac{1}{R}} = \left(1 - \frac{1}{R}\right) \sum_{i=1}^M \frac{A_i I_{iT}^{1-\frac{1}{R}}}{\sum_{i=1}^M A_i I_{iT}^{1-\frac{1}{R}}} \frac{\mathcal{H}_{t,T}^{I_i}}{I_{iT}} \chi_i = \left(1 - \frac{1}{R}\right) \sum_{i=1}^M P_{iT} \frac{\mathcal{H}_{t,T}^{I_i}}{I_{iT}} \chi_i \quad (33)$$

$$\mathcal{D}_t \log I_{iT}^{1-\frac{1}{R}} = \left(1 - \frac{1}{R}\right) \frac{\mathcal{H}_{t,T}^{I_i}}{I_{iT}} \chi_i \quad (34)$$

where,

$$\mathcal{H}_{t,T}^{I_i} \chi_i = \int_t^T \left(\mathcal{D}_t \mu_t^{I_i} - \left(\mathcal{D}_t \sigma_t^{I_i} \right)' \sigma_t^{I_i} \right) ds + \int_t^T \left(\mathcal{D}_t \sigma_t^{I_i} \right)' d\omega_s \quad (35)$$

and since the exogenous dividend processes have constant volatilities we get that

$$\mathcal{D}_t \log D_T = \sigma_T^D.$$

By plugging these results to (32) we get the desired result.

Stocks' volatilities are obtained by taking the Malliavin derivative of the stock price, in (30),

$$S_{jt} \sigma_{jt}^S E_t [M_T] + S_{jt} E_t [\mathcal{D}_t M_T] = E_t [\mathcal{D}_t M_T D_{jT}] + E_t [M_T \mathcal{D}_t D_{jT}],$$

using the result for θ_t from (31), the result for the stock price, S_{jt} , from (30) and rearranging we get that

$$\sigma_{jt}^S = \theta_t + \frac{E_t [\mathcal{D}_t M_T D_{jT}]}{E_t [M_T D_{jT}]} + \frac{E_t [M_T \mathcal{D}_t D_{jT}]}{E_t [M_T D_{jT}]}.$$

The next step is obtained by plugging the Malliavin derivative of M_T , (32), and of D_{jT} , which equals to

$$\mathcal{D}_t \log D_{jT} = \sigma e_j,$$

into the above equation and get that

$$\sigma_{jt}^S = \sigma e_j + \theta_t - R \frac{E_t [M_T D_{jT} \sigma_T^D]}{E_t [M_T D_{jT}]} + (R-1) \sum_{i=1}^M \frac{E_t \left[M_T D_{jT} P_{iT} \frac{\mathcal{H}_{t,T}^{I_i}}{I_{iT}} \right]}{E_t [M_T D_{jT}]} \chi_i.$$

By plugging θ_t from (20) we get the desired result. Following Karatzas et al. (1987) we show that

$$\begin{aligned}\sigma_{jt}^S &\geq \sigma e_j - RE_t \left[\frac{M_T D_{jT}}{E_t [M_T D_{jT}]} \sigma_T^D \right] - (R-1) \sum_{i=1}^M E_t \left[\frac{M_T}{E_t [M_T]} P_{iT} \frac{\mathcal{H}_{t,T}^{I_i}}{I_{iT}} \right] \chi_i \\ &\geq \sigma e_j - R\sigma \mathbf{1} - (R-1) \kappa \sum_{i=1}^M \chi_i \\ &\geq \sigma e_j - R\sigma \mathbf{1} - (R-1) \kappa M \mathbf{1} = \sigma e_j - K \mathbf{1},\end{aligned}$$

where κ is finite since $\frac{1}{I_{iT}}$ and $\mathcal{H}_{t,T}^{I_i}$ have finite drifts and norms of volatilities and $P_{iT} \leq 1$. Therefore, by Lemma 1 have finite expected value. We conclude that the stock volatility matrix is non degenerate and can be inverted; the conjectured market completeness assumption is fully justified. Before we derive the **optimal portfolios** we rewrite the wealth of agent i , (29), as

$$W_{it} E_t [M_T] = E_t \left[A_i I_{iT}^{1-\frac{1}{R}} \left(\frac{D_T}{\sum_{i=1}^M A_i I_{iT}^{1-\frac{1}{R}}} \right)^{1-R} \right] = A_i E_t \left[I_{iT}^{1-\frac{1}{R}} M_T^{1-\frac{1}{R}} \right]. \quad (36)$$

By taking the Malliavin derivative of this equation we get that

$$\sigma_t^S \pi_{it} W_{it} E_t [M_T] + W_{it} E_t [\mathcal{D}_t M_T] = A_i E_t \left[\mathcal{D}_t I_{iT}^{1-\frac{1}{R}} M_T^{1-\frac{1}{R}} \right] + A_i E_t \left[I_{iT}^{1-\frac{1}{R}} \mathcal{D}_t M_T^{1-\frac{1}{R}} \right].$$

Moreover, we already established $\mathcal{D}_t I_{iT}^{1-\frac{1}{R}}$ in (34) and that

$$\begin{aligned}\mathcal{D}_t M_T^{1-\frac{1}{R}} &= \left(1 - \frac{1}{R}\right) M_T^{1-\frac{1}{R}} \mathcal{D}_t \log M_T = \left(1 - \frac{1}{R}\right) M_T^{1-\frac{1}{R}} \left(-R\sigma_t^D + (R-1) \sum_{i=1}^M P_{iT} \frac{\mathcal{H}_{t,T}^{I_i}}{I_{iT}} \chi_i \right) \\ &= M_T^{1-\frac{1}{R}} \left((1-R) \sigma_t^D + \frac{(R-1)^2}{R} \sum_{i=1}^M P_{iT} \frac{\mathcal{H}_{t,T}^{I_i}}{I_{iT}} \chi_i \right).\end{aligned}$$

By plugging these Malliavin derivatives and using the result for W_{it} from (36) we get that

$$\begin{aligned}\sigma_t^S \pi_{it} W_{it} E_t [M_T] + W_{it} E_t [\mathcal{D}_t M_T] &= \left(1 - \frac{1}{R}\right) W_{it} E_t \left[I_{iT}^{1-\frac{1}{R}} M_T^{1-\frac{1}{R}} \frac{\mathcal{H}_{t,T}^{I_i}}{I_{iT}} \right] \chi_i \\ &+ (1-R) A_i E_t \left[I_{iT}^{1-\frac{1}{R}} M_T^{1-\frac{1}{R}} \sigma_T^D \right] + \frac{(1-R)^2}{R} A_i \sum_{j=1}^M E_t \left[I_{iT}^{1-\frac{1}{R}} M_T^{1-\frac{1}{R}} P_{jT} \frac{\mathcal{H}_{t,T}^{I_j}}{I_{jT}} \right] \chi_i.\end{aligned}$$

Lastly, by plugging W_{it} , (36), and using the result for θ_t , (31), we get the desired result. Specifying the result for a **geometric index** is obtained by observing that

$$\mathcal{H}_{t,T}^{I_i} \chi_i = \sigma I_{iT} \chi_i$$

and for a **linear index** by observing that

$$\mathcal{H}_{t,T}^{I_i} \chi_i = \sigma D_T \chi_i$$

□

Lemma 2. *For any investor i and for any time $t \in [0, T]$ there is a one to one map between stocks and their corresponding dividend news processes, such that*

$$S_{jt} \in S_t^{I_i} \iff D_{jt} \in I_{it}, \quad \forall j \in \{1, \dots, N\}, \quad \forall t \in [0, T].$$

Proof of Lemma 2. Suppose by contradiction that there exist a time $t \in [0, T]$ and investor i such that $S_{jt} \notin S_t^{I_i}$ but $D_{jt} \in I_{it}$. Due to the fixed composition of $S_t^{I_i}$ and I_{it} we get that at the terminal time, T , $S_{jT} \notin S_T^{I_i}$ but $D_{jT} \in I_{iT}$, which contradicts the no arbitrage condition stating that $S_T^{I_i} = I_{iT}$. The same argument follows if there exist a $D_{jt} \notin I_{it}$ but $S_{jt} \in S_t^{I_i}$. □

The notion of indistinguishability is key to understanding the following results. We say that X and Y are two indistinguishable processes if they have the same sample paths probability almost surely. More technically, there exist an event A , with probability one, such that

$$X_t(\omega) = Y_t(\omega), \quad \forall \omega \in A, \quad \forall t \in [0, T].$$

Proof of Proposition 2 (Asset Prices Effects). Let us start with the Sharpe ratio. By looking at the difference between the Sharpe ratios, $\theta'_t e_l$ and $\theta'_t e_k$, and plugging the definition of σ_T^D from (2) we get that

$$\theta_{tk} - \theta_{tl} = R\sigma E_t \left[\frac{M_T}{E_t[M_T]} \left(\frac{D_{kT}}{D_T} - \frac{D_{lT}}{D_T} \right) \right] - (R-1)\sigma \sum_{i=1}^{n_k - n_l} E_t \left[\frac{M_T}{E_t[M_T]} P_{iT} \right] \frac{1}{n_i} \quad (37)$$

and by comparing the variables state by state we get that

$$0 = R\sigma \left(\frac{D_{kT}}{D_T} - \frac{D_{lT}}{D_T} \right) < (R-1)\sigma \sum_{i=1}^{n_k - n_l} P_{iT} \frac{1}{n_i}.$$

The inequality holds since $P_{iT} > 0$, $R > 1$ and $\frac{D_{kT}}{D_T}$ is indistinguishable from $\frac{D_{lT}}{D_T}$. Therefore, if we multiply by M_T and take conditional expectations we get that $\theta_{tk} \leq \theta_{tl}$.

By using the evolution of M_t , (28), and the evolution of D_{jt} , (1), we get that

$$\frac{d(M_t D_{jt})}{M_t D_{jt}} = (\mu_t^M + \mu + \sigma \sigma_t^{M'} e_j) dt + (\sigma_t^{M'} + \sigma e'_j) d\omega_t, \quad (38)$$

where $j = k, l$.

First we observe that $M_0 D_{l0} = M_0 D_{k0}$, the volatility components are indistinguishable,

$$\sigma_t^{M'} d\omega_t + \sigma d\omega_{lt} = \sigma_t^{M'} d\omega_t + \sigma d\omega_{kt}$$

and the first two components of the drifts are identical for $j = l, k$. Second, by unfolding the last component of the drift according to (28) we get that

$$\sigma \sigma_t^{M'} e_k = \sigma^2 \left(-R \frac{D_{kt}}{D_t} + (R-1) \sum_{i=1}^{n_k} P_{it} \right) > \sigma^2 \left(-R \frac{D_{lt}}{D_t} + (R-1) \sum_{i=1}^{n_l} P_{it} \right) = \sigma \sigma_t^{M'} e_l, \quad (39)$$

since $\frac{D_{kt}}{D_t}$ and $\frac{D_{lt}}{D_t}$ are indistinguishable, $n_l \leq n_k$, which translates into $\sum_{i=1}^{n_l} P_{it} \leq \sum_{i=1}^{n_k} P_{it}$, and $R > 1$. Therefore, we conclude that

$$M_T D_{kT} \geq M_T D_{lT}$$

and by taking expectation we get the desired result. □

Proof of Proposition 3 (Symmetry). Identical benchmark sizes implies that I_{1T} is indistinguishable from I_{2T} , which further implies that $A_1 = A_2$. Suppose by contradiction and without loss of generality that $A_1 > A_2$. In this case,

$$E \left[\left(D_T^{-R} \left(A_1 I_{1T}^{1-\frac{1}{R}} + A_2 I_{2T}^{1-\frac{1}{R}} \right)^R I_{1T} \right)^{1-\frac{1}{R}} \right] > E \left[\left(D_T^{-R} \left(A_1 I_{1T}^{1-\frac{1}{R}} + A_2 I_{2T}^{1-\frac{1}{R}} \right)^R I_{2T} \right)^{1-\frac{1}{R}} \right]$$

By the indistinguishability we can replace I_{1T} and I_{2T} . By doing this exercise on the left and right hand sides we get

$$E \left[\left(D_T^{-R} \left(A_1 I_{2T}^{1-\frac{1}{R}} + A_2 I_{1T}^{1-\frac{1}{R}} \right)^R I_{2T} \right)^{1-\frac{1}{R}} \right] > E \left[\left(D_T^{-R} \left(A_1 I_{2T}^{1-\frac{1}{R}} + A_2 I_{1T}^{1-\frac{1}{R}} \right)^R I_{1T} \right)^{1-\frac{1}{R}} \right]$$

By analogy if the institutions switch indices we then have $A_1 < A_2$ which translates into

$$E \left[\left(D_T^{-R} \left(A_1 I_{2T}^{1-\frac{1}{R}} + A_2 I_{1T}^{1-\frac{1}{R}} \right)^R I_{2T} \right)^{1-\frac{1}{R}} \right] < E \left[\left(D_T^{-R} \left(A_1 I_{2T}^{1-\frac{1}{R}} + A_2 I_{1T}^{1-\frac{1}{R}} \right)^R I_{1T} \right)^{1-\frac{1}{R}} \right],$$

which is a contradiction. The same exercise is done for assuming $A_1 < A_2$. Therefore we can conclude that $A_1 = A_2$. Showing that the equilibrium quantities are equivalent is done by observing

that the relevant quantities are indistinguishable:

$$\begin{aligned} M_T D_{jT} &= M_T D_{iT}, \\ M_T \frac{D_{jT}}{D_T} &= M_T \frac{D_{iT}}{D_T}, \\ M_T P_{1T} &= M_T P_{2T}, \end{aligned}$$

$$\begin{aligned} M_T D_{jT} \frac{D_{jT}}{D_T} &= M_T D_{iT} \frac{D_{iT}}{D_T}, \\ M_T D_{jT} P_{1T} &= M_T D_{iT} P_{1T}, \\ M_T D_{jT} P_{2T} &= M_T D_{iT} P_{2T}, \end{aligned}$$

where j and i correspond to *type10* and *type01* assets respectively. □

Proof of Proposition 4 (Optimal Index Configuration). The first order condition for institution i , (23), is

$$\frac{W_{iT}}{I_{iT}} = \lambda_i S_{m0} \frac{\xi_T^{-\frac{1}{R}} I_{iT}^{-\frac{1}{R}}}{E \left[\xi_T^{1-\frac{1}{R}} I_{iT}^{1-\frac{1}{R}} \right]},$$

plugging this result to the utility function, (4), leads to

$$v_0(I_{iT}) = \frac{(\lambda_i S_{m0})^{1-R}}{1-R} \left(E \left[\xi_T^{1-\frac{1}{R}} I_{iT}^{1-\frac{1}{R}} \right] \right)^R.$$

By Theorem 1 we know that

$$\xi_T = \frac{M_T}{E[M_T]}.$$

By plugging this to the value function we get

$$v_0(I_{iT}) = \frac{(\lambda_i S_{m0} E[M_T])^{1-R}}{1-R} \left(E \left[M_T^{1-\frac{1}{R}} I_{iT}^{1-\frac{1}{R}} \right] \right)^R.$$

By setting

$$\kappa = \frac{(\lambda_i S_{m0} E[M_T])^{1-R}}{1-R}$$

we get the value function in (15). We solve for the optimal benchmark by first postulate an index configuration and, derive \hat{M}_T and $\hat{\kappa}$. Second, we plug these values into the value function and find the maximal index for each institution by iteration over possible values. If the maximal index configuration is then equal to the postulated index configuration, then the postulated index configuration is optimal. □

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