

The Leverage Ratchet Effect

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Abstract

This paper explores the implications of a “leverage ratchet effect” whereby conflicts of interest with creditors lead shareholders to resist all forms of leverage reduction even when reducing leverage would increase firm value while generally favoring an increase in leverage even if it destroys firm value. The leverage ratchet effect is present under perfect market conditions, but is exacerbated by standard frictions. Unlike theories based on asymmetric information, the leverage ratchet effect explains shareholders' resistance to earning retentions and rights offerings as ways to reduce leverage.

In a dynamic context, the leverage ratchet effect creates an additional agency cost of debt, in that prior leverage decision will distort future leverage choices. Because leverage is effectively irreversible, firms may limit leverage initially but then “ratchet up” in response to shocks. We show that even in a simple tradeoff model, the resulting leverage dynamics are complex and likely to produce cross-sectional distributions and comparative statics that are at odds with conventional wisdom.

Finally, we consider the impact of covenants or regulation designed to limit leverage via a leverage ratio. When forced to reduce leverage, by creditors or by regulation, firms can buy back debt using proceeds obtained either by selling assets or from issuing new equity. It can also purchase new assets funded by new equity. We present conditions under which shareholders are indifferent among these alternatives, considering all equally undesirable. We then analyze how various frictions affect shareholders' choice among leverage-reduction methods.

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1. Introduction

Firms are often reluctant to issue new shares. For example, a firm risking financial distress can, as long as it is still solvent, reduce the costs of the distress by issuing new shares, but this is rarely seen. When required either by market pressures or by regulation to reduce their leverage, firms appear more inclined to deleverage by selling assets even if this means that the assets are sold at “firesale” prices. Banks and financial institutions, which are among the most highly leveraged firms in the economy, seem particularly resistant to issuing new shares and also actively resist rules that require retentions of earnings.

The resistance of shareholders and managers to issuing new shares is often explained by dilution costs induced by asymmetric information along the lines of Myers and Majluf (1984).¹ The Myers-Majluf argument, however, is limited to new common share issues. It does not apply to new share issues that take the form of rights offerings, nor can it explain the resistance to earnings retentions. None of these involve shareholder losses from undervaluation in the market.² Further, when leverage reductions are imposed by regulation, adverse selection becomes irrelevant. Any “dilution costs” for the shareholders of firms with above-average return prospects would be matched by benefits for the shareholders of firms with below-average return prospects.³

In this paper we show that the resistance of shareholders – and managers acting on their behalf – to any kind of leverage reduction is to be generally expected, even under conditions of perfect information. Our analysis applies to all forms of leverage reduction, not just those involving the issuance of new shares. We also show that shareholder resistance to leverage reductions has profound effects on the dynamics of firms’ funding choices and the evolution of firms’ capital structures over time. These effects call into question the relevance of the traditional tradeoff theory of capital structure in a dynamic context. Standard comparative statics from tradeoff theory can easily be reversed in our model, potentially shedding light on their empirical failure.

Key to our analysis is the fundamental conflict of interest between shareholders and debt holders that results from debt overhang. The debt overhang phenomenon was introduced in Myers (1977), who studied leverage-induced distortions in investment. In contrast, we focus on the distortions created by high existing leverage on future leverage choices.

As is well known, debt overhang effects are present even in a world without frictions in which the Modigliani-Miller (1958) irrelevance result holds. In that setting, the tradeoff theory would suggest that any mix of funding by debt and equity might be observed, as the total value

¹ See for example Bolton and Freixas (2006) and Kashyap, Stein, and Hansen (2010).

² Indeed, Myers and Majluf (1984) emphasize that, with the information asymmetries they consider, raising funds by retaining earnings should be preferred to new borrowing.

³ In other words, removing discretion also mitigates any negative signal associated with recapitalizations (see Admati et al (2013, Section 6) and Kashyap, Hansen and Stein (2011, p. 10)). Of course, managers might want to protest increased equity requirements in an attempt to show that *their* firm is undervalued in the market. However, relative to the average price, some firms are actually overvalued by investors, and their informed managers may well know it.

of the firm is unaffected by the leverage choice. But now suppose that shareholders of a leveraged firm were to buy back some debt so as to reduce the total face value of outstanding debt. By the Modigliani-Miller Theorem, the debt buyback does not change the total value of the firm. Upon announcement of such a buyback, however, we should expect the market value of debt to rise given the reduced possibility of bankruptcy. But then, because the total value of the firm is unchanged, the net effect of the buyback on shareholder wealth must be negative. Hence, for highly leveraged firms with a non-negligible probability of default, we should expect shareholders to resist any reduction of leverage through a debt buyback in the open market. Shareholders would, however, love to raise new debt of equal priority in order fund a payout to themselves.

We are not the first to observe this effect or make the above observations. Already, Black and Scholes (1973) observed that, even in the absence of frictions shareholders cannot gain by giving up their default option. Also Leland (1994) observed a similar resistance in the context of a model in which debt is homogeneous and presumed fixed, bankruptcy costs are proportional, and only small marginal changes of *pari passu* debt are permitted.

However, the pervasiveness of the effect and its implications for dynamic capital structure theory are not generally recognized. This paper shows that shareholder resistance to leverage reductions is to be expected quite generally -- even when, due to frictions, reductions in leverage would substantially raise the value of the firm. When there are additional agency problems, the effects are even stronger. We refer to the phenomenon as *the leverage ratchet effect*: once they are highly indebted, firms will avoid value-improving leverage reductions. And while there is always an incentive to issue *pari passu* debt (due to the mechanical dilution of existing creditors), we show that with frictions, shareholders almost always gain issuing new *junior* debt even when doing so reduces total firm value.

The leverage ratchet effect creates an irreversibility in leverage that undermines the relevance of the traditional tradeoff theory of capital structure in a dynamic context. Because past leverage decisions distort future leverage choices, capital structure will become strongly history-dependent, and firms may accumulate debt more slowly to avoid these costs. As a result, we show that standard comparative statics from tradeoff theory need no longer apply. Finally, we show that when such firms are forced to reduce leverage, either by covenants or regulations, they will often choose to do so in inefficient ways, such as liquidating assets (possibly even at fire sale prices) rather than simply recapitalizing (for example by retaining earnings or a rights offering).

The leverage ratchet effect suggests that, absent strict covenants to the contrary, leverage begets more leverage and can become “addictive.” Shareholders will seek to increase leverage whenever the opportunity arises, or is accentuated by shocks that increase the benefits of debt (such as a tax increase). On the other hand, equity holder will resist leverage reductions even when lower leverage becomes more attractive – due, for example, to a decline in cash flows or tax rates – and may even seek to increase leverage at these times, in complete contrast to tradeoff

theory predictions. Evidence of such an asymmetric response in the context of tax rate changes is provided in recent work by Heider and Ljungqvist (2014).

Going beyond the observation that debt overhang affects funding choices at a given date, we note that the leverage ratchet effect is central to understanding capital structure dynamics. In addition to its direct effects, the leverage ratchet affects behavior through its effects on investors' expectations. Initial investors must anticipate that subsequent funding choices will be affected by debt overhang creating shareholder resistance to recapitalizations and shareholder preference for using new debt to make payouts. Creditors will try to defend themselves against such behavior, by covenants or by setting the price of debt to reflect the risks that subsequent debt issues may impose. Understanding how the leverage ratchet effect influences the dynamics of borrowing is critical for understanding capital structure dynamics and for designing and implementing effective covenants or regulation of leverage when such regulation is desired.

After exhibiting the great generality of the effect, we will therefore consider a dynamic model of leverage choice. We first do so in a setting without regulation or commitment. Even in a simple tradeoff model, we find that leverage dynamics can be complex and highly history-dependent. Firms will generally "ratchet up" their leverage over time, with the pace of such changes driven both by economic shocks, the tax benefits of debt, and the sensitivity of the debt price to total leverage. While firms are likely to initially be under-levered relative to the tradeoff theory, they will almost always become over-levered in time.

In reality, of course, some level of commitment, e.g., through debt covenants, is possible, but shareholders retain discretion with respect to both changes in leverage and asset transactions. We analyze how shareholders would choose to change leverage if covenants or regulation force them to reduce it, or if covenants or regulation allow them to increase it. Specifically, reducing leverage can be done by (1) selling assets and using the proceeds to reduce debt levels (asset reduction or "deleveraging"), (2) issuing new equity to buy back debt (pure recapitalization), or (3) issuing new equity to increase assets backing liabilities (asset expansion). We demonstrate an important neutrality result, namely that under certain conditions shareholders are indifferent between such modes of leverage changes. Specifically, we show that if there is one class of debt outstanding, if assets are homogeneous, and if sales or purchases of assets do not, by themselves, generate value for shareholders, then shareholders are indifferent among all ways leverage can be changed; they consider them all equally undesirable (in the case of leverage reduction) or desirable (in the case of leverage increases). We do not consider this result to be descriptive of the world, but serves rather as a benchmark that enables us to understand the actual shareholder preferences between strategies that we do observe in terms of deviations from the assumptions of our neutrality result.⁴

In this spirit, we examine a number of factors that influence shareholders' choices of how to change leverage when the conditions of this equivalence result do not hold. For example, we

⁴ This is similar to the role of the Modigliani-Miller Theorem as a benchmark telling us to think about capital structure choices in terms of the frictions that invalidate its assumptions.

show that when there are multiple classes of debt and in the absence of covenants to the contrary, shareholders will buy back the most junior debt before repurchasing debt with higher priority. When shareholders have the ability to buy back junior debt in this way, they will tend to prefer deleveraging through asset sales over the other two approaches. The reason is that asset sales funding a buyback of junior debt is a mechanism that imposes some of the cost of the deleveraging on the remaining senior debt holders whose expected returns are reduced when there are fewer assets and less junior debt to bear losses before they do. These distributive effects may dominate even if asset sales, possibly at fire-sale prices, are inefficient and reduce the value of the firm.

In our analysis, as in Myers (1977), shareholders resist leverage reductions because they are unable to prevent creditors from appropriating benefits that are created at shareholder expense. However, in contrast to the Myers (1977) underinvestment problem, which only occurs when the net present value of the project is not large enough for the shareholders' share of the benefits to cover its cost, shareholders' resistance to leverage reductions can persist no matter how much leverage reduction would increase the total value of the firm. Indeed, we show that shareholder resistance can *increase* even when the benefit of leverage *decreases* as a result of agency costs. This resistance arises because shareholders generally do not capture the benefits of reduced leverage.

We assume throughout that creditors are small and dispersed so that conflicts of interest cannot be dealt with by collective bargaining. Therefore reductions of leverage cannot be achieved through a renegotiation of outstanding contracts and any reduction in debt levels must be carried out through debt buybacks in the open market. In such a buyback, each creditor can choose whether to sell his claims back to the firm or hold on to them. The price at which debt is repurchased must therefore reflect the value that can be obtained by retaining a marginal amount of debt. If the leverage reduction reduces the borrower's probability of default, the debt's value will be raised by the buyback, which means that the price at which debt is repurchased is higher than the price that the debt would have if there were no buyback.⁵

In the literature on dynamic capital structure, it is common to explore shareholders' decisions with respect to payouts and default without allowing changes in the capital structure (prior to default). Papers that allow adjustments in capital structure often assume that it is prohibitively costly to reduce leverage in distress, or that debt can only be recalled at par or at a premium. By contrast, our analysis assumes that debt must be bought back in the market at

⁵ Such effects are well known from the literature concerning market-based solutions to the sovereign debt crisis of the 1980s. See, for example, the contributions in Frenkel et al. (1989) and Bulow and Rogoff (1990). The theory developed in that literature was confirmed in the Bolivian debt buyback of 1988 and more recently in the Greek debt buyback of 2012. By contrast, van Wijnbergen (1991) showed that, in the 1990 buyback of Mexican debt under the Brady plan, which involved collective bargaining, creditors were forced to agree to terms under which they neither gained nor lost from the buyback. The importance of the difference between collective bargaining and unilateral actions of the debtor is also stressed by Strebulaev and Whited (2012). However, they do not consider buybacks in markets, but study callable debt, where the call option requires a repayment of the amount that was originally borrowed, plus a premium.

competitive prices.⁶ In addition, we allow funds to be raised either by selling assets or by issuing equity through common share or rights offerings. Unlike much of the literature, our key results do not depend on any assumptions about exogenous transactions costs.

Our paper is related to a number of papers that have looked at various ways that changes in a firm's capital structure and its outstanding liabilities can affect the value of the firm's creditors and shareholders. Dangl and Zechner (2007) analyze the dynamics of leverage and the choice between long- and short-term debt. They observe that with long-term debt shareholders do not have the incentives to reduce leverage when the firm has poor performance. Short-term debt requires the firm pay off all its debt frequently (at par) and effectively causes the firm to start afresh with new tradeoffs a-la Modigliani and Miller each time this debt matures. The frequent issuance of short-term debt entails higher transactions costs, which must be traded off against any benefits created. An important assumption in Dangl and Zechner (2007) is that there are covenants in place that prohibit the issuance of any new debt that would increase the total face value of debt outstanding. This assumption rules out the ratchet effect that we explore in this paper.

Another closely related paper is Brunnermeier and Oehmke (2013), which shows that shareholder incentives potentially lead to a "maturity rat race," since under certain informational conditions shortening the maturity structure of a firm's liabilities dilutes the firm's longer-term creditors. The key assumption in Brunnermeier and Oehmke (2013) is that, although the firm can commit to a total amount of debt, it cannot commit to a particular maturity structure of that debt. They observe that this inability to commit is especially applicable to financial institutions with frequent funding needs and opaque balance sheets. Our paper is similar to Brunnermeier and Oehmke (2013) in that we assume that it is impossible or too costly for the firm to make binding commitments about all the details of the firm's capital structure, but instead of focusing on the maturity structure of a fixed amount of debt, we focus on the firm's leverage choices when covenants do not completely restrict firm leverage.

Finally, Bizer and DeMarzo (1992) demonstrate that in the presence of agency costs of debt, lack of commitment leads borrowers to choose excessive leverage. Their setting is focused on the case of a risk averse borrower or sovereign, rather than a firm, but the desire to increase leverage once existing leverage is in place mirrors our finding regarding resistance to leverage reductions. We also use solution methods developed in Bizer and DeMarzo (1994) for our analysis of the dynamic equilibrium.

Leverage ratchet effects are particularly strong for highly levered firms, such as banks. They are therefore critical to understanding the capital structure dynamics of financial institutions that obtain most of their funding from debt. If their debts are explicitly or implicitly

⁶ Some papers make the assumption that new debt can be issued *pari passu* with existing debt, which can help overcome the underinvestment problem identified in Myers (1977). Unless existing creditors benefit from additional investments, issuing such debt can reduce the value of existing debt and violate the creditors' seniority. In the spirit of Myers (1977), we assume for most of our analysis that violating the seniority of existing creditors is not possible.

guaranteed, banks' creditors have fewer incentives to put in place debt covenants that might mitigate the leverage ratchet. Leverage choices can therefore become extremely inefficient, especially since banks have many ways to issue debt that is effectively more senior to prior claims (e.g., because it has shorter maturity or is backed by collateral). Moreover, banks' distress or default can have significant negative external effects. Since the market fails to correct the social inefficiency, effective regulation is essential to correct the resulting distortions.

In an earlier paper (Admati et al. 2013) we considered banks' total funding costs and argued that banks will choose socially inefficient levels of leverage. The funding benefits that a bank derives from high leverage are due to debt subsidies (e.g. taxes and government guarantees) and these come at taxpayers' expense. The social benefits of high leverage are at best very small, while the social costs borne by third parties are large as witnessed in the financial crisis of 2008. In this paper we show that high leverage is likely to be *privately* costly when viewed from the limited perspective bank's investors (i.e., the bank's shareholders and the bank's creditors) in addition to socially costly. This is because leverage ratchet effects, which are particularly pronounced for banks, drive banks to increase leverage to levels that are potentially inefficient even for their own investors. The results of this paper therefore strengthen our conclusion that, in the context of banking, effective capital regulation is essential. In addition to reducing third-party damage from systemic risk associated with bank failures, such regulation can also mitigate the leverage ratchet effect, providing a substitute for ineffective *ex ante* covenants. Our analysis of shareholder preferences over various modes of leverage reduction also throws light on how banks react to such regulation.

The paper is organized as follows. Section 2 presents the basic model and preliminary results on shareholder resistance to leverage reductions. Section 3 considers agency costs of leverage on investment as well as future leverage choices, showing the key ratchet effect of leverage. Section 4 develops a dynamic equilibrium model of leverage, and demonstrates that firms will limit leverage initially but "ratchet up" indefinitely in response to shocks. In Section 5 we consider alternative ways for a firm to reduce leverage other than pure recapitalization. Section 6 discusses the application of our analysis to banking and the role of capital regulation. Section 7 provides concluding remarks.

2. Debt Overhang and Resistance to Recapitalization

In this and the following section, we develop a simple but general reduced-form, static tradeoff theory model in order to highlight the logic of the leverage ratchet effect. The analysis shows that the debt overhang effect illustrated in the introduction is in fact very robust to changes in the setting. Subsequently, in Section 4, we will embed this static model in a dynamic model and study the full dynamics of the leverage ratchet effect..

We consider a firm that has made an investment in risky assets and has funded itself with debt. We begin with a simple "tradeoff" model of capital structure based on taxes and net default

costs, which we will generalize later as we examine additional frictions. For our basic argument, we make the following assumptions:

Firm Investment: The firm has made a real investment A in the past (“date 0”). Investment returns are realized in the future (“date 2”) and are given by a random variable $\tilde{x}A$.

Firm Liabilities: We assume that the firm is funded by equity, and a total debt claim of D against the firm that is due at date 2, the date at which the asset return of $\tilde{x}A$ is realized. If $\tilde{x}A \geq D$, debt claims are honored in full.

We begin by considering three “frictions” that affect the payouts of the firm’s securities at date 2. These are taxes, bankruptcy costs, and third party (government) subsidies.

Taxes: We assume that a tax may be applied to those returns earned on the firm’s assets that exceed what is paid to the debt holders. The tax benefits are determined by the firm’s total debt outstanding and are given by $t(\tilde{x}, A, D) \in [0, \tilde{x}A - D]$ when $\tilde{x}A > D$. We assume that no tax is paid when $\tilde{x}A \leq D$. Finally, we assume that the total tax liability is weakly decreasing in D , i.e. $t_D(\tilde{x}, A, D) \leq 0$.⁷

Default costs (net of subsidies):

If $\tilde{x}A < D$, the firm is unable to fulfill its obligation to debt holders and must default unless it receives a subsidy from the government or some other third party. Let $n(\tilde{x}A, D)$ be the *net* default costs for the firm, which is the difference between the bankruptcy cost and any third party subsidy. In the event that $\tilde{x}A > D$, there are no subsidies and no bankruptcy costs and thus $n(\tilde{x}A, D) = 0$. If $\tilde{x}A < D$, we assume that $\tilde{x}A - n(\tilde{x}A, D) \in [0, D]$. Note that the net default costs could be negative if the subsidy exceeds the bankruptcy cost – which means that the firm’s debt holders will receive *more* than $\tilde{x}A$ – but we assume that, at best, subsidies bring the available funds up to the amount that is needed to avoid default

⁷ Note that there may be other effects of leverage on equity holders that can be included in the function t . For example, if debt plays a “disciplining role” as in Jensen (1986) or we can think of t as capturing any losses resulting from a lack of discipline. Alternatively, there may be ex ante costs to equity associated with leverage, such as increased wages as in Berk, Stanton, and Zechner (2010). The key assumption is that on the margin, tax shields and disciplining benefits, net of any costs, are weakly increasing in D .

Given these assumptions, the payoffs on the firm's debt and its equity are those given in the following table:

	If $\tilde{x}A < D$	If $\tilde{x}A \geq D$
Payoff to Shareholders	0	$\tilde{x}A - t(\tilde{x}, A, D) - D$
Payoff to Debt Holders	$\tilde{x}A - n(\tilde{x}A, D)$	D

Pricing at Date 1: All securities are traded in perfect Walrasian markets. The prices of securities at date 1 are equal to the expectations of their payoffs with respect to the *risk-neutral* distribution function F of the return on the firms' asset, \tilde{x} . The distribution function F has full support on $[0, \infty)$. We assume that the firm takes F as given and independent of its leverage choice.⁸ Throughout the paper, when we refer to probabilities, it is with respect to this risk-neutral measure.

Given our assumptions about payouts and pricing, it follows that at date 1 the values of the firm's debt and its equity are:

$$\begin{aligned} \text{Total value of debt} &= V^D(D, A) \\ &= \int_{D/A}^{\infty} D \, dF(x) + \int_0^{D/A} (xA - n(\tilde{x}A, D)) \, dF(x) \end{aligned} \quad (1)$$

and

$$\text{Value of equity} = V^E(D, A) = \int_{D/A}^{\infty} (xA - t(x, A, D) - D) \, dF(x). \quad (2)$$

We also define the value of the firm as usual as $V^F(D, A) = V^E(D, A) + V^D(D, A)$.

Suppose that given outstanding debt with face value D , the firm considers buying back a portion of this outstanding debt. In this section we hold fixed the assets of the firm, so the debt buyback implies a leverage reduction. The cash used for the buyback may be raised through a rights offering to existing shareholders, or a market offering of equity or other equity-like

⁸ The existence of such a distribution (or pricing kernel) F follows from the absence of arbitrage opportunities. We assume the firm acts as a price-taker with respect to this pricing kernel. Thus, as is standard in the corporate finance literature, we are ignoring any general equilibrium consequences of the individual firm's security choices on the equilibrium pricing kernel. Finally note that we can without loss of generality normalize the risk-free interest rate to zero or alternatively interpret the prices as future values.

securities (such as preferred shares). Alternatively, the firm may use cash on hand that it would either pay out as a dividend or retain to buyback debt. We will show that, independent of the source of funds or the potential benefit to total firm value, shareholders have a strict preference to avoid a recapitalization.⁹

We begin by determining the price of the firm's debt. Equation (1) above implies that, without the buyback, the date 1 market price of debt per unit of nominal face value is equal to:

$$q(D, A) = \frac{V^D(D, A)}{D} = 1 - \underbrace{F(D/A)}_{\text{Probability of Default}} \left(1 - \underbrace{E \left[\frac{x_A - n(\tilde{x}_A, D)}{D} \middle| x_A < D \right]}_{\text{Expected Recovery Rate}} \right). \quad (3)$$

Suppose that the firm considers buying back debt with a nominal claim equal to Δ . At the announcement of the recapitalization, the value of the firm will change from $V^F(D, A)$ to $V^F(D - \Delta, A)$. If the firm is currently inefficiently over-levered, then this change will be positive. It is natural to expect that some fraction of this gain will be captured by the firm's debt holders, diminishing the benefit to equity holders, but the standard intuition is that equity holders will still gain if the benefit is sufficiently large. As we establish below, however, the fraction of any gain captured by the firm's debt holders is always greater than 100%, deterring equity holders from ever voluntarily recapitalizing.

To see why, note that if the firm wants to buy back debt in the open market, it cannot do so at the price given in (3). The repurchase price must be such that debt holders are at the margin indifferent between selling debt and holding on to it. The buyback price of the debt must therefore be equal to the market price $q(D - \Delta, A)$ that prevails at the post-buyback debt level.¹⁰ We assume that that the firm's managers and shareholders assess such a buyback only on the basis of what it does to the shareholders' wealth.¹¹ This assessment depends only on whether the difference between the market value of the firm's equity with and without the buyback,

⁹ We take up whether the recapitalization is feasible and consistent with limited liability in Section 5. Throughout our analysis we assume that decisions are made on behalf of shareholders and we do not consider the governance issues associated with who can make the decision to issue shares or make a rights offering. Under U.S. law, a rights offering can be made without shareholder approval, though it may still fail if investors do not find it in their interest to acquire the new shares. Again, this issue is addressed in Section 5 when we characterize the conditions required for the recapitalization to be feasible.

¹⁰ Thus, upon announcement of the recap, the value of the firm's debt will rise to $[V^D(D - \Delta) + \Delta q(D - \Delta, A)]$. For extensive discussions of creditors ability to hold up debtors, see Frenkel et al. (1989) and Bulow and Rogoff (1990).

¹¹ Whereas, in the US, rights offerings can be decided by management without shareholder approval, in most other countries, rights offerings must be approved by shareholder meetings.

$V^E(D - \Delta, A) - V^E(D, A)$, exceeds the cost $q(D - \Delta, A) \times \Delta$.¹² The following proposition shows that the answer to this question is unambiguously negative.

Proposition 1 (Shareholder resistance to Recapitalization): *Equity holders are strictly worse off issuing securities to recapitalize the firm and reduce its outstanding debt. Losses to equity holders arise from the loss of their default option, the reduction in dilution of existing debt, and higher taxes. The loss to equity holders is mitigated by bankruptcy costs, and increased by the presence of taxes or default subsidies.*

Proof: Using (2), we can write the gain to shareholders from changing from debt D to $D - \Delta$ as

$$\begin{aligned}
G(D, D - \Delta) &\equiv V^E(D - \Delta, A) - V^E(D, A) - \Delta \times q(D - \Delta, A) \\
&= \int_{(D-\Delta)/A}^{D/A} (xA - D) dF(x) \\
&\quad + \Delta \times \left(1 - F\left(\frac{D - \Delta}{A}\right) - q(D - \Delta, A) \right) \\
&\quad + \int_{D/A}^{\infty} t(x, A, D) dF(x) - \int_{(D-\Delta)/A}^{\infty} t(x, A, D - \Delta) dF(x)
\end{aligned} \tag{4}$$

The first term in (4) captures the *default option effect*: the loss of equity's default option given final asset values between $D - \Delta$ and D . This term is strictly negative given our assumption that F has full support.

The second term in (4) captures the *dilution effect*: the portion of the debt repurchase price that compensates debt holders for their share of any recovery in default. From (3), we can see that this term is negative and decreases with the (ex post) expected recovery rate of the debt:

$$\begin{aligned}
\Delta \times \left(1 - F\left(\frac{D - \Delta}{A}\right) - q(D - \Delta, A) \right) &= \\
-\Delta \times \underbrace{F\left(\frac{D - \Delta}{A}\right)}_{\text{Probability of Default}} \times \underbrace{E\left[\frac{xA - n(\tilde{x}A, D - \Delta)}{D - \Delta} \middle| xA < D - \Delta\right]}_{\text{Expected Recovery Rate}} &\leq 0.
\end{aligned} \tag{5}$$

Note that this term would be zero if the debt repurchased were strictly junior to any remaining debt, as in that case the expected recovery rate would be zero for the “marginal” dollar of junior debt. When the debt repurchased has equal priority with the remaining debt, as assumed thus far, and the remaining debt has a positive recovery rate, this term is negative. Note also that default subsidies will raise, and bankruptcy costs will lower, this cost.

¹² Recall that we are assuming that security values are determined by the pricing kernel F , which is unaffected by the firm's leverage. Thus the impact on investor wealth is sufficient to determine shareholder preferences.

Finally, the third term in (4) is the *tax effect*: it is negative because taxes are non-increasing in D . Thus, combining these three effects, we see that

$$V^E(D - \Delta, A) - V^E(D, A) - \Delta \times q(D - \Delta, A) < 0 \quad (6)$$

and thus that shareholders must always lose from a recapitalization. ■

Proposition 1 restates and generalizes observations that have been made elsewhere in the literature. Black and Scholes (1973) note that shareholders will lose by repurchasing debt in a setting with perfect markets. Equation (4) shows this result is due to the loss of the default option and the expectation of a positive recovery rate on the debt. Leland (1994) demonstrates a similar result (which he describes as “surprising”) in the context of a continuous-time tradeoff model with linear taxes and a particular model of default costs, without providing intuition for the result. However, to the best of our knowledge, the full generality of the result in Proposition 1 has not been clearly articulated nor fully appreciated in the capital structure literature.¹³

We assumed above that the firm has only a single class of debt outstanding. If the firm has several classes of debt, shareholders will naturally find it most attractive to buy back the cheapest class first, which will be the most junior class outstanding. Note that the only difference in these classes of debt are their expected recovery rates. Because the expected recovery rate is always non-negative, however, the above logic still applies and we have the following immediate generalization:

Proposition 2 (Shareholder Resistance to buying back any debt class): *Equity holders are strictly worse off issuing securities to recapitalize the firm to repurchase any class of outstanding debt. The loss increases with the seniority of the debt.*

Note that shareholders’ resistance to a recapitalization does not depend on the tax benefits of leverage. More strikingly, note that shareholders will resist a recapitalization *no matter how large the potential gain to firm value* due to a reduction in default costs. Indeed, the magnitude of default costs has the opposite effect: by reducing the expected recovery rate and therefore the buyback price of the debt, default costs *raise* the cost to shareholders of reducing leverage. It follows that debt overhang can give rise to situations in which shareholders and debt holders jointly would benefit from a recapitalization, yet shareholders would not find it in their interest to recapitalize. The benefits from the debt buyback are due to the reduction of bankruptcy costs. However, with debt already in place, all of the benefits produced by a debt buyback accrue to debt holders. Since shareholders are unable to appropriate any of the gains due to reduced

¹³ Indeed, resistance to recapitalization is often justified by appealing to transactions costs or lemons costs associated with equity issues (see e.g. XXX). Of course, such explanations do not explain the failure of recapitalizations via rights offerings or when firms have cash available to pay out as dividends, whereas Proposition 1 immediately applies.

bankruptcy costs, and since they must buy back the debt at a price that reflects the reduced risk of debt holders after the buyback, shareholders will resist a recapitalization.

The observation that shareholders resist a recapitalization even when it would raise the value of the firm stands in contrast to the standard tradeoff theory of capital structure, where firms choose their debt levels so as to maximize total firm value given the countervailing frictions of tax benefits and distress and agency costs associated with leverage. In the standard tradeoff theory, where capital structure decisions are taken *ex ante*, before any debt has been issued, shareholder value maximization and firm value maximization lead to the same results. However, once there is debt overhang, shareholder value maximization and firm value maximization may be in conflict as shareholders do not take sufficient account of the effects of their choices on debt holders.¹⁴

The consequences of debt overhang in the context of recapitalization are stronger than those in the context of equity-financed investment as described in Myers (1977). When a firm must issue equity to undertake a valuable project, the loss to the shareholders due to the wealth transfer to risky debt holders brought about by the reduction in leverage can be more than offset by the positive net present value (NPV) of the project, a portion of which the shareholders capture. Thus, if the NPV of the project is large enough, Myers's underinvestment problem disappears, and the outcome is efficient. By contrast, when a debt buyback would increase the total firm value, debt overhang always results in a loss of efficiency. Again, no matter how large the gain in firm value, shareholders will always resist the recapitalization.

Matters would be different if there were collective bargaining about the price of debt in the buyback.¹⁵ For example, if debt contracts had collective action clauses, the firm's management, acting on behalf of shareholders, could negotiate a buyback agreement with debt holder representatives. In such negotiations, and with the no-buyback outcome as a default option, debt holders would end up sharing their gains from the buyback with the shareholders. This sharing of gains cannot be achieved in a market buyback. And even in a negotiation, if debt holders are dispersed, holdouts could be likely. In other words, at terms for which shareholders would not resist a recapitalization, we would expect (at least some) debt holders to resist, precluding a purely voluntary leverage reduction.

The difference between a buyback through collective bargaining and a buyback through the market is due to the fact that the buyback through the market itself raises the market price.

¹⁴ This point is central to the literature on dynamic theory of capital structure, see for example Strebulaev and Whited (2012). However, despite its name, this literature is more concerned with the dynamics of default and investment decisions for a given capital structure than with the evolution of capital structure through new issues and repurchases of debt and equity. Moreover, leverage changes are often restricted exogenously; e.g. Bhamra et al. (2010, p. 1499) state "In common with the literature, we assume that refinancings are leverage increasing transactions since empirical evidence demonstrates that reducing leverage in distress is much costlier."

¹⁵ In a different setting the impact of collective bargaining on debt dynamics is also noted by Strebulaev and Whited (2012).

Return prospects per unit of debt improve, because the default probability goes down and because, in the event of default the available asset value, net of bankruptcy costs, is split among fewer claimants. Because return prospects improve, the market price of the debt must go up. An exception to this rule occurs only if the buyback has no effect on the default probability and, in the event of default, debt holders do not get anything.¹⁶

Our results thus far establish the resistance of shareholders to pure recapitalizations for all equity-based sources of funding. In Section 5 we will consider leverage reduction modes that involve either asset sales or the acquisition of new assets using equity funding. In that context, we also discuss the compatibility of leverage reduction with limited liability of existing shareholders.

3. Leverage and Agency Costs

Our analysis thus far has focused on the tradeoff between tax benefits of leverage and bankruptcy costs. Bankruptcy costs alone, however, are not the only potential detrimental consequence of leverage for firm value. In this section we consider agency costs of debt overhang, and find an important feedback: Agency costs increase both the benefit of, and resistance to, recapitalizations. Moreover, existing leverage distorts future leverage decisions to further exacerbate debt overhang.

3.1. Investment Distortions Increase Resistance

It is well known that the presence of leverage in the firm creates debt-equity conflicts related to investment. In particular, leverage may induce equity holders to increase the risk of the firm's assets via asset substitution (as in Jensen and Meckling, 1976), or to fail to undertake new investment opportunities (as in Myers, 1977). Indeed, these costs are often presumed to be even more significant than explicit bankruptcy costs in the determination of optimal leverage from the perspective of tradeoff theory.

In this section we generalize our analysis to allow for both asset substitution and underinvestment. These agency frictions raise the cost of leverage for total firm value, and thus increase the potential benefit of a recapitalization. Yet we will show that despite this benefit, future debt-equity conflicts only *increase* shareholder resistance to any recapitalization.

To see the intuition for this result, consider first the case of asset substitution. Suppose the distribution of asset returns, x , may be affected by actions taken by shareholders (or

¹⁶ A recent example of this effect can be seen in the buyback of Greek debt in 2012. When about one half of the debt was repurchased. The market price in August 2012, before the news about the buyback plans transpired, was about 17c per euro of debt, whereas the buyback was transacted in different batches between 30c and 40c per euro. This was an almost exact replication of the Bolivian experience of 1988, which was documented and analyzed by Bulow and Rogoff (1990). In contrast, in the Mexican debt buyback in 1990, collective bargaining seems to have been used to prevent the private creditors from obtaining any windfalls; see van Wijnbergen (1991).

managers acting on behalf of shareholders). We denote these actions by θ , and the resulting asset returns by x_θ , which has distribution $F(x|\theta)$. In this setting, it is natural to extend our notation and define the value of equity as follows:

$$\begin{aligned} V^E(D, A) &\equiv \max_\theta V^E(D, A, \theta) \\ &= \max_\theta \int_{D/A}^{\infty} (xA - t(x, A, D) - D) dF(x|\theta) \end{aligned} \quad (7)$$

We assume in (7) that the actions θ are taken to maximize the value of equity. Let θ^* be the action choice at the target level of debt, $D - \Delta$, i.e.,

$$\theta^* \equiv \arg \max_\theta V^E(D - \Delta, A, \theta) \quad (8)$$

To see that asset substitution increases shareholder resistance to a recapitalization, note that

$$\begin{aligned} V^E(D - \Delta, A) - V^E(D, A) &= V^E(D - \Delta, A, \theta^*) - \max_\theta V^E(D, A, \theta) \\ &\leq V^E(D - \Delta, A, \theta^*) - V^E(D, A, \theta^*) \end{aligned} \quad (9)$$

Thus, the increase in the value of equity post-recapitalization is even smaller now than in the setting without agency costs (that is, with θ fixed at θ^* , the level of risk that shareholders would choose given lower leverage).

As the above argument reveals, the result that agency costs increase shareholders' resistance to recapitalization follows directly from their most basic consequence for the equity value function. Thus, we can apply the same argument to demonstrate that *any* shareholder discretion over future firm investment will lead to a similar result.

For example, suppose that in addition to determining asset risk θ , management (on behalf of shareholders) has the opportunity to invest in additional assets a by raising capital k from shareholders (or reducing planned equity payouts). Moreover, suppose these decisions will be made at a later date and conditional on some future information z that is relevant to both asset returns and the profitability of the investment opportunity. Specifically, letting $k(a, z)$ be the cost of making investment a given information z , the equity value function *conditional* on the investment policy functions $a(z)$ and $\theta(z)$ can be written as

$$\begin{aligned} V^E(D, A, \theta, a) &\equiv \\ E_z \left[\int_{D/(A+a(z))}^{\infty} (x(A+a(z)) - t(x, A+a(z), D) - D) dF(x|z, \theta(z)) - k(a(z), z) \right] \end{aligned} \quad (10)$$

Equity holders choose the policies (θ, a) to maximize (10) given outstanding debt D .

In this case, in addition to asset substitution, leverage may lead to future underinvestment due to the traditional debt overhang problem identified by Myers (1977). The next result demonstrates that, once again, the possibility of future underinvestment and risk shifting, while detrimental to total firm value, will only increase the cost to shareholders from a current recapitalization.

Proposition 3 (Agency Costs): *Although shareholder-creditor conflicts regarding investment may raise the benefits of a leverage-reducing recapitalization for total firm value, they also raise the costs of a recapitalization for shareholders relative to a setting in which investments were fixed at the optimal policy given lower leverage.*

Proof: See [appendix](#). ■

The intuition for **Proposition 3** follows the same logic as in (9): Agency costs mitigate the decline in the value of equity as leverage increases, as shareholders take actions that transfer wealth from creditors.¹⁷ But this effect implies that equity holders also gain less from a leverage reduction, and they must pay more for the debt in anticipation that such wealth transfers will diminish. Thus, even though agency costs raise the cost of leverage, they impede shareholders incentive to reduce it.

3.2. Leverage Distortions: The Ratchet Effect

The standard “tradeoff theory” of capital structure posits that firm’s choose debt in order to maximize total firm value given the countervailing frictions of tax benefits and distress and agency costs associated with leverage. Our prior results suggest, however, that once leverage is already in place, debt overhang will create a powerful dynamic that will distort shareholder incentives. In particular, we show that not only will the shareholders not choose to reduce leverage, they will *always* prefer to *increase* leverage if they have the opportunity to do so, and even if this additional leverage further reduces firm value. In other words, leverage begets additional leverage, creating a leverage ratchet effect.

The observation that shareholders can gain by issuing new debt that has equal (or higher) priority to its existing debt is well known, and results from the fact that the new debt will dilute the claims of existing creditors. This problem of sequential borrowing is often presumed to be eliminated via strict seniority rules, so that any new debt issued must be junior to existing debt claims.¹⁸ However, as Bizer and DeMarzo (1992) have shown, additional borrowing with *junior* debt can still be detrimental to more senior claims because of its influence on future firm actions, such as risk-shifting, underinvestment, or earlier default. In other words, by increasing future

¹⁷ Essentially, we follow the proof of **Proposition 1** state-by-state conditional on z .

¹⁸ See for example, Fama and Miller (1972), who stress this point. Brunnermeier and Oehmke (2013) show that a similar effect arises if shareholders can issue new debt with a shorter maturity than existing debt, as its earlier maturity gives it effective seniority.

agency costs, new junior debt can harm existing senior creditors. Equity holders do not internalize this harm, distorting their decision to engage in additional borrowing. See Figure 1. Hence, as we will show below, *an additional agency cost of leverage is that it will distort future leverage decisions of the firm.*

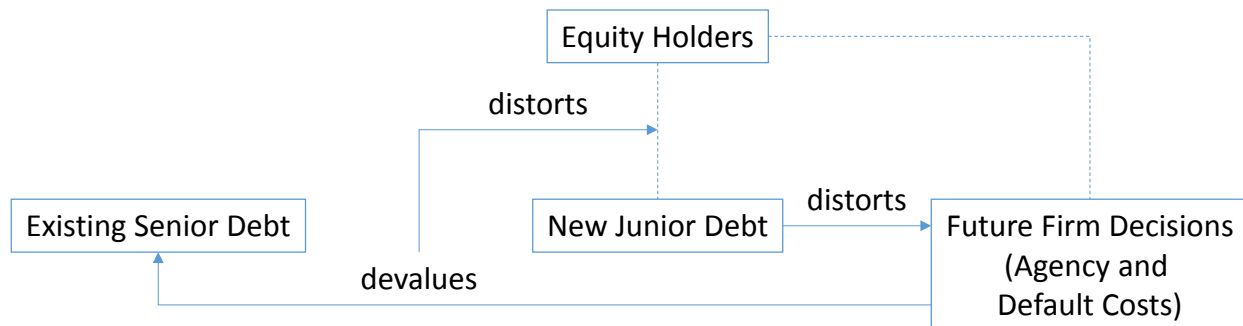


Figure 1: Sequential Borrowing Agency Distortions

To demonstrate the leverage ratchet effect, consider our setting with taxes, default costs, and asset substitution, and suppose that any existing debt is protected so that any new debt issued is junior to all other outstanding debt claims. This assumption avoids any motivation to issue new debt in order to directly dilute existing creditors. Nonetheless, we show that under broad conditions, a levered firm will *always* find it optimal to increase its leverage. To formalize this result, let $G(D, D')$ be the gain to shareholders when a firm with existing debt D increases its debt to $D' \geq D$, by issuing new junior debt with face value $D' - D$:

$$G(D, D') = V^E(D', A) - V^E(D, A) + (D' - D)q^J(D, D', A) \quad (11)$$

where $q^J(D, D', A)$ is the price at which the new junior debt is sold. Then we have the following key result:

Proposition 4 (Leverage Ratchet Effect): *Given initial debt D , suppose the firm has the opportunity to adjust its debt on a one-time basis. Then,*

- *If the firm has no initial debt, then the amount of debt D to issue that maximizes shareholders' gain $G(0, D)$ also maximizes the total value of the firm.*
- *If the firm has outstanding debt $D > 0$, shareholders never gain by reducing leverage. Moreover, if the probability of default is continuous at D and the marginal expected tax benefit of debt is positive, it is always optimal for shareholders to increase leverage by issuing new debt ($\arg \max_{D'} G(D, D') > D$), even if this new debt must be junior to existing claims, and even if it reduces total firm value.*

Proof: See [appendix](#). ■

The first statement in Proposition 4 is obvious – absent pre-existing debt, shareholders internalize any costs to creditors via the price they will receive for the new debt, and hence will choose leverage to maximize total firm value. This observation is the basis for the standard optimality prediction of the tradeoff theory.

The second statement in Proposition 4, however, makes clear that when there is a marginal tax benefit from debt, this prediction must fail if the firm makes new leverage decisions once existing debt is in place. It implies that even if the firm is already excessively levered (relative to the tradeoff theory optimum), equity holders will still be tempted to increase leverage further.

While the proof of this result is somewhat complicated by technicalities related to differentiability and continuity, the intuition is straightforward. All of standard *costs* associated with leverage in the tradeoff theory – default costs, investment-related agency costs, and even distortions of future leverage choices – are optimality decisions in the hands of equity holders (e.g., equity holders optimally determine when to exercise their put option to default). Thus, by a standard envelope argument, these costs have no first order impact on the value of new debt, and thus the only first order effect is the incremental tax benefit. Intuitively, from (7), letting θ be the optimal level of future investment decisions by shareholders given debt D ,

$$\begin{aligned}\frac{\partial}{\partial D} V^E(D, A) &= 1 - F(D/A|\theta) - \int_{D/A}^{\infty} t_D(x, A, D) dF(x|\theta) \\ &= q'(D, D, A) - \int_{D/A}^{\infty} t_D(x, A, D) dF(x|\theta)\end{aligned}\tag{12}$$

and thus from (11),

$$G_2(D, D) = - \int_{D/A}^{\infty} t_D(x, A, D) dF(x|\theta) > 0.\tag{13}$$

That is, the marginal gain from new incremental debt is equal to the associated incremental tax shield.¹⁹ Thus, independent of the amount of debt already in place, shareholders always have a positive incentive to increase debt further until its interest tax shields are fully exploited.

We illustrate the result of **Proposition 4** in Figure 2, which shows $G(0, D)$ and $V^E(D)$ for different levels of debt D . If the firm is initially unlevered, equity holders would prefer debt D^* that maximizes total enterprise value (equity plus debt). Once debt D^* is in place however, equity holders face the potential gain $G(D^*, D)$, and so would ideally choose D^{**} if given a one-

¹⁹ We note in passing that this observation provides some justification for the standard practice in capital budgeting valuation to consider only incremental tax benefits associated with new debt while ignoring the impact of bankruptcy and agency costs. Once existing debt is in place this approach correctly captures the marginal impact to the firm's shareholders.

time opportunity to issue new junior debt. Note that total enterprise value is lower at D^{**} , but equity holders gain because the value of the senior debt declines.

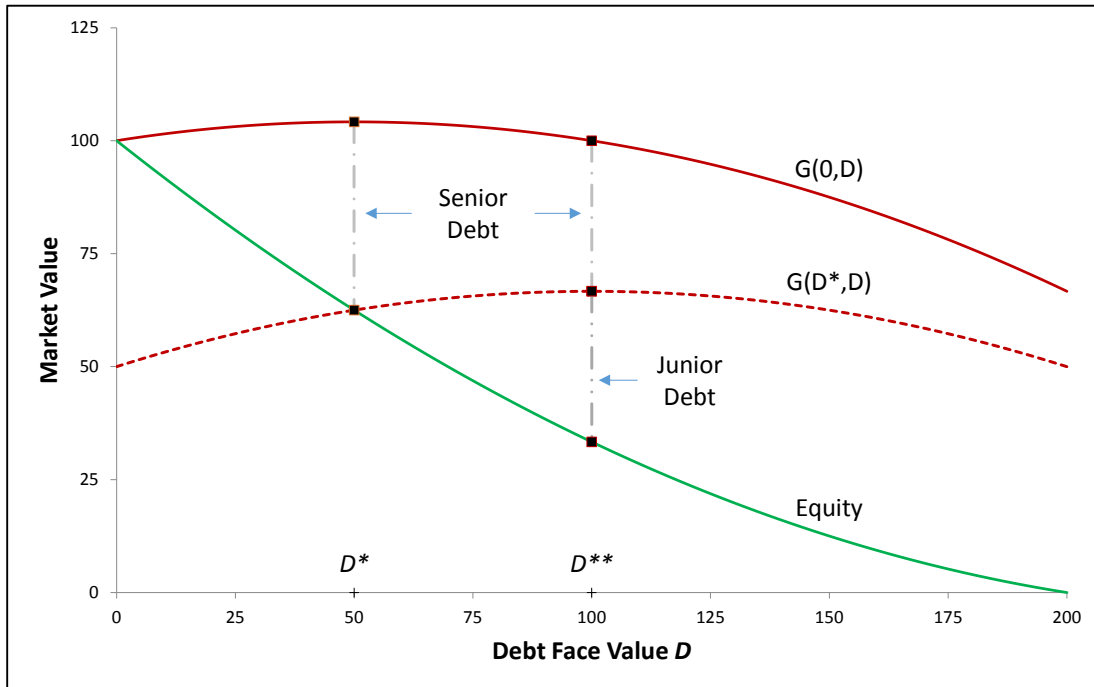


Figure 2: The Leverage Ratchet Effect

4. Leverage Ratchet in Dynamic Equilibrium

Our results thus far demonstrate that under shareholder control, leverage is “irreversible” once put in place and moreover creates a desire for even more leverage. Our analysis, however, has been restricted to one-time static adjustment of the firm’s debt level. In a dynamic context, creditors will include the cost of such future distortions in the price they are willing to pay for the debt upfront. Furthermore, this price adjustment will no doubt affect the firm’s optimal initial leverage choice. In this section we develop a simple and tractable dynamic model to explore and highlight key consequences of the leverage ratchet effect.

We begin in Section 4.1 by developing a simple stationary dynamic model of a levered firm, in which the firm earns taxable income at a fixed rate until the random arrival of an exit event. Because interest on debt is tax deductible, the firm has a tax benefit from leverage, but we assume debt distorts the payoff at exit either due to bankruptcy or agency costs. We then illustrate the leverage ratchet effect in the context of this model and consider the equilibrium debt choice when the firm has repeated opportunities to issue debt in Section 4.2. In Section 4.3

we allow for shocks to the income and value of the firm and show how these shocks will lead leverage to increase over time. Finally, in Section 4.4 we discuss the potential role of covenants in mitigating the leverage ratchet.

4.1. A Simple Dynamic Tradeoff Model

Suppose the firm generates earnings before interest and taxes at a constant rate y until the arrival at random time τ of a liquidation event. (This setting could correspond, for example, to an investment in a fixed income security or portfolio.) The interest rate r , liquidation arrival rate λ , and tax rate t , are constant, and the firm issues debt with fixed face value D and constant coupon rate c such that $cD \leq y$. The value of the firm at liquidation is given by Y_θ , which is independent of τ , and where the parameter θ reflect investment or strategy choices that are chosen to maximize the value to equity holders. In that case, the value of the firm's equity is given by²⁰

$$\begin{aligned} V^E(D) &= \max_\theta E \left[\int_0^\tau e^{-rs} (y - cD)(1-t) ds + e^{-r\tau} (Y_\theta - D)^+ \right] \\ &= \frac{r}{r+\lambda} \frac{(y-cD)(1-t)}{r} + \frac{\lambda}{r+\lambda} \max_\theta E[(Y_\theta - D)^+]. \end{aligned} \quad (14)$$

For $D \geq 0$, we define the function

$$\phi(D) \equiv \max_\theta E[(Y_\theta - D)^+] \quad (15)$$

to represent the expected payoff to equity in liquidation, and note that by standard arguments ϕ must be nonnegative, strictly decreasing and weakly convex in D for $\phi > 0$, with $\phi'(0) \geq -1$. For technical convenience we assume Y_θ is continuously distributed and ϕ is twice differentiable. Finally, we write $\theta(D)$ to represent the argmax in (15).

We assume that in the event of default, bankruptcy costs are such that debtholders are unable to recover any of the asset value and receive a payoff of zero. This assumption simplifies but is not necessary for our results (and we will develop shortly an equivalent setting without bankruptcy costs). Given a fixed face value, the total value of debt is therefore

$$V^D(D) = E \left[\int_0^\tau e^{-rs} cD ds + e^{-r\tau} D \mathbb{1}[Y_{\theta(D)} \geq D] \right] = \frac{r}{r+\lambda} \frac{cD}{r} + \frac{\lambda}{r+\lambda} D \Pr(Y_{\theta(D)} \geq D). \quad (16)$$

From (15) and the fact that Y_θ is continuously distributed,

²⁰ Here we use that fact that given arrival rate λ , $E[e^{-r\tau}] = \lambda / (r + \lambda)$.

$$\phi'(D) = -\Pr(Y_{\theta(D)} \geq D) = -\Pr(Y_{\theta(D)} > D). \quad (17)$$

Hence the debt has a price per dollar of face value of

$$p(D) = \frac{r}{r+\lambda} \left(\frac{c}{r} \right) - \frac{\lambda}{r+\lambda} \phi'(D). \quad (18)$$

The above setting includes both moral hazard (in the choice of θ) and bankruptcy costs (given a zero recovery rate). We note however, that for any function ϕ , there exists a corresponding pure agency model as well as a pure bankruptcy cost model, described as follows:

1. *Pure Agency*: With probability θ , the firm has a successful exit with value $g(\theta)$, and is worthless otherwise. That is, $Y_\theta \in \{0, g(\theta)\}$ and $\Pr(Y_\theta = g(\theta)) = \theta$. In this case,

$$\phi^A(D) = \max_{\theta \in [0,1]} \theta(g(\theta) - D)^+.$$

2. *Pure Bankruptcy Costs*: The liquidation value is independent of θ ; that is, $Y_\theta = Y$. However, in the event of default, debtholders recover nothing. In this case,

$$\phi^B(D) = E[(Y - D)^+].$$

In the context of our model, these two settings are isomorphic – debtholders receive nothing in default, and equity holders receive an expected payoff of $\phi(D)$ in liquidation. The following result demonstrates that we can construct the corresponding exit value g , or distribution for Y , to match any payoff function ϕ :

Proposition 5 (Equivalence of Agency and Bankruptcy Costs): *Given any ϕ from (15), there exists an exit value g in the pure agency model, and a distribution for Y in the pure bankruptcy cost model, such that $\phi = \phi^A = \phi^B$.*

Proof: See [appendix](#). ■

Example 1: Consider a pure bankruptcy cost model in which Y is uniformly distributed on $[0, \bar{Y}]$. Then

$$\phi(D) = \frac{(\bar{Y} - D)^2}{2\bar{Y}} \quad (19)$$

Applying the construction in the proof of **Proposition 5**, this payoff function is equivalent to a pure agency model with

$$g(\theta) = \bar{Y} \left(1 - \frac{1}{2}\theta \right),$$

so that higher probabilities θ of success are associated with lower success outcomes $g(\theta)$, but higher expected payoffs $\theta g(\theta)$. ■

We assume the firm exhausts the tax benefits of debt if the coupons cD exceed the cashflows y , or equivalently if $D > \bar{D} \equiv y/c$. Given initial debt $D \leq \bar{D}$, we can calculate the gain to equity holders from a permanent change to debt level $D + d \leq \bar{D}$ as follows:²¹

$$\begin{aligned} G(D, D+d) &= V^E(D+d) - V^E(D) + dp(D+d) \\ &= \underbrace{\frac{r}{r+\lambda} \frac{tcd}{r}}_{\text{Incremental Tax Shield}} - \underbrace{\frac{\lambda}{r+\lambda} (\phi(D) - (\phi(D+d) - d\phi'(D+d)))}_{\text{Incremental Agency/Bankruptcy Costs}}. \end{aligned} \quad (20)$$

Note that by the concavity of ϕ , the incremental agency or bankruptcy costs borne by shareholders (via the debt price) are nonnegative for any choice of d (positive or negative). Thus, as in **Proposition 4**, they are second order at $d = 0$, so that we have the following analog to (13):

$$G_2(D, D) = \frac{tc}{r+\lambda} \mathbb{I}[D < \bar{D}]. \quad (21)$$

In other words, the gain from a marginal dollar of new debt is simply equal to the value of its incremental tax shield. The optimal quantity of new debt can be found by setting $G_2(D, D+d) = 0$ which implies $d = \min(d^*, \bar{D} - D)$ where

$$d^* = \frac{tc}{\lambda \phi''(D+d^*)}. \quad (22)$$

Thus, the optimal incremental debt increases with the tax shield and decreases with the likelihood of liquidation and the intensity of the agency or bankruptcy costs (measured by ϕ''). Using the quadratic parameterization in **Example 1** with $\bar{Y} > \bar{D}$, we have

$$d^* = \frac{tc}{\lambda} \bar{Y}. \quad (23)$$

4.2. Stable Leverage without Commitment

We have shown that if shareholders are granted a one-time opportunity to issue debt, they will always have an incentive to issue new debt (even if the new debt is junior to any existing

²¹ Recall that because recovery rates are zero, this calculation applies independent of the priority of the new debt. In other words, the incentives we are identifying apply even if existing debtholders can enforce seniority with respect to any new debt.

claims) up to the point that tax shields are exhausted. But if shareholders are unconstrained in their ability to issue junior debt, and so have this opportunity repeatedly, creditors will recognize this incentive and price the debt appropriately in anticipation of future leverage changes. In this case, it is unclear what level of leverage will be sustained in equilibrium.

To analyze this possibility, we consider the following dynamic game. At each time s , the firm has existing debt D_s . It then announces a new quantity of junior debt to issue (or repurchase). The price p_s of the incremental debt is set competitively in the market; in equilibrium, of course, this price will reflect creditors' anticipation of the firm's future leverage choices given the new debt level. While this new debt must be junior to any existing debt, it is otherwise unconstrained. Absent commitment, what will be the equilibrium leverage of the firm?

To determine the equilibrium leverage choice, consider first the case in which $D_s \geq \bar{D}$. In that case, the firm has exhausted the debt tax shield, and has no incentive to issue additional debt. From our earlier results in Section 2, equity holders will also not choose to repurchase debt. Thus, a debt level \bar{D} or higher is "stable" – once it is attained, equity holders will not benefit from any change.

Next, suppose D_s is such that equity holders will gain by adjusting debt to \bar{D} . That is suppose $G(D_s, \bar{D}) > 0$. Then it is clear that D_s cannot be a stable outcome, as equity holders would gain by issuing new debt until \bar{D} is reached. Buyers of the new debt would be willing to pay $p(\bar{D})$ since they know the firm will not change debt from that level once it is attained.

Finally, suppose the current debt level D_s is the maximal debt level below \bar{D} such that $G(D_s, \bar{D}) \leq 0$. Then equity holders could not gain by issuing any new debt, since if they did, by the previous logic creditors would anticipate that they would keep issuing until the debt had face value \bar{D} . But at the price $p(\bar{D})$, issuing new debt would not be profitable for shareholders.

Repeating this logic, we obtain the following construction for an equilibrium. There exists a set of stable debt levels, $D^0 > D^1 > \dots$, defined recursively by $D^0 = \bar{D}$ and

$$D^{n+1} = \max\{D < D^n : G(D, D^n) \leq 0\}. \quad (24)$$

Note that given the continuity of G , (24) implies $G(D^{n+1}, D^n) = 0$; that is, equity holders are just indifferent between sequential stable debt levels. We then have the following equilibrium, in which the firm's leverage "ratchets up" to the next stable debt level:

Proposition 6 (Stable Leverage Equilibrium): *The following strategies represent a subgame perfect dynamic leverage equilibrium: Given any debt level D , shareholders immediately increase leverage to the next highest stable leverage level D^n , defined as $D^+(D) \equiv \min\{D^n : D^n \geq D\} \cup \{D : D \geq D^0\}$. The price of debt is given by $p(D^+(D))$.*

Proof: See [appendix](#). ■²²

Example 2: Let ϕ be as in **Example 1** with $\bar{Y} \geq \bar{D}$. Then for $D \leq \bar{D}$, $G(D - \hat{d}, D) = 0$ implies

$$\hat{d} = 2tc\lambda^{-1}\bar{Y} = 2d^*. \quad (25)$$

Therefore $D^n = \bar{D} - n\hat{d}$. We illustrate an example in Figure 3, with $t = 40\%$, $r = c = 5\%$, $\lambda = 10\%$, $y = 10$, and $\bar{Y} = 220$. Note that tax shields are exhausted with debt level $\bar{D} = y/c = 200$.

From (23), the debt level D^* that maximizes $G(D, D^*)$ is $D^*(D) = D + 44$. From (25), the debt level \hat{D} that makes equity holder indifferent so that $G(D, \hat{D}) = 0$ is $\hat{D}(D) = D + 88$.²³ The stable debt levels are therefore $D^0 = \bar{D} = 200$, $D^1 = 200 - 88 = 112$, and $D^2 = 112 - 88 = 24$. Figure 3 plots D , $D^*(D)$, $\hat{D}(D)$ and finally the stable equilibrium $D^+(D)$ which is a step function showing the jump to the next stable point.

With these parameters, given no initial debt, the firm would choose $D^+(0) = 24$, which is lower than the firm value maximizing level $D^*(0) = 44$. Note that if cash flows were lower so that $y = 8$, however, then $D^0 = \bar{D} = 160$, and $D^+(0) = 72$. In that case, the firm would choose higher leverage than the firm value maximizing level (which is still 44). ■

²² The subgame perfect equilibrium in **Proposition 6** need not be unique. In particular, because equity holders are indifferent between debt D^n and D^{n-1} , mixing between these choices is possible, or the firm could move from D^n to D^{n-1} after some period of time. That said, the equilibrium we describe is the unique subgame perfect equilibrium given pure strategies in D . Moreover, if debt quantities are discrete (e.g. there is a minimum increment to the face value of \$1), then the equilibrium we describe is generically unique (since then $G(D^n, D^{n-1}) < 0$).

²³ $\hat{D}(D) = D + 88$ up to the point $D = D^1$. For $D > D^1$, because there are no tax shields above D^0 ,

$\hat{D}(D) = D + \sqrt{(D^0 - D) \times 88}$.

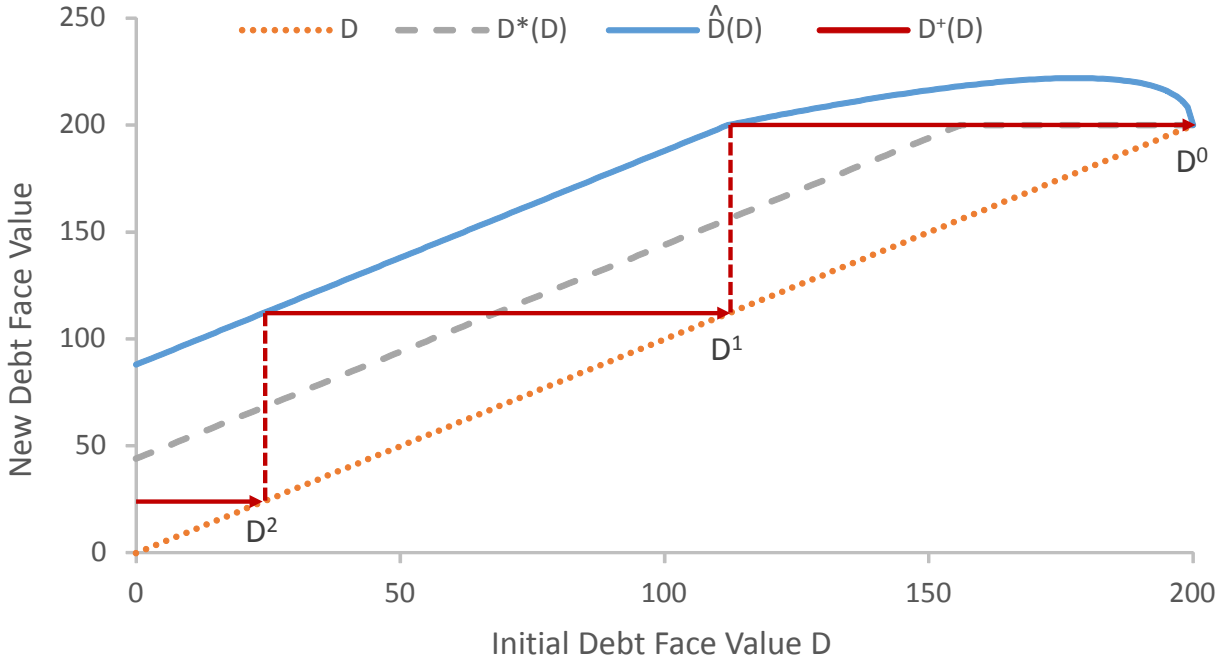


Figure 3: Stable Leverage Equilibrium

As **Example 2** illustrates, even starting with zero leverage, absent commitment the leverage ratchet effect can dramatically distort the firm’s initial debt choice away from the tradeoff theory optimum. Comparing (25) with (23), regardless of the firm’s initial debt level, we see that the firm’s equilibrium adjustment may result in it being over-levered or under-levered by up to 100% relative to d^* .

Of course, the equilibrium described in **Proposition 6** is unnaturally stark. In particular, the result that the firm makes a one-time adjustment to its debt and then debt remains stable from that point onward depends upon our assumption that all of the parameters of the firm are stationary over time. More realistically, we should expect the firms’ cash flows, likelihood of and value in liquidation, as well as macro factors such as interest rates and tax rates, to be time varying. In that case, permanently stable debt levels as described in **Proposition 6** cannot be expected, as we demonstrate next.

4.3. Shocks and Leverage Ratchet Dynamics

Thus far, we have allowed the firm to adjust its leverage in an environment without shocks. In that case, once a stable leverage level is attained, there is no reason for the firm to adjust leverage further. However, suppose the firm is now subjected to shocks to its cash flows, tax rate, or the intensity of bankruptcy or agency costs. How will the firm’s leverage respond to these shocks?

Intuitively, starting from any stable debt level prior to the shock, it is unlikely that this debt level will remain stable after the shock. Thus, as in our prior analysis, we would expect the firm to increase leverage to the next stable debt level given the new parameters. If shocks are repeated, then after each shock we will see leverage ratchet upward until the point that all tax shields have been exhausted or the firm defaults.

We formalize this by extending our prior model to allow for the Poisson arrival of a regime shift. We index regimes by $j \in \{1, \dots, J\}$. In regime j , tax rates, interest rates, debt coupons, and cash flows are given by (t_j, r_j, c_j, y_j) , and there is a random arrival of a liquidation event with arrival intensity λ_{j0} and payoff ϕ_j . In addition, there is an independent random arrival with intensity λ_{jk} of a shock that moves to regime k . The debt is priced and leverage decisions are made in anticipation of these potential shocks. We apply the same logic as in **Proposition 6** to establish the following:

Proposition 7 (Leverage Ratchet Dynamics): *There exists a subgame perfect equilibrium of the following form: For each regime j there will be a set of stable debt levels*

$$D_j^0 = \bar{D}_j \equiv y_j / c_j > D_j^1 > \dots > D_j^n > \dots$$

Upon entering regime j with current debt D , the firm will increase leverage to the next stable level

$$D^{j+}(D) = \min\{D_j^n : D_j^n \geq D\} \cup \{D : D \geq D_j^0\}.$$

Proof: See [appendix](#). ■

We show in the proof of **Proposition 7** how to construct the stable points for each regime. As before, starting from D_j^n , we find the next lower debt level such that equity holders would just be indifferent between not issuing additional debt, and issuing up to the point D_j^n . The equity value function and debt price are calculated given this issuance policy.

Note that over time, regime shocks will cause the equilibrium debt level to increase monotonically as the firm ratchets up to the next stable point with each transition to a new regime. As long as the stable points for each regime do not coincide, then **Proposition 7** implies that the debt level of the firm will continue to ratchet up over time. Indeed we have the following immediate result:

Corollary (Limit Values): Suppose the regimes are recurrent (i.e., starting from any regime there is a positive probability of ultimately transitioning to any other regime). Then starting from debt level D , the debt level will increase over time until the next universal stable point in the set $\cap_j \{D_j^n\}$, or, if no such point exists, to $\bar{D} \equiv \max_j \bar{D}_j$.

Example 3: Consider a setting as in **Example 2** with two regimes that differ in terms of the firm’s cash flow stream with $y_1 = 10 > y_2 = 8$. Both regimes share the same (t, r, c, λ, ϕ) as in **Example 2** and Figure 3. Then we can show that the stable points for each regime will be distinct, and will alternate in magnitude, with the distance between them shrinking with the frequency of the regime shifts.

Figure 4 shows the stable points for each regime when the arrival intensity of a regime shift is $\lambda_{12} = \lambda_{21} = 66.7\%$ or on average every 1.5 years. In this example, starting with no leverage, the firm’s initial debt choice is 2 (if cash flows are low) or 10 (if cash flows are high). Then, with each subsequent change in the level of the firm’s cash flow, debt will “ratchet up” to the next stable point shown in the figure, until $D = 180$; with that level of debt the tax shields are exhausted for the low cash flow firm and it is a common stable point. ■

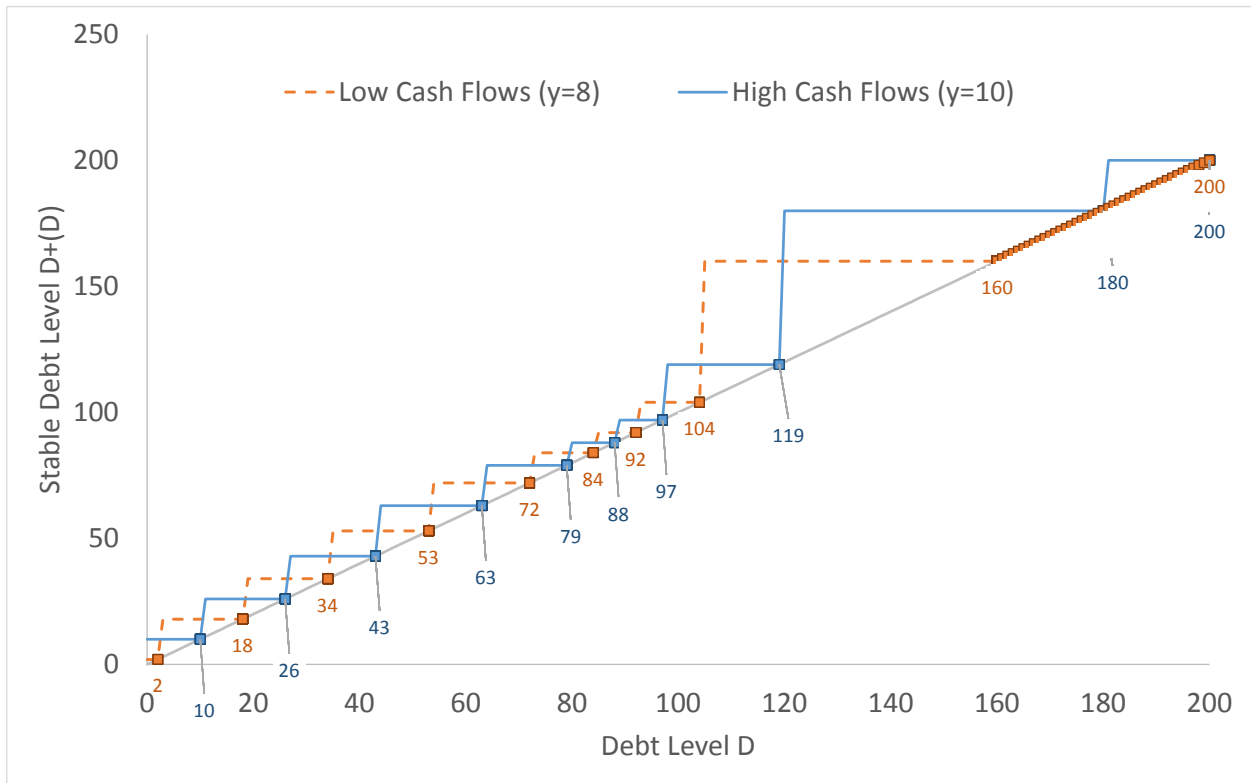


Figure 4: Leverage Ratchet Dynamics

The preceding example confirms the intuition that when there are fluctuations in factors that affect the costs or benefits of leverage, the leverage ratchet effect will induce shareholders to repeatedly “ratchet up” the leverage of the firm. (Similar results are obtained, for example, if the tax rates or default rates fluctuate over time.) Naturally, in anticipation of future ratchets, the equity holders limit the amount of additional leverage they are willing to take on today.

4.4. Leverage Ratchet and Imperfect *Ex Ante* Commitment

We have shown that leverage ratchet means that shareholders will not voluntarily reduce leverage, even if leverage reduction would increase total firm value. Instead, we have demonstrated that shareholders will prefer to repeatedly ratchet up the firm’s leverage in response to shocks. While equity holders will limit their initial use of leverage in anticipation of this future behavior, absent some form of commitment the progression to severe debt overhang seems all but assured.

Debt maturity and organic asset growth may be two forces that can help to offset the ratchet effect. Debt maturity provides an *ex ante* commitment to recapitalize the firm. Due to the leverage ratchet effect this commitment is very valuable. Indeed, extremely short maturity debt would effectively force shareholders to reevaluate leverage from the position of an unlevered firm. The tradeoff, of course, is that short maturity may also expose the firm to rollover risk or “debt runs” and thereby introduce other costs.²⁴ Asset growth provides an alternative mechanism of involuntary debt reduction, but one which is not reliable; indeed the leverage ratchet is likely to be most costly in the aftermath of a decline in asset values.

Creditors who understand that they may be subsequently harmed by the leverage ratchet can insist on debt covenants aimed at preventing shareholder actions that harm their interests (e.g. caps or restrictions on future debt issuance).²⁵ Imposing a fixed limit on debt or leverage, however, will not be sufficient to restore the tradeoff theory prediction in a dynamic context. To see why, suppose leverage is initially fixed to maximize firm value. As shocks occur to the firm and the economy over time, however, this debt level will soon be suboptimal. If optimal leverage decreases, shareholders will resist a leverage reduction, whereas if it increases, debt covenants will bind.

Of course, covenants may include some degree of shareholder discretion over leverage, or more likely, may be subject to renegotiation. But absent complete contracts, debt holders must recognize that shareholders will exercise their discretion or renegotiate the debt terms in an

²⁴ See, e.g. He and Xiong (2012). Diamond and He (2014) also show that short maturity may exacerbate debt overhang.

²⁵ Note that the common restriction that any new debt must be junior to existing creditors is insufficient to prevent the costs associated with the leverage ratchet effect. As examples thus far illustrate, even the issuance of junior debt can harm existing creditors (via default and agency costs).

asymmetric manner – increasing leverage when the opportunity arises, but not reducing leverage even if doing so would be value enhancing.

The asymmetry in shareholder leverage decisions has implications for the *ex ante* choice of debt. First, the leverage ratchet effect suggests that initial debt will trade for a lower price, as debt holders internalize the possibility of future value-destroying leverage increases combined with shareholder resistance to value-enhancing leverage reductions. This price effect will induce firms to take on less leverage initially.

The leverage ratchet effect has clear implications for leverage dynamics. It suggests that firms may have asymmetric responses to shocks in the environment that impact optimal leverage, such as changes in tax rates. Increases in the value of the debt tax shield should induce increases in leverage, but reductions in the value of the tax shield would not cause a similar fall in leverage. Such asymmetry has been documented empirically by Heider and Ljungqvist (2014).

Fundamentally, our results suggest that naïve tests of “tradeoff theory” are likely to fail in empirically. The future leverage will depend on historical debt choices, interacted with both covenants and other frictions, and driven by the asymmetric preferences of shareholders.

5. Alternative Ways to Reduce Leverage

In the paper thus far, we have consider only pure recapitalizations as a means to change leverage, holding the (real) assets of the firm fixed. But a pure recapitalization is not the only method available to change leverage. When shareholders are forced to reduce leverage, either due to covenants or regulation, or chooses to increase assets (if still consistent with covenants or regulation), leverage can be changed (assuming shareholders either must do it or choose to do it without violating covenants) through adjustments to the scale of the firm’s assets via either of the following transactions:

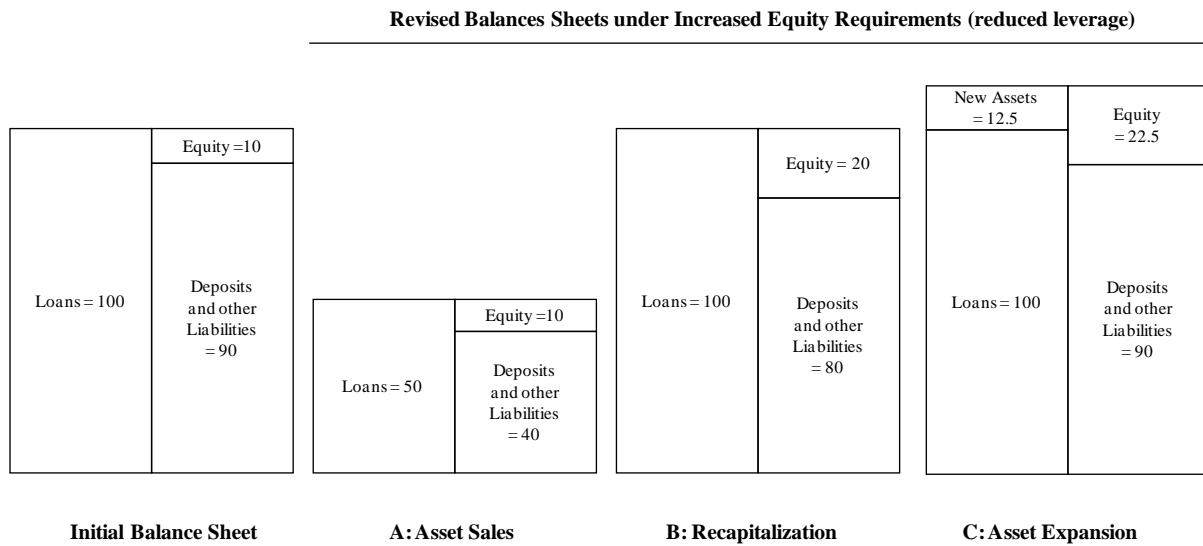
- **Asset Sales:** The firm sells assets and uses the proceeds to repurchase debt, thus lowering leverage without issuing new equity, or making payouts to shareholders, thus increasing leverage without issuing new debt.
- **Asset Expansion:** The firm issues equity and uses the proceeds to buy additional assets, thus reducing leverage without repurchasing debt, or issues debt and buys additional assets, thus increasing leverage without making payouts to shareholders.²⁶

²⁶ Asset expansion was the subject of the original analysis of debt overhang in Myers (1977). Myers shows that because existing debt holders capture some of the benefit of the new investment via reduced credit risk, shareholders may refuse to undertake a new positive NPV investment project.

One important question is whether our finding of shareholder attitudes to leverage changes through a pure recapitalization also applies to these alternative ways to adjust leverage, and what method shareholders would prefer if they have a choice.

In this section and the next one we analyze shareholders' attitude to different ways to adjust leverage that involve changes in the asset scale or composition. First, in this section, we focus on the case in which leverage is reduced. The analysis will be adapted to the case of leverage increases in the next section. The different alternatives to reduce leverage are illustrated in Figure 4. In this figure, we assume that the ratio of debt to assets must be reduced from 90% to 80%. This can be achieved by selling half of the firm's assets (asset sales), by issuing equity equal to 10% of the firm's assets and using the proceeds to buy back debt (recapitalization), or by issuing equity equal to 12.5% of the firm's assets and using the proceeds to invest in new assets (asset expansion). The figure exhibits shows how the firm's balance sheet evolves under each of these alternatives.

Figure 5: Alternative Responses to Increased Equity Requirements



In Admati et al. (2013) we observed that stricter capital requirements do not force banks to shrink as in (A) but can also be met either through recapitalization (B) or asset expansion (C). We now consider the incentives that shareholders have in choosing one course of action over the others. We first identify conditions under which all leverage reduction modes are equally undesirable to shareholders, and then relax these conditions to see how the choice of leverage reduction mode depends on various frictions associated with the types of assets or transactions involved.

5.1. An Equivalence Result

The different approaches to reducing leverage result in different sizes (assets levels) for the firm. Let D_0 be the current face value of debt and A_0 be the level of assets for the firm, so that $\delta_0 = D_0 / A_0$ is its current debt-asset ratio. Suppose that firm is required to reduce its debt-asset ratio to $\delta_1 < \delta_0$. If the firm can choose any combination of debt and assets (D_1, A_1) satisfying this debt-asset ratio – i.e., such that $D_1 = \delta_1 A_1$ – which combination will shareholders prefer?

If $A_1 \neq A_0$, then assets will be either sold or purchased as part of the leverage reduction. We assume first that the assets are perfectly homogeneous, so that each unit of the assets today will generate a payoff of \tilde{x} in the future. (We comment on the more general case of asset heterogeneity in Section 5.2.3 below.) We also assume that the frictions we have considered that are related to taxes and net bankruptcy costs are homogenous with firm size. Letting $\delta = D / A$, we assume that for all (A, D) , we have

$$t(x, A, D) = t(x, 1, \delta)A \quad \text{and} \quad n(xA, D) = n(x, \delta)A. \quad (26)$$

In addition, we assume that if agency costs due to asset substitution exist, they are also homogeneous with respect to firm size. In particular this means that

$$\theta^* \equiv \arg \max_{\theta} V^E(D, A, \theta) = \arg \max_{\theta} V^E\left(\frac{D}{A}, 1, \theta\right) \quad (27)$$

for all (A, D) .²⁷

Using the expressions for the value of debt and equity in Section 3, we see that when the assets and frictions (including those due to asset substitution) are homogeneous, the total value of the firm (equity plus debt) is proportional to its asset holdings and is given by:

²⁷ To keep the focus on how shareholders' preferences across the various modes of leverage reduction are related to changes in firm size, we do not consider the agency costs due to the Myers (1977) underinvestment problem that we discussed above. To consider the role that these underinvestment agency costs would play in the shareholders' choice among the three ways to reduce leverage, we would need to make specific assumptions about how new investment opportunities are related to the size of the firm as given by assets in place.

$$\begin{aligned}
V^F(A, D) &= \int_{\delta}^{\infty} xA - t(x, A, D) dF(x, \theta^*) + \int_0^{\delta} xA - n(xA, D) dF(x, \theta^*) \\
&= \left[\int_0^{\infty} x dF(x, \theta^*) - \int_{\delta}^{\infty} t(x, 1, \delta) dF(x, \theta^*) - \int_0^{\delta} n(x, \delta) dF(x, \theta^*) \right] A \\
&\equiv v(\delta)A
\end{aligned} \tag{28}$$

where $\theta^* = \arg \max_{\theta} V^E(\delta, 1, \theta)$.

The homogeneity of the firm's assets also implies that the average price of the firm's debt, which we denote by $q(\delta)$, depends only on the leverage ratio $\delta = D/A$:

$$\begin{aligned}
q(\delta) &= q\left(\frac{D}{A}\right) \\
&= \frac{V^D(D, A)}{D} = \int_{D/A}^{\infty} dF(x, \theta^*) + \int_0^{D/A} \frac{xA - n(xA, D)}{D} dF(x, \theta^*) \\
&= \int_{\delta}^{\infty} dF(x, \theta^*) + \frac{1}{\delta} \int_0^{\delta} (x - n(x, \delta)) dF(x, \theta^*)
\end{aligned} \tag{29}$$

Recall from Section 2 that if the firm has a single class of debt outstanding, it will be forced to pay the price $q(\delta_1)$ to repurchase its outstanding debt in the market (as this price is the value of the debt to a bondholder who refuses to tender). Thus, to reduce its debt level to $D_1 \leq D_0$, the firm must spend $q(\delta_1) \times (D_0 - D_1)$ on debt repurchases.

Assume that the firm is a price taker in the asset market and the price at which the firm can buy or sell assets is p . It follows that to move from initial balance sheet positions (D_0, A_0) to the new balance sheet positions (D_1, A_1) with $D_1 \leq D_0$, the value of equity the firm must issue is:

$$\text{New Value of Equity Issued} = N = p \times (A_1 - A_0) + q(\delta_1) \times (D_0 - D_1) \tag{30}$$

The total change in the firm's equity value from the transaction is given by:

$$\text{Change in Total Equity Value} = \nabla V^E = V^E(D_1, A_1) - V^E(D_0, A_0) \tag{31}$$

We can therefore determine the effect of the leverage change on existing shareholders by subtracting (30) from (31). Specifically, the gain or loss for existing shareholders is given by $\nabla V^E - N$.

We are now in a position to evaluate the effect on existing shareholders from alternative methods of reducing leverage. Recall that in a pure recapitalization, there is no change to the firm's assets ($A_1 = A_0$). With pure asset sales, all reductions in debt are financed by asset sales, so that $N = 0$. In a pure asset expansion, no debt is repurchased so that $D_1 = D_0$.

We can ask whether shareholder losses differ across these or other intermediate scenarios. As one would expect, the answer depends, among other things, on the relation between the price of the assets and their expected rates of return. Recall from (13) that

$$v(\delta_1) = \int_0^\infty x dF(x, \theta^*) - \int_{\delta_1}^\infty t(x, 1, \delta_1) dF(x, \theta^*) - \int_0^{\delta_1} n(x, \delta_1) dF(x, \theta^*), \quad (32)$$

is the expected payoff of the assets net of taxes and of (net) default costs. If $p = v(\delta_1)$ then, conditional on the final debt-asset ratio being equal to δ_1 , buying or selling assets does not affect the value of equity, i.e., from the perspective of shareholders, the Net Present Value (NPV) of asset sales and purchases is zero. If $p < v(\delta_1)$ then the NPV of asset purchases is positive, and if $p > v(\delta_1)$ then the NPV of asset sales is positive. Notice that, in this comparison, the NPV of asset sales and purchases depends on the debt-asset ratio because the debt-asset ratio affects taxes and (net) default costs.

We begin with the following benchmark result, which assumes the asset price $p = v(\delta_1)$. (The firm's behavior at other values of the asset price will be considered in the next subsection.) In this case, given homogenous assets and liabilities, shareholder losses are equivalent for all forms of leverage reductions:

Proposition 8 (An Equivalence Result): *Assume that $p = v(\delta_1)$, there is only one class of debt, and the firm faces no transactions costs in buying or selling assets or the securities it issues. Then shareholders find pure recapitalization, asset sales, and asset expansion equally undesirable. Specifically, starting from the initial position (D_0, A_0) , shareholder losses are equal to $(q(\delta_1) - q(\delta_0)) \times D_0 + (v(\delta_0) - v(\delta_1)) A_0 > 0$ for all (D_1, A_1) with $D_1 = \delta_1 A_1 \leq D_0$.*

Proof: After the change, the total value of equity will be:

$$V^E(A_1, D_1) = v(\delta_1) A_1 - q(\delta_1) D_1. \quad (33)$$

Therefore,

$$\nabla V^E = (v(\delta_1) A_1 - q(\delta_1) D_1) - (v(\delta_0) A_0 - q(\delta_0) D_0). \quad (34)$$

Thus, the total change in value for existing shareholders is

$$\begin{aligned}
\nabla V^E - N &= (\nu(\delta_1)A_1 - q(\delta_1)D_1) - (\nu(\delta_0)A_0 - q(\delta_0)D_0) \\
&\quad - p(A_1 - A_0) - q(\delta_1)(D_0 - D_1) \\
&= (\nu(\delta_1) - p)A_1 - (q(\delta_1) - q(\delta_0))D_0 - (\nu(\delta_0) - p)A_0 \\
&= -(q(\delta_1) - q(\delta_0))D_0 - (\nu(\delta_0) - \nu(\delta_1))A_0.
\end{aligned} \tag{35}$$

Since this does not depend on either A_1 or D_1 , it is the same for all changes that lead to a given reduction in the leverage ratio, proving the result. ■

As an immediate corollary, this proposition implies that, under the given conditions, shareholders will resist leverage reductions through asset sales or asset expansion just as they resist leverage reductions through pure recapitalization. The leverage ratchet effect that we discussed in the preceding section occurs regardless of what mode of leverage adjustment might be chosen.

At first sight, the result is perhaps surprising, but the intuition is straightforward. If asset and security sales or purchases have zero NPV, they cannot change the total value of the firm. Because debt holders gain from the decline in leverage, the shareholders must lose an equal amount. The gain for debt holders is determined by the change in the average price of the debt, which depends only on the change in the firm's leverage ratio. All of this is captured in the first term in the last line of (20). The second term represents losses on the value of existing assets due to changes in tax benefits, bankruptcy costs or subsidies resulting from the reduction in leverage.²⁸

If the reduction in leverage is mandated by regulation, against the shareholders' will, the question arises whether the regulation is compatible with limited liability of shareholders. For the move from (D_0, A_0) to (D_1, A_1) to be compatible with limited liability of existing shareholders, we must have $N \leq V^E(D_1, A_1)$; that is, the amount of equity raised cannot exceed the market value of the firm's equity after the change – as this value is the maximum value of the claim that can be given to new investors. The following result shows that, under the assumptions of Proposition 8, the validity of this condition is independent of whether the reduction of leverage occurs through asset sales, pure recapitalization or asset expansion.

Proposition 9 (Limited Liability): *Under the assumptions of Proposition 8, a move from (D_0, A_0) to (D_1, A_1) with $D_1 = \delta_1 A_1 \leq D_0$ is compatible with limited liability of existing shareholders if and only if $\nu(\delta_1) \geq q(\delta_1)\delta_0$.*

²⁸ Note however, that while the *expression* for shareholder losses is natural, it is not obvious (by inspection) that its sign should be positive. As remarked in Section 2, the standard intuition is that the sign will depend on the relative magnitude of the efficiency gains (capture by the change in ν) and the debt gains (captured by the change in q). Again, Proposition 1 implies that the latter will always dominate.

Proof: Compatibility with limited liability of shareholders requires that

$$V^E(A_1, D_1) = v(\delta_1)A_1 - q(\delta_1)D_1 \geq N = p \times (A_1 - A_0) + q(\delta_1) \times (D_0 - D_1),$$

which is equivalent to

$$v(\delta_1)(A_0 + A_1 - A_0) \geq p \times (A_1 - A_0) + q(\delta_1) \times (D_0),$$

or,

$$v(\delta_1) \geq q(\delta_1) \times \delta_0 + (p - v(\delta_1)) \times \left(\frac{A_1 - A_0}{A_0} \right) \quad (36)$$

which leads to the condition $v(\delta_1) \geq q(\delta_1)\delta_0$ when $p = v(\delta_1)$. For a pure asset sale, we also need to check that the firm can deleverage without raising new equity; that is, there exists $A_1 \in [0, A_0]$ such that

$$p \times (A_0 - A_1) = q(\delta_1) \times (D_0 - \delta_1 A_1).$$

Solving for A_1 we have

$$A_1 = \frac{(p - q(\delta_1)\delta_0)}{(p - q(\delta_1)\delta_1)} A_0, \quad (37)$$

which is in the range $[0, A_0]$ if and only if $p = v(\delta_1) \geq q(\delta_1)\delta_0$. ■

Because $q(\delta_1) \leq 1$, a sufficient condition for the leverage reduction to be feasible is $pA_0 \geq D_0$, which is the conventional condition for assessing the firm to be solvent. Under this condition, a leverage reduction can always be achieved via an asset sale from Eq. (37), and can be achieved under limited liability for any mechanism if, as we have assumed here, $v(\delta_1) = p$.

5.2. Shareholder Preferences for Different Modes of Leverage Reduction

In many settings, the conditions under which Proposition 8 holds are violated, and shareholders have a preference for one mode of leverage reduction over the others. We discuss in this section some of the major factors that can invalidate the equivalence result and lead to a firm's managers (acting in the interest of the firm's shareholders) choosing one action over the others.

5.2.1. Divergence of Internal and External Asset Values

Proposition 8 concerns the case in which $p = v(\delta_1)$. In other words, we assume that the price at which the firm's assets can be bought or sold is precisely equal to the value of the assets to the firm's investors when the leverage ratio is δ_1 . What can we say about shareholder preferences at other prices? In this analysis, we begin by taking the asset price p as parametrically given, without considering whether it is consistent with market equilibrium. This corresponds to the standard approach of analyzing the behavior of price-taking agents by considering their demand and supply choices at *any* parametrically given prices. We will introduce equilibrium considerations once we discuss the parametric analysis.

If $p > v(\delta_1)$, the market price of assets exceeds the value of those assets when held by the firm. If $p < v(\delta_1)$, the firm can increase shareholder value by purchasing assets at the market price and holding them. The change in shareholder value is:

$$\nabla V^E - N = -(q(\delta_1) - q(\delta_0))D_0 - (v(\delta_0) - v(\delta_1))A_0 - \eta(A_1 - A_0), \quad (38)$$

where $\eta = p - v(\delta_1)$. The third term shows that shareholders will prefer reducing leverage through asset sales when $\eta > 0$. Moreover, a pure asset sale may be feasible even if a pure recapitalization (or asset expansion) is not.²⁹ When $\eta < 0$, on the other hand, shareholders prefer asset purchases, and asset purchases may be feasible when a pure recapitalization is not.³⁰

Taking the asset price as given is justified if the individual firm or bank can be thought of as a price taker operating in a large market. However, when we consider what occurs when there is a policy change that affects a large number of firms, e.g., an increase in bank capital requirements, we must recognize that the price-taking assumptions may no longer be justified. Even though an individual firm acting alone may be justified in taking the market price of assets as given, when all firms change their behavior in response to changes in regulatory requirements, it can be expected that the equilibrium market price will change.

For example, in the case of banking regulation, assume that the initial capital requirements correspond to the debt-asset ratio δ_0 and that, for this debt-asset ratio, the equilibrium asset price is equal to $p_0 = v(\delta_0)$, the price at which banks with the debt-asset ratio δ_0 are just indifferent about their asset holdings. Now suppose capital requirements are tightened, so that leverage must fall to δ_1 , and that, because of a reduction in tax benefits and

²⁹ Specifically, a pure asset sale only requires $p \geq q(\delta_1)\delta_0$ whereas a recap requires $v(\delta_1) \geq q(\delta_1)\delta_0$.

³⁰ An asset expansion is feasible iff $v(\delta_1) \geq q(\delta_1) \times \delta_0 + \eta \times \left(\frac{\delta_0}{\delta_1} - 1 \right)$.

subsidies net of bankruptcy costs, we have $v(\delta_1) < v(\delta_0)$. Then, at the price $p_0 = v(\delta_0)$, all banks want to respond to the new requirement by selling assets to buy back debt. Unless there are third parties willing to hold assets at this price, the asset price $p_0 = v(\delta_0)$ will no longer clear the market. The new equilibrium price of the asset must be lower. Indeed, if there are no third parties willing to hold the assets, the new equilibrium price must fall to $p_1 = v(\delta_1)$, as we are assuming in Proposition 8. Furthermore, while a bank might initially appear solvent with $p_0 A_0 \geq D_0$, if it is the case for some banks that $p_1 A_0 < D_0$, these banks may only be revealed to be insolvent through their inability to recapitalize and satisfy the new requirements.

Throughout our discussion, we have assumed that the leverage regulation involves a debt-asset ratio D/A , which is fixed without regard to current market prices. In practice, regulations such as bank capital requirements are often based (at least to some extent) on current values, imposing an upper bound on a ratio such as $q(\delta_1)D/p_1A$ or D/p_1A . The first corresponds to a ratio based solely on market values, the second corresponds to a case where assets are marked to market but debt levels are measured at the face value of liabilities. All of our results continue to apply with either of these measures (as they simply represent rescaling of the target leverage ratio). Note, however, that if δ_1 has to be equal to either $q(\delta_1)D/p_1A$ or to D/p_1A , then, because $q(\delta_1) > q(\delta_0)$ and $p_1 < p_0 = v(\delta_0)$, the deleveraging effect is larger than it would be if δ_1 had to be equal to D/A . That is, when the leverage ratio is based on market values, rather than quantities, the effect of deleveraging is exacerbated.

5.2.2. Multiple Classes of Existing Debt

In this section we consider shareholder preferences when not all debt has the same priority. We continue to assume that the assets returns and the frictions are perfectly homogenous with firm size, but we now assume that the firm has multiple classes of *existing* debt with different levels of priority. In this case, if $D_1 < D_0$, it is optimal for the firm to repurchase the most junior debt first, as it will be the least expensive. The price at which junior debt can be repurchased depends on the precise capital structure of the firm (as well as any default costs or subsidies). Without going into the details of this dependence, we note that the price q^j at which junior debt can be repurchased should satisfy

$$\int_{\delta}^{\infty} dF(x|\theta^*) \leq q^j < q(\delta). \quad (39)$$

where, as above, $\theta^* = \arg \max_{\theta} V^E(\delta, 1, \theta)$. The lower bound in (39) reflects the fact that the price of the junior debt should be no less than the probability that the firm does not default, since

in that case it will be repaid. The strict inequality for the upper bound follows as long as seniority “matters” in the sense that there exist some states of the world in which junior debt holders have lower recovery rates in default than more senior creditors.

The fact that junior debt is cheaper to repurchase breaks the indifference obtained in Proposition 8. Now, shareholders will be better off the more junior debt that is repurchased. In particular, we have the following important result:

Proposition 10 (Multiple Classes of Existing Debt): *Assume $p = v(\delta_1)$ and (39) holds. Then,*

- i. If the firm can repurchase junior debt, shareholders find asset sales preferable to a pure recapitalization, which in turn is preferable to an asset expansion.*
- ii. In the case of asset expansion, the ability to purchase junior debt makes no difference since no debt is repurchased.*
- iii. In the case of a pure recapitalization the shareholders lose less with the ability to repurchase junior debt than they lose when there is only one debt class, but they still lose.*
- iv. In the case of asset sales, shareholders may gain if the reduction in leverage is sufficiently small.*

Proof: As before we have $\nabla V^E = (v(\delta_1)A_1 - q(\delta_1)D_1) - (v(\delta_0)A_0 - q(\delta_0)D_0)$, but given the lower cost q^j of repurchasing the junior debt, the total change in value for existing shareholders is:

$$\begin{aligned}
\nabla V^E - N &= (v(\delta_1)A_1 - q(\delta_1)D_1) - (v(\delta_0)A_0 - q(\delta_0)D_0) \\
&\quad - p(A_1 - A_0) - q^j(\delta_1)(D_0 - D_1) \\
&= (v(\delta_1) - p)A_1 - (v(\delta_0) - p)A_0 - (q(\delta_1) - q(\delta_0))D_0 \\
&\quad + (q(\delta_1) - q^j(\delta_1))(D_0 - D_1) \\
&= -(q(\delta_1) - q(\delta_0))D_0 - (v(\delta_0) - v(\delta_1))A_0 + (q(\delta_1) - q^j)(D_0 - D_1)
\end{aligned} \tag{40}$$

Note that for a pure asset expansion, we have $D_0 = D_1$, and thus the loss to shareholders in (40) is identical to that in the case of a single debt class. However, this loss is reduced in the case of a recapitalization or of an asset sale, since in that case $(q(\delta_1) - q^j)(D_0 - D_1) > 0$.

While shareholders’ losses are smaller in a recapitalization, we know from that shareholders still lose even if they can repurchase the junior debt at the minimal price in (39).

Next, we show that asset sales are preferable to a recapitalization. Because $p = v(\delta_1)$, i.e. the asset is priced fairly, it suffices to compare the repurchase prices for the most junior classes of debt in the two cases. Because the firm’s final leverage ratio is the same, under our homogeneity

assumption the probability of default is the same in either case. Thus, the only change to the payoff to a debt holder who “holds out” is that, as more debt is repurchased, the proportion of the remaining debt that is senior to it will (weakly) increase. As a result, the repurchase prices of the most junior debt classes will (weakly) decrease as their relative seniority, and therefore their expected recovery rates, declines. Thus, because more debt is repurchased at the same or lower price under an asset sale versus a recapitalization, shareholders will prefer an asset sale.

To show that shareholders may gain with asset sales if they can repurchase junior debt, it is sufficient to consider the case in which there are no frictions. To simplify notation we normalize the initial asset level to be $A_0 = 1$ and similarly assume (without loss of generality) that the price at which the assets will be sold is normalized to 1 so that $p = \int_0^\infty x dF(x) = 1$. Consider now a decrease in leverage from δ_0 to $\delta_1 (< \delta_0)$ accomplished through asset sales. Shareholders will gain if

$$\nabla V^E = A_1 \int_{\delta_1}^\infty (x - \delta_1) dF(x) - \int_{\delta_0}^\infty (x - \delta_0) dF(x) > 0 \quad (41)$$

The derivative of (41) with respect to (δ_1, A_1) at the point $(\delta_1, A_1) = (\delta_0, A_0)$, we have

$$dA_1 \int_{\delta_0}^\infty (x - \delta_0) dF(x) - d\delta_1 \int_{\delta_0}^\infty dF(x) \quad (42)$$

Recall that for a pure asset sale, the value of the assets sold must equal the cost to repurchase the debt, so that

$$1 - A_1 = (\delta_0 - \delta_1 A_1) q^J \quad (43)$$

where q^J is the average price of the junior debt repurchased. Taking the derivative of (43) at the point $(\delta_1, A_1) = (\delta_0, A_0)$, we have

$$q^J d\delta_1 = (1 - q^J \delta_0) dA_1 \quad (44)$$

Combining (42) and (44), we have

$$\begin{aligned} dA_1 \int_{\delta_0}^\infty (x - \delta_0) dF(x) - d\delta_1 \int_{\delta_0}^\infty dF(x) &= dA_1 \left[\int_{\delta_0}^\infty (x - \delta_0) dF(x) - (1/q^J - \delta_0) \int_{\delta_0}^\infty dF(x) \right] \\ &= dA_1 \left[\int_{\delta_0}^\infty x dF(x) - (1/q^J) \int_{\delta_0}^\infty dF(x) \right] \end{aligned}$$

Because $dA_1 < 0$, the derivative above is positive and shareholders initially gain from asset sales if the following expression is positive:

$$\int_{\delta_0}^\infty dF(x) - q^J \int_{\delta_0}^\infty x dF(x) \quad (45)$$

Now, the value of the junior debt can be written

$$q^J = \int_{\delta_0}^\infty dF(x) + \alpha \int_0^{\delta_0} x / \delta_0 dF(x)$$

where $\alpha \in [0, 1]$ is the expected recovery rate of the junior debt relative to average recovery rate of the firm’s debt (which is strictly positive given our assumption that F has full support). If the

debt is fully prioritized so that all debt repurchased is junior to any debt retained, then $\alpha = 0$, whereas if the debt is pari passu then $\alpha = 1$ and $q^J = q$. Substituting this value for q^J in (45), we get

$$\begin{aligned} & \int_{\delta_0}^{\infty} dF(x) \left(1 - \int_{\delta_0}^{\infty} x dF(x) \right) - \alpha \int_0^{\delta_0} x / \delta_0 dF(x) \int_{\delta_0}^{\infty} x dF(x) \\ & = \int_{\delta_0}^{\infty} dF(x) \int_0^{\delta_0} x dF(x) \left(1 - \frac{\alpha}{\delta_0} E[x | x > \delta_0] \right) \end{aligned} \quad (46)$$

where we use the fact that $E[x] = \int_0^{\infty} x dF(x) = 1$. Thus, (46) is positive and shareholders gain from an asset sale if the debt repurchased is sufficiently junior so that its relative recovery rate satisfies $\alpha < \delta_0 / E[x | x > \delta_0]$. ■

This result explains why shareholders may choose to engage in asset sales or “deleveraging” (as opposed to recapitalization or asset expansion) if a decrease in leverage is imposed by regulation and there are no covenants protecting senior debt holders. While it is well known that an asset sale that is used to fund payouts to equity holders will benefit shareholders at the expense of creditors, our result (iv) above states that equity holders can gain even if the proceeds are used to buy back junior debt. While total leverage declines and junior creditors gain, senior debt holders lose even more (as their claims are backed by a smaller pool of assets). If allowed, shareholders therefore prefer this form of deleveraging over a pure recapitalization or asset expansion.

Note that in our analysis of asset expansion we have assumed that $A_1 = D_0 / \delta_1$ so that $D_1 = \delta_1 A_1 = D_0$. Increasing assets further would necessitate issuing new debt in order to achieve the target leverage ratio δ_1 . If this new debt could be issued at an equal priority to the firm’s existing debt (so that it would command the same average price), asset expansion with $A_1 > D_0 / \delta_1$ will be no more costly than it is with $A_1 = D_0 / \delta_1$. In many cases, however, any new debt would be required to be junior to the existing debt. In this case, it would command a lower price, and additional asset purchases beyond $D_0 \times (1 / \delta_1 - 1 / \delta_0)$ would impose further losses on shareholders. In other words, we have the following straightforward extension of Myers (1977) debt overhang result:

Proposition 11 (Asset Expansion with Additional Debt): *Assume $p = v(\delta_1)$. If $D_1 = \delta_1 A_1 > D_0$ then:*

- i. *shareholders are indifferent to any choice of A_1 if the new debt is of equal seniority to existing debt;*

- ii. if new debt must be junior to existing debt, then shareholders are worse off choosing $A_1 > D_0 / \delta_1$; and
- iii. if new debt can be senior to existing debt, then choosing $A_1 > D_0 / \delta_1$ makes shareholders better off.

Proof: By the same logic as in (40), the impact on shareholders' payoff associated with increased asset purchases which are funded by increasing debt beyond D_1 is given by

$$(q^{New} - q(\delta_1))(D_1 - D_0)$$

where q^{New} is the average price of the new debt issued. If new debt is equal priority to existing debt, then $q^{New} = q(\delta_1)$ and shareholder are indifferent to any choice of A_1 and $D_1 = \delta_1 A_1$. But if new debt is junior to existing debt and $D_1 = \delta_1 A_1 > D_0$, then $q^{New} < q(\delta_1)$ and shareholders are worse off. Alternatively, if new debt is senior to existing debt, $q^{New} > q(\delta_1)$ and shareholders gain by choosing $D_1 > D_0$, effectively by usurping the priority of the initial creditors. ■

This result extends Proposition 8 by showing that that irrelevance to scale continues to hold if new debt is of equal seniority to existing debt. Shareholders would not choose to expand if any of the new debt issued must be junior to existing debt. An interesting case is one where the new debt can be senior to existing debt. This case might be relevant for financial institutions, which rely on significant amounts of short term debt. Short term debt is effectively senior to the bank's long-term debt.³¹ Proposition 11 suggests that shareholder losses are *decreasing* in the scale of the firm in this case. This result suggests that in cases when new debt can be senior, shareholders might prefer additional asset expansion.

5.2.3. Heterogeneous Assets

Proposition 8 treats the firms' assets as though they are homogeneous, with each asset unit having return of x so that the total return on all assets is simply xA . In reality, assets are heterogeneous, with differing risk and return. Nevertheless, the results of Proposition 8 continue to apply even when assets are heterogeneous as long as any asset sales or purchases correspond to a "representative portfolio" and so have the same risk and return as the average asset in the firm.

Of course, given the option, shareholders will generally have preferences with respect to which assets to sell or purchase. If a firm deleverages through asset sales, shareholders prefer to

³¹ On the shortening of debt maturity due to lack of commitments and as a way to dilute existing creditors, see Brunnermeier and Oehmke (2013). Also relevant is the bankruptcy exemption of repos and derivatives, which can encourage excessive leverage in ways that allow creditors to take advantage of the exemptions. See Jackson and Skeel (2012) and Bolton and Oehmke (2013).

sell relatively safe assets. In contrast, they will prefer to purchase relatively risky assets if the firm expands. This preference is just another manifestation of the asset substitution agency problem that we have discussed above.

As a concrete example, suppose the firm holds a mix of risky assets and safe assets. In particular, suppose it holds quantity A_r of risky assets with return \tilde{x}_r and A_s of safe assets with a riskless return. Note that we can normalize quantities so that each “unit” of assets (risky or safe) has market price p . Thus the firm has total assets $A = A_r + A_s$ with aggregate return \tilde{x} given by

$$\tilde{x} \equiv \frac{\tilde{x}_r A_r + p A_s}{A}.$$

Suppose the firm considers reducing leverage by selling safe assets and using the proceeds to buyback debt. At the conclusion of the asset sale, the firm’s leverage ratio δ_1 satisfies

$$p a_s = q(\delta_1)(D_0 - D_1) = q(\delta_1)(D_0 - \delta_1(A - a_s)).$$

That is, the value of the assets sold must equal the value of the debt repurchased. We then have the following immediate corollary to Proposition 8, showing the equivalence of “selective” asset sales and asset substitution:

Corollary (Asset Sales and Asset Substitution): *Reducing leverage via the sale of safe assets is equivalent, in terms of shareholder payoff, to recapitalizing the firm (to the same leverage ratio) and simultaneously selling safe assets and purchasing risky ones.*

Proof: Suppose the firm first exchanges its holdings a_s of safe assets for risky ones at the market price p , and then sells those risky assets to reduce leverage through an asset sale. This transaction clearly has the same shareholder payoff as simply selling the safe assets directly. But since the firm’s assets are homogenous after the asset exchange, by Proposition 8 this has the same shareholder payoff as an asset exchange followed by a pure recapitalization. ■

Note that the equivalence between asset sales and asset substitution ignores potential transactions costs. Once these are considered, asset sales are likely to be strictly preferred, as it avoids both the need to purchase risky assets and to issue equity. We discuss additional impacts of transactions costs in Sections 5.2.4 and 5.2.5 below.

In the context of capital regulation for banks, an attempt is made under Basel II and Basel III to address the problems created by asset substitution and risk shifting. This is done by assigning risk weights to assets and formulating capital requirements in terms of the size of the risk-weighted asset base. If the risk weighting system worked perfectly and completely removed the ability of bank managers and shareholders to engage in asset substitution and risk shifting when assets are sold or purchased, asset heterogeneity would not necessarily undermine the equivalence result given in Proposition 8. In particular, if risk weighting effectively means that the value of debt depends only on leverage as measured by the risk weighting system, so that

$q(\delta_1)$ will be the same no matter what the mode of leverage reduction, then the conditions for Proposition 8 to hold are potentially restored even with heterogeneous assets.

In practice risk weighting falls short of removing the ability of banks to increase risk and engage in asset substitution. Indeed, the regulations often involve transparently inappropriate risk weights, e.g., a zero risk weight for sovereign debt or for highly rated securities even when they clearly carry some potentially significant risks. Making matters worse is the fact that in practice the implementation of the risk weighting system relies in part on the banks' own internal risk models and is therefore highly manipulable. When risk weights are imperfect, the same logic as the preceding result implies that banks will have an incentive to reduce leverage by selling assets that are safer than their risk weight implies, and holding on to assets that are riskier than their assigned weights. Again, such selective sales are another mode of asset substitution that banks may engage in when capital regulations only impose capital ratios rather than specifying the mechanism by which they should be achieved.

5.2.4. Transactions Costs

Proposition 8 is based on the assumption that the firm faces no transactions costs in changing the scale of its assets or in issuing and retiring securities. Not surprisingly the introduction of transactions costs can lead to one alternative being preferred over the others, since the three ways of changing leverage that we consider involve different pairs of transactions as shown below:

	The firm purchases:	The firm sells:
Asset Sales	Debt	Assets
Recapitalization	Debt	Equity
Asset Expansion	Assets	Equity

Asset expansion will be the preferred alternative if the transactions costs involved in repurchasing debt are particularly large relative to the other transactions, but this is unlikely to be the case. The transactions costs involved in equity issuance and asset sales are likely to be more important. If equity issuance costs are large relative to those in asset transactions, then asset sales, since they involve no equity transactions, will be the preferred alternative. If the transactions costs involved in selling assets are particularly large compared to equity issuance costs (e.g., the firm faces extreme "firesale conditions" in liquidating assets), then recapitalization or asset expansion will be preferred. Without making specific assumptions about the magnitude of the various transactions costs, little more can be said about what approach will be most advantageous for shareholders.

5.2.5. Asymmetric Information

A key component of transactions costs in settings such as the ones we are considering is due to the possibility that the firm's managers have private information about the firm's assets

and growth opportunities. Managers will want to sell assets that the market is overvaluing and similarly will want to issue equity if they perceive the market is overpricing the firm's shares. The possibility that managers will make strategic choices based on their private information can account for a significant part of the bid/ask spread for transactions involving the firm's assets and securities. Information asymmetries can be particularly important in asset sales and equity issuance and this explains why transactions costs for these are likely to be larger than those associated with debt buybacks.

Asymmetric information factors that would affect the valuation of the firm's assets in the asset sales approach clearly also give rise to asymmetric information issues affecting the market valuation of the firm's equity when the firm issues equity directly (as opposed to a rights offering) to recapitalize or expand its assets.³² It is clear that if there is asymmetric information about the value of the assets in place, there must be asymmetric information about the value of the firm's equity. If a recapitalization must be done through a new share issuance as opposed to a rights offering, it is not immediately obvious whether it will be more expensive for the firm's shareholders to sell assets and deleverage or sell equity and recapitalize.

In some circumstances asymmetric information about asset values makes the shareholders indifferent between deleveraging and recapitalizing. Assume that the market undervalues the firm's assets in the following sense: while managers know that the realized value on the firm's assets will be $\tilde{x}(1+\omega)A$ for $\omega > 0$, the market assumes that the realized value of the assets will only be $\tilde{x}A$. Essentially this means that for each asset unit that the market perceives, the firm effectively has $1+\omega$ units and this difference is perceived by the firm's managers.

Proposition 12 (Equivalence with Asymmetric Information): *Assume that there is only one class of debt, the firm faces no transactions costs in buying or selling assets or the securities it issues other than that implied by the market's undervaluation of its assets and the firm must decrease its leverage from δ_0 to $\delta_1 < \delta_0$. Then for all $\omega \geq 0$, shareholders find pure recapitalization through a common share offering and asset sales equally undesirable.*

Proof: See [appendix](#).

³² Note that in Myers and Majluf (1984) the asymmetric information that makes management reluctant to issue equity relates to the value of assets in place as well as the value of the investment opportunity the equity issuance would finance. The key assumption in the Myers-Majluf analysis is that the firm can only raise equity through an offering of common shares and not, for example, through a rights offering. With symmetric information, as in **Proposition 8**, it does not make a difference whether new equity is raised through an offering of shares to the market or through a rights offering. With asymmetric information, it does make a difference. In a sale of new shares to the market, the market's assessment of the firm directly impacts the amount of money raised by the firm. In a rights offering, if it succeeds, the market's assessment of the firm does not affect the amount of money raised by the firm, but only the value of shares and therefore the value of the rights. The attitude of existing shareholders to a rights offering then depends on whether they are short-term investors, who are interested in the current share price, or long-term investors à la Myers-Majluf, who are interested in returns and share prices in the future, when the market will have learned about the underlying values.

Asymmetric information imposes costs on the current shareholders in both the asset sale and pure recapitalization cases because the firm is selling assets or equity at prices below their values. Although a greater dollar amount of assets must be sold in the asset sales approach than the dollar amount of equity that needs to be issued to effect a recapitalization, the underpricing of equity is larger in percentage terms because of leverage, and this is just sufficient to make the loss due to underpricing the same.³³

If the firm's assets are heterogeneous, the situation involving asymmetric information becomes more complex. Transactions costs due to asymmetric information are likely to be lowest on the least risky assets. As discussed above, asset substitution considerations indicate that the shareholders will want to sell low-risk assets when deleveraging, but will want to buy high-risk assets in the asset expansion approach to reducing leverage. This means that transactions costs concerns and asset substitution will tend push shareholders toward the deleveraging alternative. With deleveraging, incentives associated with asset substitution and transactions cost minimization are aligned. This is not the case with asset expansion.

Note, however, that deleveraging is not always the preferred alternative from a transactions costs perspective. If most assets are hard to value by outsiders and managers can pick the assets they sell, then the adverse selection effects can be greater with asset sales than they are when equity is sold. This is because equity represents a claim on a portfolio of assets rather than an adversely selected subset. The transactions costs associated with issuing equity can be lower than those involved in selling hard to value assets. This could tip the balance in favor of recapitalization.

Finally, it should be noted that one way that leverage can be reduced that involves almost no transactions costs due to asymmetric information is for the firm to retain earnings and build equity "internally." Adverse selection costs can also be eliminated by raising equity through a rights offering. Shareholder resistance to these ways of reducing leverage is entirely due to debt overhang.

³³ One might wonder why the results we obtain for asymmetric information differ from those presented above in 5.2.1, where we assume that the market price for the firms' assets differs by η from the value of the assets when they are held by the firm. Since in section 5.2.1 we assume that there is symmetric information about the value of the assets when they are held by the firm, it follows that when that value differs from the market price, there is uniform agreement that the firm should either be selling assets if $\eta > 0$ and buying assets if $\eta < 0$. Whether the firm should grow or shrink is unambiguous, and this makes the preferred mode of leverage reduction depend on the *amount* of assets sold or bought. With asymmetric information the situation is quite different. When equity is issued, the price is based on the market's perception of the *total* value of the assets and any losses are due the market's undervaluation of that total. As discussed above, the same amount of assets is effectively sold at undervalued prices when equity is issued as when assets are sold directly. This means that while in the analysis of 5.2.1 the losses or gains are based on the amount of assets sold, while in the case of asymmetric information the losses are based on the market's valuation of all the assets. It does not matter whether the assets are directly sold or indirectly sold through issuance of equity — the loss is the same. A long-term investor, who is patient enough to wait until the market has learned the correct value for the assets, would however take a different view if equity were raised through a rights offering; see the preceding footnote. Such an investor would prefer the rights offering to a sale of assets.

6. Implications for Banking Theory and Policy

While, in principle, our analysis applies to corporations in all industries, it is particularly relevant for banks. Over the past century, leverage in banking has steadily increased, and today banking institutions are by far the most highly leveraged corporations in the economy. Whereas nonfinancial firms are deemed to be highly levered if their borrowing reaches seventy percent of their assets, and many of them borrow hardly at all, banks often fund significantly more than ninety percent of their assets by borrowing.

The increase of borrowing was particularly pronounced in the years before the financial crisis of 2007-2009.³⁴ Some of the increase was actually difficult to detect because it involved derivatives and because important commitments were left off the reported balances sheets of banks.

Much of bank borrowing, and much of the recent increase in bank borrowing, has involved short-term debt. Traditionally, short-term bank borrowing involved mainly deposits of retail customers. Today banks also borrow a lot through wholesale markets involving derivatives, asset-backed commercial paper, or repo contracts.

The high leverage of financial institutions played an important role in the financial crisis of 2007-2009 and meant that many of these institutions were unable to absorb the losses they suffered. Even those that did not become insolvent lost significant fractions of their equity, which induced them to deleverage by selling assets, putting further pressure on asset prices and on other institutions. Eventually, the chain reactions led to major breakdowns in funding and credit across the globe. Contagion was very intense not only because institutions were highly interconnected but also because the weakness of institutions caused them to react strongly to the adverse developments that affected them.³⁵

The academic literature on banking has sought to explain the observed high leverage as a result of optimal contracting. One line of argument focuses on the idea that, if the bank's debt is constantly in need of being rolled over, then bank managers will always be on their best behavior in order to forestall a breakdown of funding. Another line of argument focuses on the desire of investors to have assets that are "liquid" and can be turned into cash whenever they wish. As we have explained in some detail elsewhere, both lines of argument, as well as arguments using asymmetric information to justify high leverage and resistance to leverage reduction, have

³⁴ Among the twenty largest European banks, in 1998, equity of less than four percent of total assets (under IFRS accounting rules, which disallow netting) was the exception in 1998 but had become the rule by 2007. For details on this point and more, see Advisory Scientific Committee (2014)

³⁵ For detailed accounts see Hellwig (2009) as well as Admati and Hellwig (2013a, Chapter 5).

serious conceptual and empirical weaknesses that render them inadequate for explaining banks' actual behavior and for guiding policy.³⁶

By contrast, our analysis suggests that the observed high leverage of banks and the growth of this leverage over the past century may reflect the leverage ratchet rather than optimal contracting. The leverage ratchet effect suggests that banks will generally resist leverage reductions and will try to increase leverage whenever they have the opportunity to do so. And this is in line with much of what we observe.

For example, banks often seek to make payouts to shareholders in order to maintain or increase their leverage. This behavior is contrary to the "pecking order" theory of finance, which claims that retained earnings are unaffected by market frictions such as asymmetric information and are therefore the most preferred source of funding for any corporation. The banks' preference for payouts to shareholders is however fully in line with our analysis.

The observation that bank leverage has been going up over time is also consistent with our analysis, particularly in light of the ever expanding system providing "safety nets" for banks in the form of explicit and implicit guarantees of their debt. For example, in the 2000s a significant expansion of short-term bank borrowing occurred through a dramatic increase in borrowing through repo contracts. Repo borrowing, which legally is not borrowing but a combination of a sale and repurchase, is effectively a way to issue new debt ahead of any incumbent debt, jumping the queue of claimants in default, getting ahead even of depositors because the repo collateral is not available to repay them or other creditors.

The growth of repo borrowing accords with our result that, once significant leverage is in place, shareholders have an incentive to increase leverage if they need not internalize the consequences of additional leverage on existing creditors. This incentive is particularly strong if new debt can usurp the priority of existing claims.

While outright dilution may be ruled out by covenants prohibiting the issue of new debt that is senior or equal in status to incumbent debt, such covenants can be circumvented if the new debt matures earlier than the incumbent debt. Brunnermeier and Oehmke (2013) have referred to this development as a "maturity rat race". The "maturity rat race" is fully in line with the logic of our analysis. The effect is strengthened if the bank is free to secure the new debt with collateral. The collateral that is used for repo borrowing or for asset-backed commercial paper is not available to incumbent debt holders in bankruptcy.

In the case of nonfinancial corporations, many of these effects are weakened or eliminated because there are few lenders and these lenders are in a position to exert effective control, through covenants and through direct interference with borrowers' decisions if they don't like them. By contrast, banks usually get money from many lenders. These lenders tend to

³⁶ See Admati et al. (2013, Sections 5-7), Admati and Hellwig (2013a, b, c), Admati (2014, Section 4), Hellwig (2014), and Pfleiderer (2014).

be small, and none of them has the incentives or the power to control their borrower's behavior. Shareholders, or managers acting on behalf of shareholders, are therefore much more in a position to take advantage of debt holders.

The decisions that are thus taken can be and generally are *socially inefficient* even when there is no further damage to third parties (e.g., the rest of the economy). These decisions would not be taken if contracts that prohibit them were feasible to write and were enforceable. In the context of banking, the scope for writing and enforcing such contracts is very limited because, individually, debt holders are not in a position to impose their will on managers and shareholders and, collectively, they are not sufficiently well coordinated.

These concerns are reinforced when debt holders feel they are protected by a government safety net in the form of explicit and implicit debt guarantees. With such protection, creditors have weak incentives (if any) to protect themselves by *ex ante* contracting or by *ex post* monitoring and control. Investor anticipations of bank bailouts perversely encourage further risk taking and leverage which increase the value of the implicit government guarantees. The leverage ratchet effect is reinforced. In this interpretation, the observed high leverage of banks cannot be presumed to be efficient. Indeed, even when only considered from the narrow perspective of the banks' investors, it may be highly inefficient. Also the prominence of short-term debt in bank funding may reflect the maturity rat race, the desire of the borrower to use short-term borrowing as a way to jump the priority queue and the desire of lenders to protect themselves against such jumping of the priority queue by new lenders.

In contrast to the academic literature that tries to explain observed funding patterns of banks as being efficient, our analysis suggests that the observed high leverage of banks, the growth of this leverage over time, and the prominence of short-term funding should be seen as resulting from failures of commitment and therefore as highly inefficient. Regulation aimed at reducing leverage can therefore be beneficial not only for the overall economy, but even for the banks themselves.

The case for government regulation of firms in an industry usually rests on the presence of significant externalities: the decisions of firms in the industry can adversely affect third parties. In the case of banks, particularly large banks, the external effects are indeed important because the failure of a large bank can cause severe damage to the entire economy.³⁷ Here we have the added consideration that government policies reducing the force of the leverage ratchet effect can improve market outcomes for the banks and their investors themselves.

Regulatory limits on bank leverage increase the ability of banks to absorb losses and reduce the intensity of contagion in a crisis. Third-party damage from bank failures and their systemic implications are thereby contained. Moreover, from an *ex ante* perspective, the

³⁷ For example, the billions of dollars, euros or pounds that were lost by the creditors in the bankruptcy Lehman Brothers, declared in September, 2008, were dwarfed by the trillions of dollars that were lost by the subsequent disruption of economic activity.

institutions themselves may even be better off because regulatory limits on bank leverage also provide a remedy for the inability of banks to commit their future funding policies.

However, the implementation of such limits must be executed with care. Our results in Section 5 suggest that, if regulation forces a bank to decrease its leverage, shareholders will try to impose part of the cost on incumbent senior creditors or on the deposit insurance system. They can do this by selling relatively safe assets and buying back junior debt. The reduction in assets worsens the senior debt holders' prospects; moreover, the effect is stronger the safer are the assets that are sold. By focusing on junior rather than senior debt, shareholders both minimize the cost of the buying back the debt and devalue any remaining claims. In fact, shareholders might gain by this form of deleveraging if the dilution of senior creditors (and the deposit insurance system) is sufficiently large.

This prediction from our analysis is again in line with what we observe. For an example consider what happened in the fall of 2011 when European authorities mandated banks to increase their core equity up to nine percent of risk-weighted assets by June 30, 2012. Many banks responded to this by using revenues from asset sales to buy back the most junior kinds of debt they had.³⁸

Asset sales and reductions in banks' sizes are not necessarily undesirable. However, if policymakers are concerned that, if many banks are selling assets at the same time and pushing asset prices down, there may be adverse consequences for the overall economy, they should be sure to introduce the new regulation by setting targets for equity in terms of absolute amounts (derived, for example, by multiplying the new ratios with the asset positions on a fixed date) rather than ratios. In this case, banks can only fulfil the new requirements if they increase their equity, either by retaining earnings or by getting new equity funding from investors.

Banks often claim that they *cannot* raise additional equity because the supply of equity in the market is too small. Quite often, this just means that they merely do not *want* to raise equity, which is consistent with our analysis of the pervasive resistance to leverage reduction by highly leveraged firms that we studied in Sections 2-5. If a bank is profitable, it can always raise equity by retaining earnings.³⁹ A bank that is listed on a stock exchange can raise equity by selling new shares, e.g., through a rights offering. Reluctance to do so is likely to be due to the leverage ratchet effects we studied, reinforced by concerns that corporate taxes might be higher or subsidies from explicit or implicit government guarantees might be reduced if the bank replaced some debt by equity. As discussed in Admati et al. (2013, Section 4), none of these private considerations should be a concern from the perspective of public policy.

³⁸ In particular, they repurchased so-called *hybrid* debt, which before Basel III, had to some extent been counted as "capital", namely the so-called "Tier 2 Capital". The scope for counting such hybrid debt as bank capital was much reduced under Basel III.

³⁹ It is incongruous that, for fear of transition problems, Basel III gives the banks until 2018 to satisfy the new requirements but at the same time, there is no restriction on dividend payments and share buybacks.

An inability of a publicly traded bank to raise new equity can be taken by itself as evidence that the bank might be insolvent. Proposition 9 presented conditions resembling solvency tests often applied under the law for a corporation to be able to reduce its leverage. The proposition implies that if a bank satisfies this condition, it can always meet a stricter equity requirement by issuing new shares and using the proceeds to invest in tradable assets.

If a bank is in fact *unable* to raise equity, this fact in itself should be a cause for concern. It is important that hidden insolvencies should not be allowed to persist. Weak, insolvent banks have strong incentives to engage in reckless lending and risky asset purchases and “gamble for resurrection,” or avoid recognizing losses by continuing to lend to insolvent debtors while rejecting loan applications from new entrepreneurs.⁴⁰ Such banks should not be allowed to continue operating without regulatory intervention. Regulatory forbearance can exacerbate distortions in lending and increase the social costs when insolvencies are eventually dealt with.⁴¹

For the same reasons, it is important that “liquidity” supports to banks should be accompanied by measures strengthening the banks’ equity positions and reducing their leverage. Central bank lending to banks against “good enough” collateral or government support in the form of preferred equity alleviate the immediate distress, but if the claims of central banks or governments are senior to the claims of shareholders on the banks’ assets, these measures effectively increase the banks’ leverage and distort the incentives of shareholders (and managers). Distortions such as resistance to recapitalization, excessive risk taking, and underinvestment due to debt overhang, are thus exacerbated rather than alleviated by such bailouts.⁴²

7. Concluding Remarks

In this paper we analyzed an agency cost of debt that stems from subsequent capital structure choices of firms that already have debt in place. This agency cost, which we call the

⁴⁰ The Savings and Loans crisis in the US in the 1980s illustrates the first problem and the Japanese crisis of the 1990s illustrates second. Consistent with these warnings, neither the LTRO program nor TARP resulted in significant increases in (business) lending. On the LTRO program, see Acharya and Steffen (2013), who refer to the LTRO as “the greatest carry trade ever.” Anecdotal evidence that the program did not improve lending includes such stories as Louise Armistead, “ECB’s LTRO plan flops as banks cut lending,” *Telegraph*, March 28, 2012. Cole (2012) shows that banks receiving capital injections from the TARP failed to increase their small-business lending, and instead decreased their lending by even more than other banks. Additional examples of distortions in lending include real-estate loans and mortgages in the United States, Ireland and Spain, or shipping loans in Germany with loan-to-value ratios kept low by relying on historical real estate or ship prices rather than current market prices.

⁴¹ See ASC Report 01/2012 and references given there and Admati and Hellwig (2013a, especially Chapter 11)..

⁴² Examples are the European Central Bank’s Long Term Refinancing Operation (LTRO) of 2011/2012, which provided cheap loans to banks, and the Troubled Assets Relief Program (TARP) in the US of 2008-2009, which provided funding in the form of preferred equity. Acharya and Steffen (2013) see the ECB’s LTRO as a basis for the “greatest carry trade ever,” poorly capitalized banks borrowing at percent from the European Central Bank and investing the funds in their own sovereigns at four or five percent.

leverage ratchet effect, biases the shareholders of leveraged firms against reducing and towards increasing leverage..

In the absence of full commitments and complete contracts, the agency conflicts created by the presence of debt tend to increase the *ex ante* costs of funding mixes that include significant borrowing. Debt covenants that try to deal with the ratchet effect might try to address the problem, in some cases even going as far as to forbid all borrowing until the debt is paid, but such covenants may unduly reduce the subsequent flexibility of the firm.

Because it reflects an additional agency cost of debt, the leverage ratchet effect may help explain why some firms choose very low leverage. It is well known that low leverage gives firms more flexibility to take advantage of investment opportunities without constraints from covenants and helps them avoid the negative effects of subsequent debt overhang and asset substitution problems. The leverage ratchet effect strengthens this rationale for low leverage by the observation that low leverage helps firms avoid the inefficiencies associated with excessive subsequent leverage. This observation may contribute to our understanding of the so-called zero-leverage puzzle.⁴³

As we discussed, the leverage ratchet effect applies most obviously to banks and other financial institutions whose creditors, particularly depositors and other creditors who believe they will be paid by the government if not by the banks, do not constrain subsequent leverage increases through contracts. Because high leverage exacerbates the other agency costs of debt and increases the likelihood of costly default or bankruptcy, banks' high leverage is a source of inefficiency, including social inefficiency if there is collateral damage of distress and default.

Moral hazard problems associated with explicit and implicit government guarantees exacerbate these problems. Regulation that allows a form of commitment to a more efficient capital structure with lower leverage can therefore play an important role. The analysis in this paper reinforces the conclusions of Admati et al (2013) that equity requirements significantly higher than those currently considered would provide large social benefits at little if any social cost.

Politicians and regulators tend to shy away from imposing stricter capital requirements on banks. Despite narratives suggesting that the regulations have undergone major reforms, the actual changes since the final crisis have been minor. Basel III, the new international accord on banking regulation, still allows banks to fund up to 97% of their assets with debt and have as little as 3% equity to total assets.⁴⁴ This reluctance to impose stricter capital requirements on banks reflects a fear that stricter capital requirements might induce banks to reduce their lending and this would harm economic growth.

As we have seen in Section 5 of the current paper, firms that are forced to reduce leverage may prefer to do so via sales or reductions of safe assets and reductions of their most junior debt,

⁴³ See, for example, Strebulaev and Yang (2013).

rather than by raising new equity. However when banks maintain higher equity levels on a regular basis, losses represent smaller fractions of the equity. Moreover, if banks are profitable regulators can avoid undesired spillovers from asset sales by requiring that leverage reductions be achieved via retained earnings or rights offerings, thereby mitigating shareholders' ability to impose further losses on creditors when they reduce leverage.⁴⁵ Well-designed capital regulations, which take into account the effects identified in this paper, will improve the quality of bank lending, as well as the stability of bank lending and the financial system.

⁴⁵ Admati and Hellwig (2013, Chapter 11) outline how capital regulations in which equity requirements are significantly higher than current levels can be designed and implemented to achieve their objectives. Admati et al (2013, Section 9) discuss in more detail the relation between capital requirements and lending.

Appendix: Remaining Proofs

Proof of Proposition 3:

Note that the expectation in (10) is with respect to the information z . Then, using the same argument as in Proposition 1, holding the policy functions fixed,

$$\begin{aligned}
& V^E(D - \Delta, A, \theta, a) - V^E(D, A, \theta, a) = \\
& E_z \left[\Delta \times \left(1 - F \left(\frac{D - \Delta}{A + a(z)} \middle| z, \theta(z) \right) \right) + \int_{(D - \Delta)/(A + a(z))}^{D/(A + a(z))} (x(A + a(z)) - D) dF(x|z, \theta(z)) \right. \\
& \quad - \int_{(D - \Delta)/(A + a(z))}^{\infty} t(x, A + a(z), D - \Delta) dF(x|z, \theta(z)) \\
& \quad \left. + \int_{D/(A + a(z))}^{\infty} t(x, A + a(z), D) dF(x|z, \theta(z)) \right] \\
& < E_z \left[\Delta \times \left(1 - F \left(\frac{D - \Delta}{A + a(z)} \middle| z, \theta(z) \right) \right) \right] \\
& = \Delta \times \Pr \left[x_{\theta(z)}(A + a(z)) > D - \Delta \right]
\end{aligned} \tag{47}$$

As in Proposition 1, the inequality follows because shareholders forfeit their default option for final asset values between $D - \Delta$ and D , and have a higher expected tax burden. The last equality states that the increase in the value of equity per dollar of debt repurchased is less than the ex-ante probability of no default at the lower level of leverage.

The proof then follows using exactly the same argument as in (9) above. Let θ^* and a^* be the optimal risk and investment policy functions for equity holders given debt $D - \Delta$:

$$V^E(D - \Delta, A) = \max_{\theta, a} V^E(D - \Delta, A, \theta, a) = V^E(D - \Delta, A, \theta^*, a^*) \tag{48}$$

Then,

$$\begin{aligned}
V^E(D - \Delta, A) - V^E(D, A) &= V^E(D - \Delta, A, \theta^*, a^*) - \max_{\theta, a} V^E(D, A, \theta, a) \\
&\leq V^E(D - \Delta, A, \theta^*, a^*) - V^E(D, A, \theta^*, a^*) \\
&< \Delta \times \Pr \left[x_{\theta^*(z)}(A + a^*(z)) > D - \Delta \right] \\
&\leq \Delta \times q^D(D - \Delta, A)
\end{aligned} \tag{49}$$

The first inequality follows since we have fixed the investment policy functions at a level that may not be optimal with higher leverage (due to agency costs), the second follows from above, and the third follows since the repurchase price of the debt will be at least the no default

probability (and will be strictly higher if the debt has a non-zero recovery rate in any default states). ■

Proof of Proposition 4:

Note that $q^J(0, D, A) = q(D, A)$, and therefore $Dq^J(0, D, A) = V^D(D, A)$. That is, proceeds from issuing debt are equal to the total value of the firm's debt. Hence,

$$\begin{aligned} G(0, D) &= V^E(D, A) - V^E(0, A) + Dq^J(0, D, A) \\ &= V^E(D, A) + V^D(D, A) - V^E(0, A) \end{aligned} \quad (50)$$

Thus, D maximizes $G(0, D)$ if and only if it maximizes total firm value.

For the second result, note that our earlier results already establish that shareholders lose if the firm reduces debt ($D' < D$) regardless of the seniority of the debt that is repurchased. Therefore, it is enough to establish that the marginal benefit of an increase in leverage from its current level is positive. Specifically, we need to show the right-hand derivative of G at $D' = D$,

$$\left. \frac{\partial G(D, D')}{\partial D'} \right|_{D'=D_+} = \left. \frac{\partial (V^E(D', A) + (D' - D)q^J(D, D', A))}{\partial D'} \right|_{D'=D_+}, \quad (51)$$

is positive. Let θ^* be the optimal risk choice with debt level D . From the definition of V^E , and using the fact that holding the risk choice fixed at θ^* only reduces the gain to equity holders, we have

$$\begin{aligned} \left. \frac{\partial V^E(D', A)}{\partial D'} \right|_{D'=D_+} &\geq \left. \frac{\partial}{\partial D'} \left[\int_{D'/A}^{\infty} (xA - t(x, A, D') - D') dF(x|\theta^*) \right] \right|_{D'=D_+} \\ &= - \int_{D_+/A}^{\infty} dF(x|\theta^*) - \int_{D_+/A}^{\infty} t_D(x, A, D) dF(x|\theta^*) \\ &> - \int_{D_+/A}^{\infty} dF(x|\theta^*) = -\Pr(xA > D|\theta^*) \end{aligned}$$

where the final inequality follows from the assumption that tax benefits are positive.

Next, for $D' \geq D$, define $\pi(D')$ to be the proceeds raised from the new debt:

$$\pi(D') \equiv (D' - D)q^J(D, D', A) = (D' - D) \int_{D'/A}^{\infty} dF(x|\theta') + \int_0^{D'/A} (xA - n(\tilde{x}A, D') - D)^+ dF(x|\theta').$$

Then let $\hat{\pi}(D') \equiv (D' - D) \int_{D'/A}^{\infty} dF(x|\theta')$. Because $\pi(D) = \hat{\pi}(D) = 0$ and $\pi(D') \geq \hat{\pi}(D')$ for $D' > D$, we have

$$\pi'(D_+) \geq \hat{\pi}'(D_+) = \lim_{\varepsilon \downarrow 0} \frac{\hat{\pi}(D + \varepsilon) - \hat{\pi}(D)}{\varepsilon} = \lim_{D' \downarrow D} \int_{D'/A}^{\infty} dF(x|\theta') = \lim_{D' \downarrow D} \Pr(xA > D'|\theta').$$

That is, the marginal price per dollar of junior debt is at least the probability of no default (and could be higher in the presence of default subsidies). Thus we have shown

$$\left. \frac{\partial G(D, D')}{\partial D'} \right|_{D'=D_+} > \lim_{D' \downarrow D} \Pr(xA > D'|\theta') - \Pr(xA > D|\theta^*) = 0 \quad (52)$$

where for the final equality we use the fact that the probability of default is continuous at D . ■

Proof of Proposition 5:

For the pure bankruptcy cost model, define the distribution for Y such that

$$\Pr(Y \leq D) = 1 + \phi'(D) = 1 - \Pr(Y_{\theta(D)} > D) = \Pr(Y_{\theta(D)} \leq D).$$

Then we have

$$\phi^B(D) = -\Pr(Y > D) = -\Pr(Y_{\theta(D)} \leq D) = \phi'(D).$$

Because the derivatives match and they share the same limit, we have $\phi^B = \phi$.

For the pure moral hazard model, define

$$\theta(D) = -\phi'(D).$$

Next define g as

$$g(\theta(D)) = D + \frac{\phi(D)}{\theta(D)}$$

on the range of $\theta(D)$ and zero elsewhere. Then we can rewrite the shareholders' optimization as

$$\phi^A(D) = \max_{\theta} \theta(g(\theta) - D)^+ = \max_{\hat{D}} \theta(\hat{D})(g(\theta(\hat{D})) - D)^+$$

Now

$$\begin{aligned} \theta(\hat{D})(g(\theta(\hat{D})) - D) &= \theta(\hat{D})(\hat{D} - D) + \phi(\hat{D}) \\ &= \phi(\hat{D}) + \phi'(\hat{D})(D - \hat{D}) \\ &\leq \phi(D) \end{aligned}$$

where the last inequality follows from the convexity of ϕ . Hence we have $\phi^A = \phi$. ■

Proof of Proposition 6:

To verify an equilibrium, note first that given the equilibrium leverage strategy, the debt pricing is rational for creditors since the firm is expected to maintain leverage permanently at $D^n = D^+(D)$. Next note that if $D < D^n = D^+(D)$, then $G(D, D^n) > 0$ from (24). Thus, shareholders gain from increasing debt to D^n . Moreover, it is suboptimal to delay this increase in debt, as it would delay earning the gain $G(D, D^n)$.

Finally, we need to establish that shareholders would not prefer some alternative debt choice or sequence of choices. From the prior argument, it is sufficient to consider only changes to some other stable point $D^m \neq D^n$. Note that for $D^m < D$, $G(D, D^m) < 0$ since shareholders both lose tax benefits and bear (via the debt price) incremental agency or bankruptcy costs when buying back debt. For $D \leq D^n < D^m$, note that because $p(D^m) \leq p(D^n)$,

$$G(D, D^m) \leq G(D, D^n) + G(D^n, D^m) \leq G(D, D^n),$$

where the last inequality follows since $G(D^n, D^{n-1}) \leq 0$ by (24). ■

Proof of Proposition 7:

Let $V_j^E(D)$ and $p_j(D)$ be the payoff to equity and the price of debt if the firm has stable debt D until the next shock arrives, and let λ_j be the total arrival rate $\sum_k \lambda_{jk}$. Then if the firm enters regime k with debt D , the equity value and debt price will be

$$\hat{V}_j^E(D) = V_j^E(D^{j+}(D)) + (D^{j+}(D) - D)p_j(D^{j+}(D)), \text{ and} \quad (53)$$

$$\hat{p}_k(D) = p_k(D^{k+}(D)),$$

where

$$V_j^E(D) = \left(\frac{(y_j - c_j D)(1 - t_j) + \lambda_{j0}\phi_j(D) + \sum_{k>0} \lambda_{jk}\hat{V}_k^E(D)}{r_j + \lambda_j} \right)^+, \text{ and} \quad (54)$$

$$p_j(D) = \frac{c_j - \lambda_{j0}\phi_j'(D) + \sum_{k>0} \lambda_{jk}\hat{p}_k(D)}{r_j + \lambda_j} \times 1[V_j^E(D) > 0].$$

Note that in (54), we account for equity's option to default when $D > \bar{D}_j$. By the identical logic as **Proposition 6**, the stable points are defined by

$$D_j^{n+1} = \max\{D < D_j^n : G_j(D, D_j^n) \leq 0\} \quad (55)$$

where

$$G_j(D, D') \equiv V_j^E(D') - V_j^E(D) + p_j(D')(D' - D). \quad (56)$$

Note that we can calculate the equilibrium value function and stable points via backward induction on the debt level D , beginning from debt level $\bar{D} \equiv \max_j \bar{D}_j$. Once $D = \bar{D}$, there will be no further increases in debt, and the system (53) and (54) can be solved using standard methods. ■

Proof of Proposition 12:

Let $q(\delta)$ is the market value of a unit of debt (face value is equal to 1) when the perceived leverage is δ and let $e(\delta)pA$ be the *total* market value of equity when the market value of assets is equal pA and the (perceived) leverage is δ .

In a recapitalization the firm must issue equity sufficient to buy back Δ_D units of debt so that

$$\frac{D - \Delta_D}{A} = \delta_1, \text{ or } D - \Delta_D = \delta_1 A \quad (56)$$

The true value of current equity holders' claim after recapitalization will be:

$$\left(1 - \frac{q(\delta_1)\Delta_D}{pA - q(\delta_1)(D - \Delta_D)}\right) e\left(\frac{\delta_1}{1 + \omega}\right) p(1 + \omega)A \quad (57)$$

The *total* value of equity (from the perspective of the informed insiders) is $e(\delta_1^{True})p(1 + \omega)A$ where $\delta_1^{True} = \delta_1^{Market} / (1 + \omega)$ and $p(1 + \omega)A$ is the managers' assessment of the value of the assets. Note that true leverage as perceived by the managers is less than the market perceived leverage since the market is undervaluing the assets. The fraction of the total equity claim retained by current shareholders is based on the amount that must be raised through issuing equity to buy back the debt, i.e., $q(\delta_1)\Delta_D$, and the market's valuation of equity after the recapitalization, i.e., $pA - q(\delta_1)(D - \Delta_D)$.

Substituting (56) into (57), we have

$$\begin{aligned}
\left(1 - \frac{q(\delta_1)\Delta_D}{pA - q(\delta_1)(D - \Delta_D)}\right) e\left(\frac{\delta_1}{1+\omega}\right) p(1+\omega)A &= \left(\frac{pA - q(\delta_1)D}{pA - q(\delta_1)(D - \Delta_D)}\right) e\left(\frac{\delta_1}{1+\omega}\right) p(1+\omega)A \\
&= \left(\frac{pA - q(\delta_1)D}{pA - q(\delta_1)pA\delta_1}\right) e\left(\frac{\delta_1}{1+\omega}\right) p(1+\omega)A \quad (58) \\
&= \left(\frac{pA - q(\delta_1)D}{1 - q(\delta_1)\delta_1}\right) e\left(\frac{\delta_1}{1+\omega}\right) (1+\omega)
\end{aligned}$$

In reducing leverage through assets sales the amount of debt bought back must solve:

$$\frac{D - \Delta_D}{pA - q(\delta_1)\Delta_D} = \delta_1 \quad \text{or} \quad \Delta_D = \frac{D - pA\delta_1}{1 - q(\delta_1)\delta_1} \quad (59)$$

Since $A - q(\delta_1)\Delta_D / p$ will be the new level of assets after the deleveraging is completed, the value of the equity claim after the asset sales is:

$$e\left(\frac{\delta_1}{1+\omega}\right) p(1+\omega) \left(A - \frac{q(\delta_1)\Delta_D}{p}\right) \quad (60)$$

Using (59), we find that the new level of assets will be:

$$\begin{aligned}
\left(A - \frac{q(\delta_1)}{p} \left(\frac{D - pA\delta_1}{1 - q(\delta_1)\delta_1}\right)\right) &= \left(\frac{pA - pAq(\delta_1)\delta_1 - q(\delta_1)D + pAq(\delta_1)\delta_1}{p - pq(\delta_1)\delta_1}\right) \\
&= \left(\frac{pA - q(\delta_1)D_1}{p - pq(\delta_1)\delta_1}\right) \quad (61)
\end{aligned}$$

This means that (60) becomes

$$\begin{aligned}
e\left(\frac{\delta_1}{1+\omega}\right) p(1+\omega) \left(A - \frac{q(\delta_1)\Delta_D}{p}\right) &= e\left(\frac{\delta_1}{1+\omega}\right) p(1+\omega) \left(\frac{pA - q(\delta_1)D_1}{p - pq(\delta_1)\delta_1}\right) \\
&= e\left(\frac{\delta_1}{1+\omega}\right) (1+\omega) \left(\frac{pA - q(\delta_1)D_1}{1 - q(\delta_1)\delta_1}\right) \quad (62)
\end{aligned}$$

Since this is precisely equal to (58), the shareholders are indifferent between recapitalization and asset sales.

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