Effects of level of investors confidence and herding behavior on stock prices and their volatilities

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Abstract

A model is presented investigating how incomplete confidence of investors regarding their evaluation of stocks affects their demand. It is shown that contrary to previous claims, the demand for stocks is of finite (negative) elasticity. Investors' incomplete level of confidence is subjected to the impact of the information they obtain from observing others and also induces them to herding behavior, that is to let the presence of information about others' choices to affect their demand. It is shown how herding, following different types of (others') information, affects the equilibrium price and its volatility. I show that in most cases herding increases volatility, but I identify some cases where it decreases it. I also show how a signal, supporting the investor's own information, which has not been hitherto considered a factor explaining herding behavior, can generate such behavior and influence demand.

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I. Introduction

Herd behavior in decision making is defined as a situation where observation of decisions made by others, influences the decision made by an individual. It has been argued that herding has several negative impacts on the market: it is believed that this phenomenon distorts the public knowledge aggregation process, exacerbates volatility of the markets, and takes part in the creation of bubbles and crashes†. Nevertheless, since there is very little argue about the major role played by structural changes and other economic processes in the generation of bubbles and crashes, one may question the true contribution of the "herding behavior" to those events.

One may wonder how much of the overpricing during the bubble or the underpricing during the crash, may be attributed to herding behavior of the investors, as opposed to the inevitable reaction of markets to the strictly structural changes in the economic setting. Moreover, one may also question the other marginal effects of herding behavior on the generation of the bubble or the crash and whether it accelerates or decelerates it. However, perhaps the most pertinent questions are: 1. what is the (marginal) contribution of the presence of herd behavior in financial markets to the volatility and to the equilibrium price; and 2. Is, or under what conditions is, herding indeed a distorting phenomenon to market efficiency?

Along with its description of the various motives to herd in financial markets, the theory describes various different processes of herding. One of the main descriptions is a phenomenon known as "informational-based herding" or "informational cascade". This type of herding describes herd behavior as created by a process of concluding information about a choice, from observing other people's choices, and reacting to it. It is this type of herding that stands at the heart of this study.

In this study I describe and map some effects of informational-based herd behavior of investors in the stock market on the stock equilibrium price and on its volatility. I present a model that shows how different processes of herd behavior,

created by different types of signals about the choices of others, would differently affect the demand curve for a stock, the stock price and its volatility. I maintain that an important factor regarding the herding effects on the market lies in the effect of the information obtained by observing others, on one's confidence in her evaluation of the market.

The study argues that signals deduced from observing others' decisions affect the observer choice in two different ways: not only they provide information about the choice (the chosen asset), as current theory assume, but they may also differently affect the investor's level of confidence in her own evaluation of the market and change her demand through that. The distinction between the confidence of a subject in a specific choice and her confidence in her ability to choose was described by Liberman and Tversky (1993). According to them, specific overconfidence is a state in which a person "overestimates the probability of an outcome" namely of a specific future event, while generic overconfidence refers to an overestimation of a person's judgment ability i.e. an overestimation of the likelihood that the choices one makes are correct. The model constructed in this study shows that when the effect of observing others' choices on one's generic confidence is taken into account, different processes of investors herd behavior may appear, each affecting the market price and its volatility in a different way.

According to the theory I suggest, both positive and negative information about others' choice regarding a stock may have a positive affect on the investor confidence in her evaluation of the market if it is consistent with her own valuation of the stock. I maintain that while the binary decision of whether the stock is worth buying or selling is determined by the investors' valuation of the stock, the amount she will buy or sell is also determined by her confidence in her abilities of evaluating the market. By distinguishing between a few different types of information about choices of others, and subsequently analyzing their different effects on investors' confidence in their own evaluation, one can map the differences in effect on market results. In this study I describe the effect of: supporting information signal (one that supports the decision the investor would have made without it) and contradicting information signal (that contradicts the decision the investor would have made without it) deduced from observing others' choices. I also describe the effect of qualitative (binary) and quantitative information about choices of others, on the
market demand for a stock, on the short run equilibrium price and on its volatility. I show how herding following some types of signals increases volatility, while herding following one other type of signal decreases volatility and may even improve efficiency.

The consideration of the effects of the investors' confidence about their evaluation of the market also provides us with new answers to some financial markets' herding puzzles. It may reconcile some of the ambiguous conclusions of studies regarding herding effects on financial markets' efficiency. It also shows how, unlike current theory suggests, informational-based herd behavior may not be generated only by a "contradicting signal" (a signal that changes the investor prior information set regarding the asset), but may also caused by a "supporting signal," a signal that although does not change the investor prior information set regarding the asset, may still affect her generic confidence.

The construction of the model has also prompted me to address the unresolved question of the elasticity of the demand curve for a stock. By including the element of uncertainty about the market, and the incomplete confidence of investors in the group of factors that determine the demand for a stock, I provide an answer to the yet unsolved question of the elasticity of that curve. According to the model I suggest, the seminal theory conclusion of a horizontal demand curve for a stock holds as long as some market participants are fully confident about their estimation of the stock price. When, however, no market participant is fully confident about his estimation, the demand curve for the stock is of finite and negative elasticity. This answer, suggested by the model, does not contradict the seminal theory, while accounting for empirical findings that were previously thought to conflict with it.

The structure of this paper is as follows: Section II provides a short review of the current theories and findings regarding the demand curve for a stock and regarding investors' confidence. A description of the model assumptions is provided in section III. Section IV describes the model, and section V concludes
II. A Short review of current study of the economic concept of confidence and of the demand curve for a single Stock:

The economic concept of confidence

When assets are risky, the investor needs to predict their future cash-flows on the basis of some estimation she makes. This prediction (of the future cash-flows expected from stocks) is strongly connected to the economic concept of "confidence". The concept as interpreted in the economic context has several different meanings and accordingly a few different definitions. One well-known meaning refers to the market participants' optimistic or pessimistic prediction of the economic future. A state of "confidence in the market" according to this economic meaning is a prediction of a positive future, and the lack of "confidence in the market" is a prediction of a bleak future. A different well-known meaning of the term refers not to the market as whole or to the way market participants perceive it, but to the subjective perception of a market participant regarding her own knowledge, prediction or ability to predict. This interpretation of the term, which was discussed in detail in the literature, defines confidence as "a subjective probability or level of accuracy assign by a person to her own prediction". According to it, different levels of the investor's confidence in her prediction reveal different subjective estimations of the prediction's standard deviation/ variance: a high confidence is a low-estimated variance of the prediction, and a low confidence is a high value of that variance. Note that this definition leads to a meaning significantly different from the economic meaning of "confidence in the market" in one major way: it refers to the level of trust one has in her own valuation as "pink" or "gray". Namely, the high confidence of an investor regarding her valuation of a stock indicates that the investor highly trusts her "pink" or "bleak" valuation of it, while a low level of it indicates that she does not put a lot of trust in her "pink" or "bleak" estimation of it.

Note also that following this meaning, confidence may be interpreted as a fully rational factor, provided that the estimation of the prediction distribution is rational. Following this interpretation's school of thought, a person may be miscalibrated (i.e. underconfident or overconfident) provided that her estimation of the prediction variance is biased, or she may be well-calibrated (i.e. confident) provided that her estimation of the variance is correct. Accordingly, while overconfident and underconfident persons are exhibiting irrational (or bounded-rational) aspects of behavior and considered to be irrational, confident persons are considered to be rational.

A new meaning of the concept, which refers to both the rational and irrational aspects of it, has recently been suggested by Akerlof and Shiller (2009). They argue that the definition of the economic term of "confidence" should be closer to its dictionary definition as "trust" or "full belief". According to them: "The very term confidence-implying behavior that goes beyond a rational approach to decision making", they claim that beside a rational prediction, emotional factors, beliefs, instincts and other irrational factors are also playing a role in determining the "confidence" in its wider definition which they suggest as "trust" or "full belief".

**The demand curve for a signal stock**

One of the most controversial conclusions derived from seminal models in finance is that the elasticity of the demand curve for a single stock is infinite. The CAPM, Modigliani and Miller Proposition 1, the NPV rule and Arbitrage Pricing Theory all assume that every financial asset has perfect substitutes in the market (in the form of another single asset, or in the form of a portfolio of assets). Likewise, the theory assumes or deduces that the amount of the stock which is chosen to be issued by a company doesn't affect its stock price, and that investors can buy and sell any amount of equity without affecting its market price. These assumptions lead to the conclusion that the demand curve for a single stock is of infinite elasticity. The stock price is determined independently of the number of shares issued, and the quantity of stock traded in the market has no influence on its price. Nonetheless, most empirical

studies that test the shape and the elasticity of demand for a stock find a negative relation between the number of shares and the price. The conclusion of many of these studies is that the demand curve for a stock has a finite elasticity and is probably downward sloping.††

The attempt to reconcile the gap between the conclusions of the prevailing theory and the empirical findings led to a number of important hypotheses. Two main hypotheses were published in the 1960’s: the "price pressure hypothesis" which pertains to the effect of large transactions and the "imperfect substitute hypothesis" suggesting that in reality most stocks have no perfect substitute.‡‡ A famous study was published in 1972 by Scholl, examining the existing hypotheses and suggesting one of his own. Scholl’s hypothesis, called the "information hypothesis" maintains that drops in the stock price that occur following large transactions are not caused by a negatively sloped demand curve for the stock but by the new information released to the market by the sale. This theory has become a third main hypothesis. A more recent study of the subject provided more empirical findings to support these hypotheses and suggested some new ways to test them; however, no new theoretical, widely accepted explanation was suggested since Scholl's information hypothesis.

Many empirical studies about the elasticity of demand for stocks were conducted following the publication of the information hypothesis, some of them tried to isolate the information influence, in order to test the true shape of the curve§§. The findings of these studies are not consistent: most define a downward sloping demand curve, like: Kaul, Mehrtra and Morack (2000) and Plerou, Gopikishnam, Gabaix and Stanley (2002), while some conclude that the demand curve is horizontal, like: Cha and lee (2001). Consequently the questions of the typical form and of the elasticity of the demand curve for a single stock remain unsolved.

†† The exception is macroeconomic studies testing the aggregate demand for all stocks in the market. These studies, which deal with the aggregate demand for investments in the stock market, have some different conclusions that are not relevant to the question of elasticity of demand for a single stock

‡‡ See Lintner (1962)
§§ See for instance Shleifer (1985)
III. The model assumptions

a. The market consists of 'n' individual participants, each making an investment decision about a stock.

b. In addition to the stock under consideration, there exist many other available assets for investment.

c. The stock market portfolio is value weighted and its size equals the size of the average portfolio of the market participants. All stocks are included in the market portfolio. The exact composition of the market portfolio is calculated by the stock exchange and published as an index. I assume that although the daily change in a stock price changes the market value of its company, the composition of the stock market portfolio as published by the stock exchange is checked and adjusted not on a daily basis, but at most, once a month (or even once every three months). The historical beta of each stock is publicly available. Individuals hold the "market portfolio" either directly, or indirectly through pension funds, insurance policies etc.

d. Every individual knows the same publicly available fundamentals regarding the stock in question and other traded stocks and their market prices. He may also possess some private information. On the basis of his set of information, the individual reaches his private estimation of the "right price" for the stock. This "subjective equilibrium price" (SEP) is defined as an individual's measure of the price yielding the same return as that available on assets having the same risk. Hence the SEP doesn't contain any excess return/abnormal profit.

e. The decision about the SEP is a private decision of each, and may vary among persons according to the information available to them and their interpretation of it.
f. When the SEP equals the stock market price, namely when according to the individual’s estimation, there is no arbitrage opportunity or no opportunity for a positive NPV, he will hold a fixed amount of the stock in his portfolio. This amount (which may differ from one person to another), derives from the proportion of the stock in the index (the "published market portfolio"), the size of the investor's portfolio and his private choice of degree of leverage. However, when the SEP equals the market price, the proportion of his whole holding of the stock relative to the individual's holding of the market portfolio would equal the proportion of the stock in the index.

g. When at any given moment, the stock price is too high or too low relative to the SEP of an individual; he identifies an arbitrage opportunity or an opportunity for a positive NPV, and would like to purchase or sell that particular stock in order to hold more or less than the quantity held under his SEP.

h. Due to the existence of uncertainty, individuals may not be completely sure that their estimates of the stocks and their right price are accurate. 'b' is a variable representing the level of confidence of the individual about his own independent knowledge and understanding of the market, and about his estimated SEP***. 'b' lies between 0 and $\infty$. When $b = 0$ the individual feels completely ignorant and doesn't put any trust in his own estimate. When $b = \infty$ the individual is completely sure about his understanding of the market and fully confident of his estimated SEP.

i. The individual is exposed to other individuals participating in the market. They are divided into two groups:

**Group A:** This group consists of close colleagues, peers, friends and other persons with whom the individual has direct contact or communication.

**Group B:** This group consists of market participants whom the individual does not know privately - people who are not in direct contact with that individual.

*** Confidence has a several different meanings in economic. This model definition of it is explained hereby, In 3.3
j. The information of an individual about the market and the stock is divided into two categories:

**Personal information set**: Three types of information are included in personal information set:

1) public information about the company, its records, financial reports, company statements, published information about the market etc.;
2) Professional private information derived from proprietary tools used for analysis; and
3) Information held by close colleagues and other individuals participating in group A.

As regarding the third type, the information held by participants in group A differs from that of group B in two major aspects. First, information an individual collects through his communications with group A is regarded to have a different influence on the investor's choice than other information may have. Since people may be influenced by their friends, colleagues and other partners to different levels and in various conscious and unconscious ways, separating one’s independent opinion from the influence of his close friend’s opinion is extremely difficult and tricky, if not impossible. Second, those colleagues and peers make choices as well, and the individual can observe their choices, but since they are close to the individual he can inquire about the data which led them to their decision, and in many cases he does not need to deduce what they know by observing their choices. In this sense, the information held by participants in group A differs from information derived from observing only the choices of others, such as in the case of group B. Note also that observing the choices of others while having no access to their information, is an essential assumption in most herding models, and the information concluded this way is considered to be the information that generates herding or cascades. Hence, I regard the information possessed by participants in group A as part of a *personal set of information* held by the investor. I assume that if someone from the close circle doesn't share his data with the individual and therefore the
individual is forced to deduce information only by observing his choice, this someone does not belong to group A, but to group B, and his information, being unavailable to the individual, is not part of the individual’s personal information set.

**Observational information**: Information that the individual obtains from observing the choices of investors in Group B, with whom he has no direct connection/communications. As in most herding models, the assumption is that the individual can not reach the information held by these others, but he may reach some conclusions about their information from the choices they make.

k. When an individual makes a choice, he employs both types of information in arriving at his decision (personal and observational from above). There are two possible scenarios about the combined set of information at an individual investor's disposal:

1) It is possible that an individual will only have a personal set of information. For example, when a certain piece of information about a particular stock arrives while the stock exchange is closed and there is no way for the individual to know what choices will be made by group B, his choice about the stock, prior to the opening of the next trading day, will be based only upon his personal set of information.

2) A more common scenario is a scenario in which the individual has both types of information: personal and observational. I assume that in an advanced financial market, the scenario of an individual having solely observational information is impossible, since it is unlikely to be able to observe other individuals’ choices while having no access to any objective information about the stock - financial reports and company's announcements for instance.

l. The individual's decision on investing in a stock at its given market price consists of two stages. In the first stage, the investor makes a *binary quality decision* of whether to invest (long or short) in the particular stock. In the following stage, the investor makes a *quantitative decision* on the amount of money to be invested in the stock.
All individuals participating in the market hold a positive amount of each stock, as they always hold the market portfolio. The stock’s proportion in the market portfolio is fixed as long as the stock exchange does not change the composition of the index. This proportion multiplied by the size of the individual holding of the market portfolio is the total quantity the investor would like to hold, provided that the stock market price equals his SEP. In this sense both the binary and quantitative decisions determine the excess quantity the individual would like to buy or sell from the stock when he thinks it is mispriced. Another way of interpreting it is to assume that investors hold the market portfolio as one part of their savings, as assumed before, but also keep an additional portfolio which may include several assets the investors expect to yield particularly high returns. The binary and quantitative decisions define this excess holding.

m. There are two types of signals an individual may receive: a binary signal indicating whether or not it is worthwhile to invest excess investment in the stock, and quantitative signals indicating the extent of desired excess investment in the stock at its given price.

n. I assume the investor cannot observe other investors’ demand curves. He may only observe their choices, or in some cases their information, but not their entire demand curve.

o. The supply of the stock is defined as number of shares issued, and hence represents a fixed supply curve.
The model

A definition of an investor's "self confidence" in her valuation of an investment

This model definition of "self-confidence" of an investor in her evaluation of the market is based both on the definition of "confidence" as "a subjective probability or level of accuracy assign by a person to her own prediction/ ability to predict" and on Akerlof and Shiller's suggestion. It uses the definition of confidence as "the subjective estimated standard deviation of the investor's prediction", but it does not assume that this estimation can be fully rational. As Akerlof and Shiller suggest, this model assumption is that both rational estimation and some irrational behavioral factor play a role in determining an investor's confidence in her evaluation of the market or an investment. This leads to the following definition: "self confidence" of an investor in her evaluation of the market/ an investment is the level of trust or full belief she has in her prediction regarding it (or evaluation of it). This level of trust is realized in her estimation of her prediction's variance which is negatively correlated to it. Her level of trust (and estimated variance) may be determined by the rational analysis of the relevant available data and by the private behavioral characteristics of the investor".

In this work, whenever I use the word "confidence" it should be referred to as a shortcut for the term that I have defined as "self confidence".

The demand curve of a single individual in the absence of information about others’ choices

Each individual's portfolio includes the market portfolio with or without leverage and may also include an additional holding of a different portfolio consisting of a small number of stocks.

Individuals will increase/decrease their ownership of a stock, within the context of the additional portfolio, by purchasing or selling the stock or short-selling it, if they think there is an arbitrage opportunity or an opportunity for positive NPV. The individuals estimate the profitability of investing in a stock according to its
known fundamentals, to what is publicly known about its industry and the entire market, and to all the other information they may possess.

The market is a well-administrated market, in which transparency is required from public companies. There is a lot of information about each and every stock and the market is characterized by a large amount of public information. However, this information is imperfect and there is always some uncertainty about the future income from each stock.

Since individuals are aware of the uncertainty and the imperfection of the information, their self-confidence about the accuracy of their evaluation of the stocks may be limited. Individuals differ in their level of confidence, which varies from a very low degree amongst individuals who feels ignorant and unable to evaluate the stocks and the market, to a very high degree amongst individuals who feels that their evaluation and understanding of the market and the stock is almost perfect.

Let:

\[ p_i^* = \text{the price evaluated by individual } i, \text{ as the right price for the stock, his SEP.} \]

\[ b_i = \text{the confidence coefficient, indicates the level of confidence of individual } i, \]

in his evaluated SEP. \( 0 \leq b_i \leq \infty \), while:

- When \( b_i = 0 \), the individual lacks any confidence as to his ability to evaluate the stock.
- When \( b_i \Rightarrow \infty \), the individual is fully confident that he can properly evaluate the stock and its price, and is of the opinion that his estimated SEP is entirely correct; and
- When \( 0 < b_i < \infty \), the individual has a positive but not complete confidence in his estimate of the price. The higher the confidence, the higher \( b \) is.

The demand of individual \( i \) for the stock is composed of two parts: The demand for the stock while its price equals his SEP, and the excess demand for the stock, when its price differs from his SEP.

The demand equation, \( Q_i \), of an individual \( i \) for the stock is:

\[ Q_i = a_i + b_i \left( p_i^* - p \right) \]
Where:

\[ Q_i \] = the total quantity of the stock that individual \( i \) wants to hold in all the portfolios he holds (measured in units; i.e. the number of shares which an individual wants to hold)

\[ a_i \] = the quantity of the stock that individual \( i \) wants to hold when there is no opportunity for excess return on the stock, namely when its price equals his SEP (his \( p_i^* \)). The value of this amount, (at the time the index is adjusted) equals the proportion of the stock in the index multiplied by the size of the investor's holding of the market-portfolio at that time. \( \dagger \dagger \dagger \)

\[ b_i^* \left( p_i^* - p \right) \] = The excess quantity of the stock that individual \( i \) wants to hold when a "price gap" exists (a gap between the individual's SEP and the market price).

Therefore:

When \( p = p_i^* \), the market price for the stock equals the individual's SEP, and the individual's holding of the stock is exactly \( a_i \).

When \( p < p_i^* \), the market price is lower than the individual's SEP, buying that stock yields abnormal/excess returns (it has a positive NPV or an arbitrage profit), and the individual wants to hold a positive excess holding of it and become a buyer.

When \( p > p_i^* \), the market price is higher than the individual's SEP, selling that stock yields abnormal/excess returns (it has a positive NPV or an arbitrage profit), and the individual wants to hold a negative excess holding of it and become a seller.

Recall that according to classical finance theory, in the case of a price gap, the individual's demand (negative demand) for the stock would become infinite. This happens because, according to the perfect substitute assumption, the gap creates a risk-free arbitrage opportunity. Practically, in this case, the individual would try to

\( \dagger \dagger \dagger \) Note that \( a_i \) is derived from the index and the investor's portfolio's size, and is not dependent on the stock's market price.
buy (sell/short sell) as much as possible of that stock, and his demand would only be limited by the finite supply (or his supply would be limited by a finite number of shares or a finite demand). Under the suggested model, this classical finance theory result holds when $b_i \Rightarrow \infty$. Namely, when the model assumptions match the seminal theory assumptions, and an individual is fully confident in his estimated SEP, the model leads to infinite growth in the demand in reaction to any price drop, and an infinite fall of the demand following any price rise. Hence, the demand curve in this case is of infinite elasticity. However, when the individual is not fully confident, the growth (fall) in demand caused by the price gap is always finite, and its size depends on the individual's confidence level, and on the size of the gap between the stock's market price and the individual’s SEP.

The model assumption is that the lack of full confidence limits the growth (fall) of demand following a price gap. When the level of confidence rises, so too does the resulted growth (fall) of the demanded quantity, on the account of a given price gap. The demand curve elasticity is, hence, a function of the individual level of confidence.

The size of the excess holding is also dependent on the size of the price gap. Under a given confidence level, the larger the gap is, the bigger the growth of the demanded quantity. One explanation for that is that when the size of the price gap increases, the excess return also increases and the NPV will be bigger. Consequently, the individual is motivated to buy (sell) more of that particular stock even if his level of confidence ($b_i$) remains the same. One more possible intuitive alternative explanation for that result is that investors assume that a bigger gap between the market price and SEP decreases the probability that there is no gap, and accordingly the probability for no excess return from buying (selling) the stock. The assumption is that in the eye of the individual, the probability of small errors in the estimated SEP is higher than the probability of big ones. Therefore, under a given confidence level, when the gap between the market's price to the SEP become bigger, the probability that there is no excess/abnormal return in buying (selling) the stock, even in the presence of an error in the individual's estimated SEP, becomes lower and as a result the individual will want to extend his holdings of that stock. Hence, a bigger price gap motivates the individual to purchase (sell) more units of that stock.
To summarize:

- The demand curve of a single individual has three exogenous constant parameters: the SEP \(= p^*_i\), which depends on the individual's information regarding the stock; the \(a_i\), which depends on the proportion of the stock in the index (and the size of the portfolio); and the \(b_i\), the individual's level of confidence coefficient.

- The variable that changes along the curve, as a result of the changes in the market price, is the number of demanded shares which depends on the gap between the constant SEP and the market price.

- The slope of the demand curve is \(-\frac{1}{b_i}\); namely: the level of confidence multiplied by -1 = \(-b_i\) is the slope of the demanded number of shares (relative to the market price); and the demand curve elasticity is finite and negative.

The following diagrammatic analysis assumes a downward sloping linear demand curve. This simplified shape was assumed by Karl Rau in 1841, and is consistent with the general down-sloping demand of Alfred Marshall 50 years later, which was derived from maximizing a utility function (under given prices, income and tastes) and which did not imply necessarily a constant slope.

Although the most common shape used in demand studies is concave, many other studies in microeconomics assume a linear demand, both for an individual and for aggregated demand in a given market (where the same market price is given to all consumers). It is always correct as a first approximation, if the sign of the slope is known (i.e. negative), and thus represents the qualitative effect, where theory usually has little to say about the rate of change of the slope (the 2\textsuperscript{nd} derivative). The linear shape is widely assumed in many specific areas of analysis, such as studies of monopolistic behavior (see Varian 1987).

In our analysis, looking at the demand equation: \(Q = a_i + b_i*(p^*_i - p)\), we see that as long as the individual confidence coefficient \(b_i\) is positive and finite (namely, the confidence is positive but not full), the demand curve is down-sloping. Like the Marshallian demand, this shape may also be derived from utility maximization, but in this case a behavioral factor of "confidence" plays a modifying role. In the seminal
finance theory, the demand for a stock as derived from maximizing utility (when no behavioral factor is involved) is infinitely elastic. However, the elasticity is finite in our model as a result of adding to the analysis this important behavioral factor. Note also that the curve derived from our model is not necessarily linear. It depends on the relations between the $b$ and the price-gap ($p_i^* - p$): If $b$ is not dependent on the size of the price-gap, it is constant throughout the curve and the curve is linear. However, if $b$ is "price-gap" dependent, the curve would not necessarily be linear. Nevertheless, it can be shown that as long as the slope is negative, the conclusions of the analysis would be similar. Hence for simplicity I present the following model using linear demand functions in the analysis and in the diagrams.

Figure 1 shows the shape of an individual's demand curve for a stock:

The aggregate demand curve in the absence of information about others’ choices

The distribution of the $p_i^*$: According to seminal theory, each and every investor will see the same ($p_i^*$). I initially work under this assumption. Then I relax this assumption and assume that different investors hold different beliefs regarding the
stock and estimate a different SEP for it. In both cases that the variance of the \( p_i^* \) of the market participants is positive.

The distribution of the \( a_i \): The model assumes that \( a_i \), will differ from one individual to another due to the differences in the portfolio size and leverage. Hence, \( a_i \) variance is also positive.

The distribution of the \( b_i \): I assume that the coefficient of confidence differs from one individual to another and the values of the different individuals' confidence coefficient level may change on a scale of zero to infinity.

The aggregate demand for the stock of all the individuals participating in the market is therefore:

\[
\sum Q = \sum a_i + \sum b_i (p_i^* - p)
\]

\[
\sum Q = \sum a_i + \sum b_i p_i^* - \sum b_i p
\]

The slope of the aggregate demand is \(-\sum b_i\) (the curve slope is \(-1/\sum b_i\)) and the demand will be a finite number as long as the confidence coefficient of every individual participating in the market is finite. I hereby briefly analyze the shape and elasticity of the aggregate demand curve in some interesting cases:

- When the market consists of many individuals, including one who has \( b_i \) that tends to infinity (one individual has full confidence): this case will result in a totally elastic aggregate demand curve in this individual's \( p_i^* \). (Since the demand of this individual to purchase the stock at any price lower than his SEP is infinite, the aggregate demand will be infinite as well. As for prices higher than \( p_i^* \), at every such price, this individual's demand for the stock would be infinitely negative (the aspiration to short sell/sell is infinite) and because the market demand is aggregate, his infinite negative demand will offset the entire investors' positive demand, namely, every other individual will be able to purchase from this individual as much as he wants at a price that is higher by an

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This relaxation of the "homothetic expectation assumption" was analyzed and studied by Miller (1977) and by some later studies that claim that the assumption is too far removed from reality and does not hold in an uncertain market.
epsilon from the individual’s SEP, so that no individual will agree to purchase the stock at a higher price and therefore the market aggregate demand curve of the stock will be totally elastic at the SEP ($p_i^*$) of that individual). Obviously, when all individuals see the same $p_i^*$, and only one individual has full confidence, the curve will be totally elastic at this price as assumed by the classic finance models.

-When the market consists of many individuals who may each see a different $p_i^*$, including two or more who have $b_i$ that tends to infinity: Using a similar analysis (such as that of the prior case), it is easy to show that this case will result in a totally elastic aggregate demand curve at the highest $p_i^*$.

-When the market consist of many individuals but no individual has $b_i$ that tends to infinity: as long as the number of individuals participating in the market is finite, the aggregate demand curve will be finite and downward sloping.

The equilibrium price and the holding distribution among the investors when all investors see the same $p_i^*$:

This case is quite trivial and it will result in $p_i^*$ being the equilibrium market price, while each and every individual holds the exact amount of $a_i$ units of that stock in their entire portfolio.

Let: $a_{MP} =$ The quantity (number of shares) of the stock in the published market portfolio, (calculated according to the published index). Since the size of the market portfolio is the size of the average investor's portfolio, and the market consists of "n" investors, in equilibrium the total supply/demand for the stock will equal $a_{MP} * n$. As for the demand side, although $a_i$ has a positive variance, and some investors possess more than the $a_{MP}$ while others possess less than it, since all investors see the same SEP, they would all want to sell the stock or they would all want to buy it at each market price different then this SEP. Now, if the price is lower (higher) then the SEP, all investors would like to increase (decrease) the proportion of their holding in the stock to be bigger (smaller) then its proportion in the market portfolio; but since all investors want to buy (or sell) and none want to sell (or buy)
this will result in an excess demand (supply) at that price. Hence, in order to have neither an excess demand nor supply, the demand of each investor would have to be his $a_i$, so that in equilibrium, the aggregate demand would have to be exactly $\sum a_i$.

As a result: in equilibrium we must have: $\sum Q_i = a_{MP}^* n = \sum a_i$, and this will hold only when $p = p_i^*$ (in any other price $\sum Q_i$ is larger or smaller than $\sum a_i$).

**The equilibrium price and the holding distribution among the investors when different investors have different $p_i^*$:**

Since $p_i^*$ varies among the investors and since there is only one market price; at every market price some investors will have a positive (or negative) price gap and would like to hold some excess positive (or negative) holding of the stock. In that case, in order to have neither an excess supply nor demand, the market has to consist of both buyers and sellers and therefore, the market equilibrium price will have to be higher than some investors' SEP ($p_i^*$), yet lower than some other investors' SEP. It will be the price that will aggregate all the excess holding - buying and selling- to zero. Since $a_{MP}$ is the number of shares being held in the market portfolio, a portfolio of an average size, in equilibrium, the demand for the stock will equal $n^* a_{MP}$, and the equilibrium price will be the price that fits this quantity on the aggregate demand curve$^{**}$

As for the distribution of the holdings, some investors will hold more than their $a_i$ and keep the stock in their additional small numbered stocks' portfolio, and some will hold less then $a_i$ (and keep a total amount of the stock that is smaller than their $a_i$).

$^{**}$ Note that although this price would be placed somewhere in the middle of the individuals' SEP range, it is not the average price, but rather the price derived from the average holding.
Figure 2: Aggregate demand and the holding distribution among the investors, in the absence of an informational signal when the SEP varies from one investor to the other.

Figure 2a: $Q_1$ = the demand of individual 1 – Buyer

Figure 2b: $Q_2$ = the demand of individual 2 – Seller

Figure 2c: The aggregate demand in the absence of informational signal ($P^*_1$ varies)
Figure 2 describes the aggregate demand and the holding distribution in this case. Figure 2.a presents the demand curve of an individual whose $p_i^*$ is higher than the market price $p_o$ - a buyer, and Figure 2.b presents the demand curve of an individual whose $p_i^*$ is lower than the market price - a seller.

The demand curve for stock in the presence of information about others’ choices - including a possible herd behavior

As mentioned before, the individual's demand for a stock may be seen as composed of two parts: a constant part $a_i$ and a price dependent part, and the individual's decision is about the second part. In practice, this is a two-stage decision: an initial qualitative-binary decision (a decision of whether or not to hold an excess holding in the stock) and a subsequent quantitative decision. Therefore, while analyzing the impact that observing others choices' would have on the individual's decision, we need to consider two types of "observing others' choices" signals ("observational information signals"): 

1) A binary qualitative signal, which merely signals whether or not it is worthwhile to hold an excess holding in the stock (without any indication as to the quantity that should be hold).

2) A quantitative signal which indicates the optimal holding quantity (excess holding) in that stock.

A definition of a binary-supporting signal and binary-contradicting signal

A supporting binary signal is a signal supporting the binary choice of the investor. If the investor holds an excess holding of the stock, and thinks that other investors are rational and may be privy to true information, a supporting observational signal shows him that others also hold an excess holding of that stock and a contradicting signal shows that others aren't doing so or are doing the opposite. Therefore, I define a binary-supporting signal as a signal that reveals a similar binary choice of other investors (and hence supports the individual's estimation of his $p_i^*$, or his SEP range as above, below or at the market price). In contrast, a binary-contradicting signal would be a signal that reveals other investors' different binary
choices and contradicts the individual's estimation of $p_i^*$ (or his SEP range as above, below or at the market price).

The impact of a binary-observational signal on the individual's choice

Let's assume that the individual estimated SEP is higher than the market price, i.e. he chose to invest a positive excess investment in this stock. In this scenario, the individual who observes others' decisions can observe two types of binary signals: a supporting signal and a contradicting signal.

The prevailing herding models assume that observational signals affect only the set of information that forms the basis for one's decision whether or not to invest. In other words, an observational signal may only affect one's decision if it changes one's set of information (affecting a sway from one choice to another).

By contrast, I suggest that an observational signal may have two different types of influences on an individual making a choice: an "information influence" - an informative contribution to the set of information about the stock, as other herding models assume; and in addition a "confidence influence" - an effect on the confidence level of the individual in his estimation of the stock. I argue that while they are observing others' choices, some individuals are not looking solely for information, as the other herding models assume, but are also seeking reassurance of their confidence in their own choices. Moreover, even individuals that are looking solely for information might get an "unlooked for" reassurance (or the opposite of it) for the confidence they have in their choice. Although the observation that other investors are making a similar binary choice to one's own, does not, strictly speaking, grant new information that might warrant a change in one's binary choice, it may however strengthen one's level of confidence in one's own opinion and thereby effect a change in his quantitative choice ****.

According to the proposed model, a binary signal about others' choice influences the level of the individual's confidence in his evaluation of the stock, and in his estimated SEP. A supporting binary signal would strengthen the individual's confidence and therefore increase his $b_1$, whereas a contradicting binary signal will

****Note that by way of symmetrical parallel, a contradicting signal will potentially both influence my set of information and decrease my confidence in my ability to estimate the stock, thereby affecting my choice in two ways.
weaken that confidence in his estimated SEP and therefore reduce his $b_i$. Since $b_i$ is the slope of the demand (relative to the price), a binary signal may change the slope and the elasticity of the investor demand. A supporting binary signal will increase the individual's confidence and make the demand more elastic whereas a contradicting binary signal will decrease that confidence and will make the demand less elastic.

Also note that a binary signal in the prevailing widely-accepted herding models generates an "all or nothing" binary influence; either it affects a change from 0 to 1 or it makes no difference whatsoever. By contrast, according to this study, a supporting binary signal that will not change the binary choice, may nevertheless have an impact on one's quantitative decision (due to increased confidence in one's binary choice) and thereby have a "partial effect" (as opposed to "all or nothing effect") on the individual choice.

**The effect of a supporting binary signal on the demand curve of a single individual**

The equation of the demand curve has three parameters: $a$, $p_i^*$ and $b_i$. Since $a_i$ is derived from the market portfolio composition as calculated by the stock exchange, and would only be changed when the proportion of the stock in the published index is changed, $a_i$ does not change following an observational signal. Nevertheless both $p_i^*$ and $b_i$ may change following the investors' observation of others' choices.

While referring to a supportive signal, since it is defined as a signal that supports the individual's estimation of the SEP, I assume that it does not cause a change in the investor's $p_i^*$ and may only affect the investor's $b_i$. Hence, a supporting binary signal increases the individual confidence in his estimated SEP (or in his binary choice) and therefore increases his $b_i$ causing a bigger growth (decrease) of the demand for the same price gap between the market price and his SEP, namely, generating a change in the quantitative choice. Figure 3 presents the demand curve for stock without observing a supporting binary signal and with it.

†††† If, for example, the individual holds the market portfolio in his pension fund savings, this holding is based on the stock index as computed by the stock exchange and will only change when the proportion of the stock in the index changes
The less elastic curve represents the individual's demand for the stock at each market price when the individual observes no information about others' choices. Note that the SEP of the individual's estimation $p_i^*$ is constant (remains the same throughout the curve).

The more elastic curve in the chart represents the demand curve of the same individual, estimating the SEP as $p_i^*$ and receiving a signal that supports this estimation. The supporting binary signal increases the $b_i$ and creates a more elastic demand curve, while maintaining the same $a_i$. That is, at any market price higher than $p_i^*$, the individual's desired quantity is smaller than it was before observing the signal, and at every price lower than $p_i^*$, it is larger than its size prior to the observation. Only if the market price equals the individual's SEP, namely, only if $p_i^* = p_o$, does the individual's demand equal $a_i$ and remains the same after observing the binary signal, as it was prior to it.

Proof:

Let:
\[ b_{A_i} = b_i \] after observing a binary signal about others' choices.
\[ b_{Bi} = b_i \] before observing a binary signal about others' choices
\[ \Delta b_i = \text{the change in } b_i \text{ due to observing the binary signal about others' choices.} \]
\[ b_{A_i} = b_{Bi} + \Delta b_i \]

The demand curve before observing the binary signal about others' choices will therefore be
\[ Q_i = a_i + b_{Bi}^* (p_i^* - p) \]
Whereas the demand curve after the observation will be
\[ Q_{A_i} = a_i + b_{Ai}^* (p_i^* - p) = \]
\[ Q_{A_i} = a_i + b_{Bi}^* (p_i^* - p) + \Delta b_i^* (p_i^* - p) \]

When the binary signal is supportive \( \Delta b_i \) will be positive, and thus:
- When the market price 'P' equals the SEP, the demand will be identical with or without a supportive binary signal and will be equal to \( a_i \)
- When the market price is lower than the SEP, the increase in the demand due to the price gap will be bigger after observing the binary signal, compared to that increase without observing it.
- When the market price is higher than the SEP, the reduction in the demand due to the price gap will be bigger after observing the signal, compared to the non-observation reduction.

To summarize: A supporting binary signal which increases the individual's confidence in his estimation of his SEP will increase the individual's demand curve elasticity compared to the elasticity of the demand curve prior to observing it, while the quantity demanded in the SEP will remain unchanged.

Note that this result is consistent with both the seminal theory and the widely-accepted models in finance. Since those assume full rationality and do not take into consideration the lack of confidence by the individual, their assumptions may be interpreted as assuming a complete confidence of the individual in his rational estimation of the SEP. In this case (under their assumption), this model's individual's demand curve will be of infinite elasticity as they suggest. Moreover, according to the
model, a supportive binary signal that strengthens the individual's confidence i.e. brings his confidence level to be closer to the full confidence assumed by these models, also affects a positive change in his demand curve elasticity, causing the curve to be closer to the curve suggested by the widely-accepted financial models.

*The effect of a contradicting binary signal*

In the case of a contradicting binary signal, one has to consider two possible effects on the investor's demand curve: its effect on the investor's $b_i$, parallel to the one in the supporting signal case, and in addition, a possible effect on his estimation of $p_i^*$. Since a contradicting signal contradicts the investor's estimation prior to the signal, it may both reduce his confidence in his estimation and also drive him to adjust his estimation of $p_i^*$. This study's assumption is that a contradicting signal either change solely the $b_i$ namely: reduces one's confidence in one's estimation without changing the estimation itself, or it would change both $b_i$ and $p_i^*$, i.e. reduce the confidence and would cause the investor to change his estimation of the $p_i^*$. (Since I assume that a change in the SEP is caused by doubts about its estimation, I do not consider a possibility that the contradicting signal will affect the estimated SEP, while having no influence on the investor's confidence regarding his prior estimation of it).

Accordingly, I have divided the analysis into two cases: the case in which the signal affects only the $b_i$, and the case in which it affects both $b_i$ and $p_i^*$. Nevertheless, despite this division, as I will show, both cases lead to the same result regarding the direction and the type of the change in the aggregate demand curve, save one difference: in the case of a multiple influence, the change will be bigger.

*The effect of a contradicting binary signal: the case where it only influences the confidence level.*

Figure 4 presents the demand curve for the stock with and without the observation of a contradicting binary signal, in this case.
The less elastic curve is the demand curve after observing the signal. A contradicting binary signal will reduce the $b_i$ and will make the demand curve less elastic while maintaining the same $a_i$ as before. Therefore, at any price higher than $p_i^*$ the desired quantity will be larger, and at every price lower than it the demand will be smaller compared to the "pre-observation" contradicting signal.

In an imperfect information setting, since the individual is not fully confident about his estimated $p_i^*$, and since a contradicting binary signal suggests that other investors have an opposite opinion or at least do not share his estimation of it, it will affect the individual confidence in his choice and therefore the individual will decrease the level of his reaction to the price gap. A positive gap will generate a smaller excess demand, and a negative gap will generate a smaller excess negative demand (amount he desires to sell).

Proof:
Recall, that according to the above definitions the demand curve before observing a contradicting binary signal will be:

\[ Q_i = a_i + b_{Bi} \ast (p_i^* - p) \]

Whereas the demand curve after it is

\[ Q_{Ai} = a_i + b_{Ai} \ast (p_i^* - p) = Q_i + \Delta b_i \ast (p_i^* - p) \]

When the binary signal is contradicting, the \( \Delta b_i \) will be negative and therefore we receive the following results:

- When the market price equals the SEP, the demand will be identical to the demand without a contradicting binary signal and will equal \( a_i \).
- When the market price is lower than the SEP, the growth in demand due to the price gap will be smaller after observing the binary signal compared to the growth prior to the observation.
- When the market price is higher than the SEP, the reduction in demand due to the price gap will be smaller after observing the binary signal compared to the reduction prior to the observation.

To summarize: A contradicting binary signal that weakens the individual's confidence in his estimation of the SEP, will affect a change in his demand curve making it less elastic than his demand curve prior to observing it, while keeping the quantity demanded at the SEP unchanged.

Note that according to the model, a contradicting binary signal which weakens the individual's confidence distances his demand curve from the curve derived from the seminal theory in finance.

The effect of a contradicting binary signal: the case when it influences both the confidence level and the estimated \( p_i^* \).

In this case, in addition to the effect described above, the curve will also move upward or downward depending on whether the investor was a buyer or a seller prior to his observation of the signal. If he was a buyer, receiving a contradicting signal will lower his estimate of \( p_i^* \), and this will cause his curve to move downward and
reduce his demand even more than in the case with no effect on his $p_i^*$. If he was a seller, receiving a contradicting binary signal will increase his estimated $p_i^*$ and his curve will move upward causing the increase of his demand to be even bigger than its increase in the case where the signal does not influence the estimated $p_i^*$. Figure 5 shows the change of the demand curve of a buyer, when the contradicting binary signal influences both $b_i$ and $p_i^*$. While the curve signaled as $Q_{A1}$ shows the change of the demand caused solely by the redaction of $p_i^*$ (while the confidence level remains the same), the curve signaled as $Q_{A2}$ shows the final demand curve including both effects.

Fig. 5: The effect of contradicting binary signal – Buyer
The case when it influences both the confidence level and the estimated $P_i^*$

- $Q_0$ = Demand curve in the absence of signal
- $Q_{A1}$ = Demand curve in the presence of signal: $P_i^*, b_i$
- $Q_{A2}$ = Demand curve in the presence of signal: $P_i^*, b_i, b_i$

The effect on the aggregate demand when different individuals have a different estimation of the SEP:

As shown above, in the case were different individuals have different SEP, namely, when the estimated $p_i^*$ has a positive variance, the equilibrium market price will be within the range of the SEP values, resulting in market equilibrium in which
some investors will hold an excess holding of the stock, while other investors will hold a negative excess holding (less than $a_i$) of it. Therefore, any binary signal that would be observed by the market participants would be supporting to some, while being contradicting to others. Accordingly, in this section I have not divided the signals into the same categories of supporting and contradicting, but rather into positive (bullish) versus negative (bearish). A positive (bullish) binary signal is a signal revealing that other investors’ choose to purchase the stock, and a negative (bearish) binary signal is one revealing that other investors' choose to sell the stock.

The effect of a positive (bullish) binary signal on the aggregate demand curve

Suppose the market consists of many investors, two of whom have different SEP-s: One's SEP is higher than the market price and the other has a SEP lower than it. Now, let's assume that they both get a positive (bullish) signal revealing that other investors have chosen to buy the stock. For the buyer, this signal would be a supporting one and for the seller it would be contradicting. Therefore, the demand curve of the buyer would become more elastic, while the demanded quantity at his SEP remains unchanged. However, the curve of the seller would have its elasticity reduced, while his demand at his SEP would also remain unchanged. Figure 6 presents the effect on both investors' demand. As it shows, this signal leads both investors to increase their demand for the stock at its market price. The buyer will increase his demand because his confidence has risen; while the seller, whose confidence has been undone, will decrease the quantity he sells, i.e. increase his demand for the stock. Consciously, this would generate a larger aggregate demand at the stock market price and will result in a higher equilibrium price for the stock.

A comparison of this result to that of the seminal herding models who claim that a signal about other people's choices will only affect the individuals who see the signal as contradicting, reveals the contribution of this model. This framework of analysis proves that a positive signal about the choices of others may increase the demand of all the market participants, both those that view the signal as contradicting and those that view it as supporting.
Figure 6: The effect of a positive binary signal on the aggregate demand curve

**Fig. 6a:** The effect of a positive binary signal on the demand curve – Buyer

- $Q_d$: Demand curve in the absence of signal
- $Q_{d1}$: Demand curve in the presence of signal

**Fig. 6b:** The effect of a positive binary signal on the demand curve – Seller

- $Q_s$: Demand curve in the absence of signal
- $Q_{s1}$: Demand curve in the presence of signal
**The effect of a negative binary signal on the aggregate demand**

Suppose the market consists of many investors, two of whom have different SEP-s: one higher than the market price and the other lower than it. Now, let us assume that they both receive a negative (bearish) signal revealing that other investors choose to sell the stock. For the buyer, this signal would be contradicting and for the seller it would be supporting. Therefore, the demand curve of the buyer would become less elastic, while his demand at his SEP is unchanged, and the curve of the seller would become more elastic, also leaving his demand at his SEP unchanged. As figure 7 shows, this will lead both investors to decrease their demand for the stock at its market price. The seller will sell more, namely decrease his demand because his confidence has risen, and the buyer, whose confidence has been reduced, would decrease the excess quantity he buys. This will generate a lower demand for the stock at the market current price and will lead to a lower equilibrium price for the stock.

In this case of a negative signal, as in the case of a positive one, the model proves the same principle argument regarding the effect of a binary signal: a negative binary signal about the choices of others may decrease the demand of all the market participants, both those that sees the signal as contradicting and those that sees it as supporting.

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**The affect of a binary signal on the aggregate demand when a contradicting signal change both \( b_i \) and \( p_i^* \)**

When we allow \( p_i^* \) to be influenced by a contradicting binary signal, we receive the same direction of influence on the market but the changes would be bigger. Since a reduction in \( p_i^* \) following a contradicting signal will lead to the same direction of change in the demand of a single individual and will only cause the change to be bigger, it will also cause the change in aggregate demand to be in the same direction but bigger. Figure 8 illustrates:
The effect of a negative binary signal on the aggregate demand curve:

Figure 7a: The effect of a negative binary signal on the demand curve - Buyer

Qd = Demand curve in the absence of signal
Qd1 = Demand curve in the presence of signal

Figure 7b: The effect of a negative binary signal on the demand curve - Seller

Qs = Demand curve in the absence of signal
Qs1 = Demand curve in the presence of signal

ΔQd: Change in demand
ΔQs: Change in supply
Figure 8. The effect of a negative binary signal on aggregate demand - the case when the estimated SEP is affected.
A short summary of the effects of binary signals

Although there are differences in the herding reaction of market participants to different types of binary signals, the detailed analysis of the different cases reveal some consistent principle conclusions regarding the effects of a binary signals about the choices of others. These effects may be classified into two types: effects on the market equilibrium price, and effects on the volatility.

- The effect of a binary signal on aggregate demand and equilibrium price: since in most cases any binary signal would be supportive to some investors, while being contradictory to other investors, the effect of a binary signal on the elasticity of the aggregate demand curve may take different forms. Nonetheless, the model does show the contribution of each type of signal to the aggregate demand elasticity: Every supportive signal provided to a participant will increase the elasticity of the aggregate demand and every contradicting signal provided to her will decrease it. Consequently, a signal that would be supportive to most investors would, in most cases, increase the elasticity while a binary signal that would be contradicting to most investors will decrease it. However, despite of it slightly complex conclusion about the effect of a binary signal on the aggregate demand elasticity, the model conclusion about the effect of a binary signal on the aggregate demanded quantity is simple, consistent and clear-cut. The model proves that a positive binary signal would increase the demand of all the market participants (those that sees it as supporting and those that see it as contradicting) at the stock market price, while a negative signal would do the opposite. This leads to the conclusion that herding following a binary positive signal would increase the demand of all participants and therefore increase the market aggregate demand and cause a rise in the equilibrium price, while herding following a binary negative signal would do the opposite and decrease the equilibrium price.

- The effect of a binary signal on the volatility: It is well-known and easy to see that a more elastic demand curve ensures smaller changes in the price in reaction to changes in the market's supply and demand. Hence, a supporting binary signal might actually have a positive influence on the market volatility as it contributes positively to the elasticity of the market
demand, and by enlarging the elasticity of the demand, it helps to reduce volatility. While on the other hand, a contradicting signal contributes to the reduction of the elasticity of the demand and thereby might influence the volatility negatively (increase it).

The effect of observing quantitative information about other investors' choices

A definition of a quantitative signal about other investors' choices

A quantitative signal about others' choice is a signal that reveals other investors' quantitative choices, i.e. it shows the investor how much other investors chose to hold (excess hold) in the stock.

The effect of a quantitative signal about the choice of other investors, on the investor’s decision-making

The effect of quantitative signal differs from that of a binary signal in the following respect: While the binary decision of the investor is only a first-stage decision in a twofold decision-making process, the quantitative decision concludes the process. Therefore, due to the sequential nature of the twofold decision process, a binary signal may play a different role than that played by a quantitative signal in the decision-making process. Moreover, imitation of a binary signal may differ significantly from the imitation of a quantitative signal. Since a binary signal leaves the final decision – the quantitative one – unsolved, imitating it involves thinking and reaching one's own decision after observing it, whereas imitation of a quantitative signal may be a relatively simple act of mimicry.

Note that such primitive mimicry, under the assumption of discrete or bounded space of the choices (e.g. discrete mutually exclusive choices) as seminal herding theories usually assume, may result, according to the known cascade model, in a scenario in which the investor simply copies the others' choice. However, in a continuous space of the choices as in this model, the primitive mimicking process may take a different form and may play only a partial role: when an investor chose to
adjust his prior choice to be closer to that of the others' but not to fully imitate their choice.

The investor's demand curve for a stock in the presence of a quantitative signal about others' choice.

An additional assumption: To simplify, in this part of the model, I add an assumption about the size of the $a_i$. Recall that according to the prior assumptions $a_i$ may vary from one individual to another, nonetheless, since $a_i$ derived from the index, the only differences between different investors' $a_i$ are due to differences in the sizes of their portfolios and in their leverage degree. Therefore, if we assume that all investors have the same size of portfolio and the same leverage degree (no leverage), all investors would have the same $a_i$. This assumption does not change the direction and the principle results that would have been reached without it, yet it allows me to present a simple model (that considers a simple direct imitation). For example, if one investor that has a portfolio of 15,000$ observes a quantitative signal about another investor with a portfolio of 4,000,000$, who chose to invest 100,000$ in the stock, presenting his imitation process might be a little complex. This complication dissipates when both investors have the same size of portfolio. Therefore, to simplify, I add this assumption about "one value of $a_i$".

When an individual observes a quantitative signal he may choose to imitate it to different levels, namely, he may choose to fully follow it, to ignore it or to change his choice partially, so as to more closely resemble the choice he observes.

Let:

$Q_i = \text{The individual's demand before he observes a quantitative signal. Note that this may be the demand that includes or excludes a binary signal.}$

$Q_{Qi} = \text{The individual's demand after he observes a quantitative signal.}$

$Q_{oth} = \text{The quantitative signal = the quantity others hold in the stock.}$

$Q_{Qi} = (1 - \alpha)Q_i + \alpha Q_{oth}$

$Q_{Qi} = (1 - \alpha)(a_i + b_i(p_i^* - p)) + \alpha Q_{oth}$

$Q_{Qi} = (1 - \alpha)a_i + (1 - \alpha)b_i(p_i^* - p) + \alpha Q_{oth}$

Where
\( \alpha \) = A coefficient represents the imitation tendency degree of the individual where \( \alpha \) size is on the scale of 0 to 1. That is, \( \alpha \) is the weight of the signal in a function determining the investor's quantitative choice (while following a quantitative signal) as a weighted average of his choice prior to observing it, and the choice indicated by the signal.

Therefore:
When \( \alpha = 0 \), the investor has no imitation tendency and the quantitative signal does not affect his choice at all.
When \( \alpha = 1 \), the investor fully imitates the other choice, completely mimicking the signal.
When \( 0 < \alpha < 1 \), the investor partially imitates the others' choice. Namely, his choice after observing the signal would more closely resemble the others' choice but would not equal it.

**The effect of a quantitative signal on the demand curve when \( Q_{oth} = a_{i} \)**

This is the case in which the quantitative signal is the average holding of the stock and also equals the individual's \( a_{i} \); Since we assume that all investors have the same \( a_{i} \), this will be the equilibrium average holding in the stock. Note that \( a_{i} \) is the average holding even when different investors see different \( p_{i}^{*} \) and hold different excess holdings of the stock. Since, in equilibrium there is no excess in the markets (the whole excess buying equals the entire excess selling), and since all the excess holdings in that case are defined relative to the same \( a_{i} \) (even when the holding of the stock varies amongst the investors), the \( a_{i} \) equals the average number of shares in a portfolio. Therefore, when the investors are alerted to the fact that the other's holding of the stock \( Q_{oth} = a_{i} \), this provides them with information about the true average holding of the stock.

Following this signal, the demand curve will change in the following way:
\[
Q_{Q_i} = (1 - \alpha)a_i + \alpha Q_{oth} + (1 - \alpha) * b_i (p_{i}^{*} - p) \\
Q_{Q_i} = a_i + (1 - \alpha) * b_i (p_{i}^{*} - p)
\]
Figure 9 shows the change in the demand curve

The less elastic curve describes the demand after observing the quantitative signal. In this case:

- If the investor \( p_i^* \) equals the market price, he would hold \( a_i \) units of the stock after observing the signal, as he did before observing it and his demanded quantity will not change.

- If his \( p_i^* \) is higher than the market price, i.e. he was a buyer before the signal; he will reduce his excess holding in the stock following the signal, in comparison with his excess holding prior to observing it. and

- If his \( p_i^* \) is lower than the market price, i.e. he was a seller before the signal; he will reduce his selling of the stock following the signal in comparison with his selling prior to observing it.

As a result: in this case, the demand curve of the investor would intersect his "pre-observation signal" demand curve at a quantity equaling \( a_i \), namely, at the market average holding, and would be less elastic around the market average holding.

The intuition behind the result is quite simple: since the investor calculates a weighted average between his own choice and the others' choice revealed by the
signal, which in this case equals the \( a_i \), his "pro-observing a signal" choice would be closer to the \( a_i \). The weighted average also gives a smaller weight to the price-dependent part of the demand, (on the account of the other part of the demand which is price-independent), making the demand less sensitive to a price gap-less elastic.

**The effect on the demand curve when \( Q_{oth} \neq a_i \)**

In this case, the signal is biased since it shows a size of holding among the other investors that is bigger or smaller than the average holding. Now, since the signal does not represent the true state of the others' holding it may be considered as a "noised signal".

- **The effect on the demand when \( Q_{oth} > a_i \)**

In this case, the demand is composed of 3 parts.

\[
Q_Q = (1-\alpha)a_i + \alpha Q_{oth} + (1-\alpha)\alpha b_i (p_i^*-p)
\]

The first two parts are constant and independent of the market price and the third part is price-dependent.

Therefore:

- When the market price equals the investor's \( p_i^* \), his holding of the stock would be a weighted average of \( a_i \) and the \( Q_{oth} \) (\( \alpha \) being the weight of the signal), that is a larger quantity of the stock than the \( a_i \) he would have held at this price, prior to observing the signal.

- The average holding between the investor's \( a_i \) and the \( Q_{oth} \), being bigger than \( a_i \), will take place as the constant part of the demand at any market price. Thus, if we compare the demand in this case to the one in the case when \( Q_{oth} = a_i \) we see that in this case, at any given market price, the constant demand will be bigger, while the dependent-price demand remains the same, so the whole demand would be bigger in this case, at any market price than that of the case when \( Q_{oth} = a_i \).

- If we compare the demand in this case to the investor's demand before he observed the signal, we see that while the constant part of it is bigger, the price-independent part of it is smaller. Although it is clear that the investor
demand in this case is bigger than in the case of true information about the average holding, it is not clear whether it is bigger than the demand the investor would have if he had not observed a signal. In fact, since the new demand $Q_{Oi}$ is an average of the prior demand $Q_i$ and the signal $Q_{oh}$, only when $Q_{oh} > Q_i$, we get $Q_{Qi} > Q_i$. Namely, the demand of the investor after observing the signal is not always closer to the true average holding as in the prior case, but closer to the signal. Thus, the investor's demand after observing this signal may become smaller or larger, depending on whether the signal is bigger or smaller than the prior demand.

As for the change in the elasticity of the investor's demand, the same intuition about the reduced sensitivity to the price gap demonstrated in the last case (in the case when $Q_{oh} = a_i$), holds here too.

Comparing the demand curve equation following the signal with that prior to the signal, reveals the following: The curve will have a bigger constant part, its slope will become less elastic relative to the price, and it will intercept the prior demand curve at a quantity bigger then $a_i$, at a quantity equaling the $Q_{oh}$. This last result may be explained intuitively as follows: since the new demand $Q_{Qi}$ is the average of the prior demand $Q_i$ and the signal $Q_{oh}$, the only case in which $Q_{Qi} = Q_i$ is when $Q_{oh} = Q_i$. Figure 10 presents the changed demand curve in this case.

Comparing the demand curves of the last two cases shows that the demand of the investor at any market price when $Q_{oh} > a_i$ would be higher than that in the case when $Q_{oh} = a_i$. 


The effect on the demand when $Q_{oth} < a_i$

Using a similar frame of analysis to the one above, it is easy to see that in this case, the demand curve would have a smaller constant (price-independent) part and would also be less elastic to the price; it will intercept the prior demand curve at a quantity smaller than $a_i$, at a quantity equaling the $Q_{oth}$, and in comparison to the case in which $Q_{oth} = a_i$, in this case at any market price the investor's demand would be smaller.

The effect of a quantitative signal on the aggregate demand curve

While aggregating the individuals' demand curves under the presence of a quantitative signal, into a market aggregate demand, we need to consider the distributions of the additional parameter- the $\alpha$, amongst the different individuals participating in the market: as I did regarding the $a_i$, here too, for the purpose of simplification, I assume that all individuals have the same $\alpha$. The meaning of this assumption is that all individuals have the same "imitative tendency", namely, they all
give the same weight to the quantitative signal in their choice. Once again, relaxing this assumption does not change the principle result, but keeping this assumption, simplifies the model.

I divide the analysis of the effect of the quantitative signal into two category cases: a true unbiased signal and a biased signal.

Recall that since the average holding in the stock is $a_i$, any quantitative signal suggesting that others hold a different amount of the stock is biased.

The effect of a quantitative signal on the aggregate demanded quantity

Let:

$E(Q_i) =$ the average holding of the stock at the market price before the quantitative signal

$E(Q_{Qt}) =$ the average holding of the stock at the market price after the quantitative signal

Since $Q_{Qt} = (1−\alpha)Q + \alpha Q_{oth}$

$E(Q_{Qt}) = E((1−\alpha)Q_i) + E(\alpha Q_{oth})$

$E(Q_{Qt}) = (1−\alpha)E(Q_i) + \alpha E(Q_{oth})$

Thus:

- When $Q_{oth} = a_i$, namely, $Q_{oth} = E(Q_i)$, $E(Q_{Qt}) = E(Q_i)$ the average demanded quantity and the total demanded quantity at the market price would remain the same.
- When $Q_{oth} > a_i$, namely, $Q_{oth} > E(Q_i)$, $E(Q_{Qt}) > E(Q_i)$ the average demanded quantity and the total demanded quantity increase.

And finally, when $Q_{oth} < a_i$, namely, $Q_{oth} < E(Q_i)$, $E(Q_{Qt}) < E(Q_i)$ the average demanded quantity and the total demanded quantity decrease.

The effect of a quantitative signal on the elasticity of the aggregate demand curve

Since in all of the cases regarding different sizes of the $Q_{oth}$, the demand curve of each investor would become less elastic as its slope becomes larger (the slope relative to the price became smaller), and since the aggregate demand curve is a

\[†††\] Given that the market consists of a large enough number of individuals and that the individuals' tendency to herd is not correlated to their SEP size.
horizontal aggregation of the individuals' demand curves; the slope of the aggregate demand curve would also become larger and the curve would become less elastic.

**The effect of a quantitative signal on the aggregate demand when** $Q_{oth} = a_i$

The composition of the two last effects on the aggregate demand curve, in the case when $Q_{oth} = a_i$, shows that the aggregate demand curve intercepts the prior demand curve at the prior market price, while its slope is larger, namely, its elasticity is reduced around the market price.

**The effect of a quantitative signal on the aggregate demand when** $Q_{oth} > a_i$

The composition of the two effects in this case, when $Q_{oth} > a_i$, shows that the aggregate demand curve intercepts the prior demand curve at a larger quantity than the prior equilibrium quantity, namely, at a price lower than the market price, while its slope is larger, namely, its elasticity is reduced around the point where it intercepts the prior demand curve. In this case, it is easy to see that the aggregate demanded quantity at the market price is increased. Thus, this case results in a higher equilibrium price as well as its increased volatility.

**The effect of a quantitative signal on the aggregate demand when** $Q_{oth} < a_i$

The composition of the two effects in this case, when $Q_{oth} < a_i$, shows that the aggregate demand curve intercepts the prior demand curve at a smaller quantity than the prior equilibrium quantity, namely, at a price higher than the market price, while its slope is larger, namely, its elasticity is reduced around the point where it intercepts the prior demand curve. In this case, it is easy to see that the aggregate demanded quantity at the market price is decreased. Thus, this case results in a lower equilibrium price as well as in its increased volatility.

**A short summary of the effects of quantitative signals**

As in the case of binary signals, the detailed analysis of quantitative signals' cases also leads to some consistent conclusions regarding their effects. These effects on the demand curve may be classified into the effects of true signals about others'
choices, on the market equilibrium price and on its volatility and the effects of biased signals.

- **The effect of a true unbiased signal about the choices of others, on the equilibrium price and on the volatility:** As the model shows, this type of signal would not change the average demand for the stock at the market price, so it will not affect the equilibrium price in the short run. However, herd behavior following this type of signal reduces the elasticity of the demand curve of the individual investor, and consequently the elasticity of the aggregate demand curve, and increases the volatility of the stock market price. As a result, although in the short run the stock price remains the same, it will become more sensitive to any change in the demand or the supply, so in reaction to any change of them, the market will exhibit (in the short term) overpricing/underpricing of the stock in comparison to the prices it would have reached, in the absence of herding following quantitative signals about the choices of others.

- **The effect of a biased signal about the choices of others:** in this case, two different effects occur: an effect on the elasticity and an effect on the price of the stock. While the effect on the elasticity of the aggregate demand curve is similar to that in the unbiased signal case, (increasing the volatility in the same way), the effect on the equilibrium price in this case is different: in addition to its effect on the volatility, herd behavior following a biased quantitative signal also changes the average demand for holding, in the same direction as the bias of the signal, namely: it would increase the demand following a "positive biased" signal and would decrease it following a "negative biased" signal. Since the supply is fixed, the amount of stock can not grow in the short run to adjust to the bigger (smaller) demand, so the result will be an immediate rise (fall) in the equilibrium price. Therefore, in this case, the damaging effect would be even bigger than in the prior case when the signal was correct, since in this case the result would be a greater volatility and overpricing (underpricing) in the future as well as an immediate change of the price (in the direction of the bias of the signal).
V. Conclusions

While the existing informational-based herding models assume that observing other investors' choices affects the investor choice by adding new information to her prior set of information regarding the choice, this study maintain that observing other investors' choices has an additional important impact on the investor - an effect on her confidence in her evaluation of the market. The model constructed in this study shows that when the effect of "observational information" on one's generic confidence is taken into account, herd behavior of investors may be caused not only by contradicting signals, signals that change the investor's private set of information regarding a choice, as the prevailing herding models argue, but also by a supportive binary signal.

Furthermore, the current herding models, by assuming a simple "one-stage" decision making, assert that when an individual is facing a choice, information about others' choice that indicates a similar choice to hers will not cause her to change her choice, i.e. to herd. however, since in reality, investing decisions in financial market present the investor with a twofold decision: the decision whether to invest in a given asset (binary choice), and also how much to invest in the asset (quantitative choice), information about binary choices of others that supports the investor's own binary choice may also drive her to herd. This information which may reinforce her prior beliefs, admittedly does not affect her binary choice, but may vicariously, by boosting her confidence in her evaluation, affect her further quantitative decision. Namely, binary supporting signal may cause the investor to positively change her quantitative decision. This last type of herding process is totally absent from existing herding theory, and to the best of my knowledge, is described here for the first time.

The model, constructed in this study, analyzes and maps the different effects of different types of information about other investors' choices, on the demand for the stock, on its equilibrium price and on price volatility. It differentiates binary signals from quantitative ones and shows that a few conclusions about the effect of the signals remain the same under various assumptions and their relaxation:
1. Herd behavior following a contradicting binary signal is accompanied by a lower level of investors' confidence and increases the volatility of the stock price.

2. Herd behavior following a supporting binary signal is accompanied by a higher level of investors' confidence and decreases the volatility of the stock price.

3. Herd behavior following a positive (bullish) binary signal increases the aggregate demand for the stock and increases the "short-run equilibrium price", whether it has a contradicting effect or a supporting effect on the prior information of the investor.

4. Herd behavior following a negative (bearish) binary signal, decreases the aggregate demand for a stock and decreases the "short-run equilibrium price" regardless of whether the signal contradicts or supports the investor's prior information.

5. Herd behavior following any type of quantitative signal increases the volatility of the stock price. Herd behavior following all types of quantitative signals, with the exception of an accurate true one, also changes the short-run equilibrium price.

These conclusions have implications that are of importance for several reasons: first, they show that the effect of herding on volatility depends on the change in the investors' confidence. According to the model observing others may cause a change in the investor confidence which drive her to herd. Nevertheless, the effect of the herding on price volatility is caused by the change in the investor confidence and not by the herding phenomenon itself. While herding accompanied by a lower level of confidence will result in increased price volatility, herding following a higher level of confidence would cause the volatility to decrease. According to the model, the volatility would be higher when the investors are less confident about their evaluation of the market, and would be lower when the investors are more confident, and thereby herding following contradicting observational signals is a distorting phenomenon to the market efficiency, while herding following supporting observational binary signals might increase the level of confidence of the market participants and may even
be beneficial to the market. This conclusion suggests that deducing information from observing others' binary choice may not always distorts efficiency and might, in some cases, play a positive role, making the markets "less nervous". Furthermore, the last conclusion may lead to further question, suggesting that the distorting phenomenon is perhaps not the herding behavior, but the lack of confidence of the participants, that in the base of it.

Second, the model shows that not all types of information generate the same type of herding and cause the same results. That explain why different studies of herding (mainly empirical) led to different and sometimes opposing results about the existence of the phenomenon in financial markets and about its effect on the markets. By mapping the effects of the main basic types of information about others' choices, on the herding of investors and on the market results following this herding, I provide some answers to the yet-unsolved question about the effect of herding on market price volatility and on the equilibrium price.

One important answer address the effects of binary signals in compare of the effect of quantitative signals about the choices of others: Binary signals and quantitative signals affect the choice of the investor in different ways and thereby cause different effects on her demand and as a result on the market. Since the binary decision of the investor is only a first stage decision in a twofold decision making process, a binary signal provides her only with additional information to be taken into account or with an addition (positive or negative) to her confidence. In that sense, a binary signal may only contribute to the set of parameters that stand in the base of the decision making process of the investor, but can not be simply mimicked by her. The model proves that deducing information about an investment by observing this kind of signal, may not always affect the market in a distorting way, and may sometimes reduce volatility and thus may even create beneficial effect. Quantitative signals, on the other hand, may have completely different effect on the investor decision making. Since the quantitative decision concludes the process of the investor decision making, a quantitative signal may be mimicked by her directly, and in the most basic primitive way "copied" by her. The model presented in this work proves that even in a continuous choice space when imitation may be partial, a simple direct imitating of a quantitative signal is always distorting. An imitation of a true quantitative signal, though may not change the equilibrium price, may still decrease elasticity and
increase volatility, while an imitation of a biased quantitative signal may both change the equilibrium price and decrease the elasticity and thereby increase the volatility as well.

Third, the conclusions of the model about the effect of binary signals show how "surprising news" about the choices of others, being contradicting to most participants, decreases the elasticity of the demand and increases the price volatility, while "predicted news" about the choices of others are "good news" in that sense since they increases elasticity and reduces volatility.

References


