האם מחירי מניות מגיבים באיחור למידע ציבורי?

אתי אינהורן

דצמבר 2014

תקציר: מחקרים אמפיריים רבים תיעדו את הנטיה של מחירי מניות להיסחף בכיוון של הפתעת הרווח במשך מספר שבועות, ואפילו חודשים, לאחר הכרזת הרווח. תבנית התמחור הזו ידועה בשם post-earnings announcement drift ונחשבת בספרות לאחת האנומליות הבולטות והמתמיהות ביותר של שווקים פיננסיים. תופעה זו על פי רוב מיוחסת לחוסר יעילות של השוק ונתפסת כמשקפת תגובה לא רציונלית של משקיעים בשוק למידע ציבורי זמין. מחקר זה נוקט בגישה שונה ומבוסס על הטענה שהתופעה יכולה להתקיים בשוק יעיל למחצה כתוצאה רציונלית של קיומו של מידע פרטי המוחזק בידי משקיעים בשוק. המחקר מראה שהתגובה האיטית לכאורה של מחירי מניות למידע ציבורי זמין יכול שהינה מונעת מהתגובה הרציונלית של המחירים למידע פרטי, אשר מתואם במידת מה עם המידע הציבורי, אך מתפזר בין השחקנים בשוק באופן הדרגתי עם חלוף הזמן.

קרית האוניברסיטה, ת.ד 39040, רמת אביב, תל אביב 6139001 טלפון: 03-6407740
Tel Aviv University, POB 39040, Ramat Aviv, Tel Aviv 6139001, Israel; Tel: 972-3-6407740
alizae@tauex.tau.ac.il | www.recanati.tau.ac.il

Do stock prices slowly adjust to public information?

Eti Einhorn[*]

Tel Aviv University

Abstract: Numerous empirical studies document the tendency of cumulative abnormal stock returns to drift in the direction of the earnings surprise for several weeks or even months following an earnings announcement. This pricing pattern, known as the post-earnings announcement drift, is considered in the literature as one of the strongest and most puzzling stock pricing anomalies. It is conventionally ascribed to market inefficiency and commonly perceived as being caused by irrational under reaction of traders to publicly available information. Taking a different tack, this study argues that the post-earnings announcement drift can appear in a semi-strong efficient market as the rational consequence of the existence of privately held information in the market. The study suggests that the seemingly slow adjustment of stock prices to observable public information could actually be driven by a slow adjustment of prices to correlated, unobservable, private information. This argument is established within a traditional rational expectations setting of trading.

Keywords: Post-earnings announcement drift; Information asymmetry; Accounting information; Asset pricing; Rational expectations models; Market anomalies; Market efficiency.

[*] Eti Einhorn, Faculty of Management, Tel Aviv University, Ramat Aviv, Tel Aviv 69978, Israel, Email: einhorn@post.tau.ac.il, Tel: 972-3-6408506, Fax: 972-3-6407738.

1. Introduction

Numerous empirical studies in accounting and finance have documented the way that stock prices respond to earnings announcements. The accumulated empirical evidence indicates the tendency of cumulative abnormal stock returns to drift in the direction of the earnings surprise for several weeks or even months following an earnings announcement. This phenomenon, known as the post-earnings announcement drift, was first noted by Ball and Brown (1968), and afterward confirmed over four decades of intensive research, using different samples and research methods (e.g., Jones and Litzenberger, 1970; Latané, Joy and Jones, 1970; Joy, Litzenberger and McEnally, 1977; Watts, 1978; Latané and Jones, 1979; Rendleman, Jones and Latané, 1982; Foster, Olsen and Shevlin, 1984; Bernard and Thomas, 1989; Bernard and Thomas, 1990; Bhushan, 1994; Chan, Jegadeesh and Lakonishok, 1996; Liang, 2003; Mendenhall, 2004; Livnat and Mendenhall, 2005; Jegadeesh and Livnat, 2006; Narayanamoorthy, 2006; Feldman, Govindaraj, Livnat and Segal, 2010; Cao and Narayanamoorthy, 2012). The continuation of stock returns empirically observed after earnings announcements challenges the traditional market efficiency hypothesis, even in its semi-strong form, as it suggests that stock prices are predictable and thus do not fully and immediately reflect the publicly available information. Empirical attempts to explain the post-earnings announcement drift within the semi-strong market efficiency paradigm as compensation for risk or as resulting from imperfections in research design have so far been unsuccessful. The post-earnings announcement drift is therefore considered in the literature as one of strongest and most puzzling stock pricing anomalies (e.g., Fama, 1998).

The post-earnings announcement drift is widely ascribed in the literature to market inefficiency and commonly perceived as being the consequence of irrational under reaction of traders to publicly available information. This common perception has led to the development of behavioral models that attribute continuation of stock returns to cognitive biases and irrational behavioral patterns of traders (e.g., Daniel, Hirshleifer and Subramanyam, 1998; Barberis, Shleifer and Vishny, 1998; Campbell and Cochrane, 1999) or alternatively to traders' incomplete knowledge of the market structure (e..g, Lewellen and Shanken, 2002). Taking a different tack, this study aims at explaining the continuation of stock returns observed after earnings announcements within the traditional rational expectations paradigm, where traders are assumed to be perfectly rational and their knowledge of the market structure is assumed to be complete. A rational expectations framework has already been employed by Dontoh, Ronen and Sarath (2003) to explain the post-earnings announcement drift. However, they ascribe the empirically observed post-earnings announcement drift to a flaw in the empirical methodology usually utilized to measure abnormal stock returns, which relies solely on price data and does not incorporate in addition other publicly available information like accounting information. This study is instead designed to demonstrate that continuation of abnormal stock returns following earnings announcements is likely to be empirically observed in a market with rational traders even when abnormal stock returns are correctly measured.

Contrary to the conventional perception, this study claims that the post-earnings announcement drift can appear in a semi-strong efficient market, where stock prices adjust to publicly available information instantly and in an unbiased manner, as the rational consequence of the existence of privately held information in the market. The study thus argues that the seemingly slow adjustment of stock prices to observable public information could actually be driven by a slow price adjustment to correlated, unobservable, private information. This argument is indirectly supported by empirical findings indicating that the post-earnings announcement drift is closely related to the level of information asymmetry in the market (e.g., Vega, 2005; Sadka, 2006; Francis, Lafond, Olsson and Schipper, 2007), as well as by empirical findings of an intensive production of private information by equity market participants following earnings announcements (e.g., Barron, Byard and Kim, 2002). The idea that lies in the basis of this study is demonstrated by analyzing a standard rational expectations model of trading in the fashion of Grossman and Stiglitz (1980), where traders exchange shares in a risky asset and a riskless asset on the basis of publicly announced information and privately held information.

As a benchmark, the analysis starts by considering the case where the market is efficient in the strong sense, and thus the equilibrium price of the risky asset is so effective in revealing privately held information that it eliminates any informational asymmetry that may have existed initially in the market. The benchmark case yields equilibrium, where the risk-adjusted return that the risky asset yields immediately after the arrival of information in the market is uncorrelated with the subsequently earned risk-adjusted return. The benchmark equilibrium thus confirms the common wisdom that continuation of risk-adjusted stock returns following public announcements of information (like other kinds of trends in stock returns) contradicts the strong form of the efficient market hypothesis. Surprisingly, however, this conclusion does not carry over to a semi-strong efficient market. To establish this argument, a semi-strong efficient market is then considered by introducing a noise into the model, which masks some of the private information held by informed traders and prevents its full revelation through the price of the risky asset. When trading occurs among traders who hold different beliefs in equilibrium, the arrival of information in the market is followed by positively auto-correlated returns on the risky asset, after adjusting for the risk premium. Contrary to the conventional perception, the continuation of the asset risk-adjusted returns that emerges in equilibrium subsequent to the information arrival does not represent a slow adjustment of the asset price to previously available public information. It rather reflects a slow price adjustment to previously held, correlated, private information. The positive autocorrelation of the asset risk-adjusted returns prevails in equilibrium in spite of the fact that all the traders in the market behave in a perfect rational manner, utilizing correctly and instantly their entire set of information. This is because the same noise that prevents the full revelation of privately held information via the price of the risky asset also prevents the full revelation of the risk premium embedded in the price. So, uninformed traders are incapable of perfectly extracting the risk premium hidden in the price of the risky asset, and thus cannot take advantage of the autocorrelation in the asset risk-adjusted returns.

The paper proceeds as follows. The next section describes the rational expectations model underlying the analysis. Section 3 presents the equilibrium outcomes that emerge from the model and shows how they reconcile with extant empirical findings regarding the behavior of stock returns following earnings announcements. The final section summarizes and offers concluding remarks. Proofs appear in the appendix.

2. Model

A standard rational expectations model of trading in the fashion of Grossman and Stiglitz (1980) is considered, where many traders exchange shares in a riskless asset (e.g., a riskless bond) and a risky asset (e.g., a firm's stock). Trading takes place at a single point of time, whereas liquidation and consumption occur at a certain subsequent time. The economy is large, so traders behave competitively, taking the market price as given. At the trading time, each trader rationally makes optimal portfolio decisions by exchanging assets on the basis of his information set. As a result of the exchange of assets, an equilibrium price for the risky asset is set. At some subsequent point in time, the risky and the riskless assets are liquidated and each trader consumes his holdings. The rest of this section details the parameters and assumptions of the model, which are all assumed to be common knowledge unless otherwise indicated.

The commonly known value of the riskless asset is the numéraire in the market and is fixed at 1.[1] The uncertain liquidation value of the risky asset is represented by the random variable $\tilde{v}$, whose realization becomes commonly known only at the consumption time. The market is composed of a large number of traders. The utility of each trader for wealth w is given by the (negative) exponential function $-e^{-aw}$, where the positive scalar a represents the degree of risk aversion of each trader.[2] The traders might vary in their initial holdings in the riskless asset and in the risky asset, but their optimal holdings in the assets when trading takes place are independent of their initial endowments due to the exponential structure of their utility function.

All traders initially believe that the uncertain liquidation value $\tilde{v}$ of the risky asset is normally distributed with mean μ and variance $1/h$. Before trading takes place, a public signal (which can be interpreted as an earnings announcement) arrives in the market, resolving part of the uncertainty that the traders face with respect to the value $\tilde{v}$ of the risky asset. The public signal, which is observable to all traders, is a noisy estimator of $\tilde{v}$ represented by the random variable $\tilde{s}_1 = \tilde{v} + \tilde{\varepsilon}_1$, where the noise term $\tilde{\varepsilon}_1$ is an independent, normally distributed, random variable with mean zero and variance $1/h_1$. A fraction $0 < \lambda < 1$ of the traders is assumed to be privately informed.

---

[1] Assuming a risk-free interest rate of zero simplifies the analysis and its exposition, but the results of the analysis can be easily generalized to a setting with a positive risk-free interest rate.

[2] For simplicity, a uniform degree of risk aversion is applied to all traders. However, the results of the analysis can be generalized to a setting where traders vary in their degree of risk aversion.

The privately informed traders additionally receive before trading a private signal of $\tilde{v}$ that enables them to more accurately evaluate the risky asset. Their private signal is represented by the random variable $\tilde{s}_2$, which takes the form $\tilde{s}_2 = \tilde{v} + \tilde{\varepsilon}_2$, where the noise term $\tilde{\varepsilon}_2$ is an independent, normally distributed, random variable with mean zero and variance $1/h_2$.[3] The parameter h represents the precision of the market's prior beliefs about the value $\tilde{v}$ of the risky asset, whereas the parameters h_1 and h_2 capture the precision of the public signal $\tilde{s}_1$ and the private signal $\tilde{s}_2$, respectively, in estimating the uncertain liquidation value $\tilde{v}$ of the risky asset. The realizations s_1 and s_2 of the random variables $\tilde{s}_1$ and $\tilde{s}_2$, respectively, represent the informational content of the two signals.

As standard in the literature, in order to mask some of the private information held by privately informed traders and prevent its full revelation via the price, the aggregate supply for the risky asset at the trading time is assumed to be uncertain due to the existence in the market of additional noise traders, whose trading is motivated by liquidity needs. The per-capita supply for the risky asset, denoted $\tilde{x}$, is accordingly considered as an independent, normally distributed, random variable with mean μ_x and variance σ_x^2. The edge case of $\sigma_x^2 = 0$ depicts a market that is efficient in the strong form, where the price of the risky asset fully reflects both the content of the public signal $\tilde{s}_1$ and the content of the private signal $\tilde{s}_2$. In all other cases, where $\sigma_x^2 > 0$, the model describes a semi-strong efficient market, where the price of the risky asset fully reflects the content of the public signal $\tilde{s}_1$ but only partially reveals the content of the private signal $\tilde{s}_2$.

---

[3] The assumption that the two noise terms, $\tilde{\varepsilon}_1$ and $\tilde{\varepsilon}_2$, are uncorrelated is a simplifying assumption, which does not qualitatively affect the results of the analysis. Also, for simplicity, it is assumed that all informed traders observe the same private signal. The results of the analysis, however, also hold when considering a setting where informed traders receive different private signals.

Equilibrium in the model consists of three functions: $P:\Re^3 \to \Re$, $D^I:\Re^3 \to \Re$ and $D^U:\Re^2 \to \Re$. The function $P:\Re^3 \to \Re$ represents the market pricing rule, where $P(s_1, s_2, x)$ is the price of the risky asset at the trading time, given that s_1 and s_2 are the realizations of the public signal $\tilde{s}_1$ and the private signal $\tilde{s}_2$, respectively, and x is the realization of the random per-capita supply $\tilde{x}$ for the risky asset. The function $D^I:\Re^3 \to \Re$ is the demand function of an informed trader for the risky asset, where $D^I(s_1, s_2, p)$ is the optimal holding in the risky asset of each informed trader at the trading time for any given price p, after observing the realization s_1 of the public signal $\tilde{s}_1$ and the realization s_2 of the private signal $\tilde{s}_2$. The function $D^U:\Re^2 \to \Re$ is the demand function of an uninformed trader for the risky asset, where $D^U(s_1, p)$ is the optimal holding in the risky asset of each uninformed trader at the trading time for any given price p, after observing the realization s_1 of the public signal $\tilde{s}_1$. In a rational expectations equilibrium, the following conditions must hold for any $s_1, s_2, x, p \in \Re$:

(i) $$D^I(s_1, s_2, p) \in \arg\max_{d\in\Re} E[-e^{-ad(\tilde{v}-p)} \mid \tilde{s}_1 = s_1, \tilde{s}_2 = s_2]$$

(ii) $$D^U(s_1, p) \in \arg\max_{d\in\Re} E[-e^{-ad(\tilde{v}-p)} \mid \tilde{s}_1 = s_1, P(s_1, \tilde{s}_2, \tilde{x}) = p]$$

(iii) $$\lambda D^I(s_1, s_2, P(s_1, s_2, x)) + (1-\lambda) D^U(s_1, P(s_1, s_2, x)) = x$$

The first equilibrium condition pertains to the demand function of the informed traders, requiring that each informed trader sets the demand for the risky asset that maximizes his expected utility at the consumption time for a given price, based on observing the realization of the public signal $\tilde{s}_1$ and the realization of the private signal $\tilde{s}_2$. The second equilibrium condition relates to the demand function of the uninformed traders, requiring that each uninformed trader sets the demand for the risky asset that maximizes his expected utility at the consumption time for a given price, based on observing the realization of the public signal $\tilde{s}_1$, and utilizing his rational expectations about the pricing rule $P:\Re^3 \rightarrow \Re$ in an attempt to most accurately elicit the content of the private signal $\tilde{s}_2$ from the price. The last equilibrium condition describes the market clearing condition, imposing the per-capita demand for the risky asset to be equal to the per-capita supply x.

As standard in the literature, the analysis is restricted to equilibria with a linear pricing rule $P:\Re^3 \rightarrow \Re$. Accordingly, the price $P(s_1, s_2, x)$ of the risky asset is assumed to be a linear function of the content s_1 of the public signal, the content s_2 of the private signal, and the realization x of the random per-capita supply for the risky asset. Linear equilibria are commonly assumed in the rational expectations literature. When combined with a negative exponential utility function for the traders and a normal distribution of the uncertain value of the risky asset and the information about it, a linear pricing function enables a tractable analysis and yields equilibrium outcomes that can be analytically characterized and intuitively explained. As linearity restrictions are commonly made in empirical research, the assumption of a linear pricing rule is also useful in linking theoretical results to empirical findings and in making theoretically based predictions that readily map into linear empirical frameworks.

3. Equilibrium Analysis

The equilibrium analysis starts by considering the benchmark case of $\sigma_x^2 = 0$, where the per-capita supply for the risky asset is commonly known to be μ_x. As has been well established in the literature (Grossman and Stiglitz, 1980), when the per-capita supply for the risky asset is commonly known, the equilibrium market price of the risky asset is so effective in revealing privately held information that it eliminates any informational asymmetry that may have existed initially in the market. The benchmark case of $\sigma_x^2 = 0$ thus depicts a market that is efficient in the strong form. While such an extreme model is not very descriptive of real financial markets, it nevertheless provides a natural theoretical point of reference to the analysis. Consistent with the earlier literature, Observation 1 formally indicates the existence and uniqueness of equilibrium with a linear pricing rule in the benchmark case of $\sigma_x^2 = 0$, denoted by the subscript B, and characterizes the form of the equilibrium pricing rule.

Observation 1. *In the benchmark case of* $\sigma_x^2 = 0$ *(a strong form of market efficiency), the model yields a unique equilibrium with a linear pricing rule. The equilibrium pricing rule* $P_B : \Re^3 \to \Re$ *takes the form* $P_B(s_1, s_2, \mu_x) = E[\tilde{v} | \tilde{s}_1 = s_1, \tilde{s}_2 = s_2] - a\,\mathrm{var}[\tilde{v} | \tilde{s}_1, \tilde{s}_2]\mu_x = \dfrac{h\mu + h_1 s_1 + h_2 s_2 - a\mu_x}{h + h_1 + h_2}$ *for any* $s_1, s_2 \in \Re$.

By Observation 1, the equilibrium market price of the risky asset in the benchmark case of $\sigma_x^2 = 0$ equals the market expectations $E[\tilde{v} | \tilde{s}_1 = s_1, \tilde{s}_2 = s_2] = \dfrac{h\mu + h_1 s_1 + h_2 s_2}{h + h_1 + h_2}$ about the uncertain liquidation value $\tilde{v}$ of the risky asset, minus a risk premium of $a\,\mathrm{var}[\tilde{v} | \tilde{s}_1, \tilde{s}_2]\mu_x = \dfrac{a\mu_x}{h + h_1 + h_2}$ associated with holding the risky asset. In order to calculate the risk-adjusted returns earned on the risky asset after the information arrival, it is useful to use the notation $\hat{P}_B(s_1, s_2, \mu_x)$ to denote the price of the risky asset after neutralizing the component $\dfrac{a\mu_x}{h + h_1 + h_2}$ in the price that captures the risk-premium. Accordingly, $\hat{P}_B(s_1, s_2, \mu_x)$ is defined as $P_B(s_1, s_2, \mu_x) + \dfrac{a\mu_x}{h + h_1 + h_2}$ and it therefore equals $\frac{h\mu + h_1 s_1 + h_2 s_2}{h + h_1 + h_2}$. Using this notation, the random variable $\tilde{v} - \hat{P}_B(\tilde{s}_1, \tilde{s}_2, \mu_x)$ represents the risk-adjusted return earned on the risky asset in the period that starts at the trading date and ends at the liquidation date (whereas $\tilde{v} - P_B(\tilde{s}_1, \tilde{s}_2, \mu_x)$ captures the raw return for the same period).[4] Similarly, the previously earned risk-adjusted return is represented by the random variable $\hat{P}_B(\tilde{s}_1, \tilde{s}_2, \mu_x) - \mu$.

In the benchmark case of $\sigma_x^2 = 0$, from observing the price of the risky asset, uninformed traders can perfectly deduce the content s_2 of the private signal $\tilde{s}_2$, which is the sole ingredient in the price that is unobservable to them. Consistent with the earlier literature, therefore, the benchmark equilibrium price of the risky asset fully reveals the information held by the informed traders to the uninformed traders, and thus the initially existing asymmetry in information among the traders totally disappears in equilibrium. Hence, the benchmark case of $\sigma_x^2 = 0$ depicts a strong efficient market, where both the content of the public signal $\tilde{s}_1$ and the content of the private signal $\tilde{s}_2$ are fully reflected in the price of the risky asset. Consequently, consistent with the common wisdom, the risk-adjusted return $\tilde{v} - \hat{P}_B(\tilde{s}_1, \tilde{s}_2, \mu_x)$, which is earned on the risky asset after the traders exchange assets on the basis of their information, is not correlated with either the public signal $\tilde{s}_1$ or the private signal $\tilde{s}_2$. It is consequently also uncorrelated with the previously earned risk-adjusted return $\hat{P}_B(\tilde{s}_1, \tilde{s}_2, \mu_x) - \mu$ on the risky asset. These results are formally presented in Observation 2.

---

[4] For simplicity, the return on the risky asset is taken throughout the paper as the change in its price, rather than the rate of the change in price.

Observation 2. *In the benchmark case of* $\sigma_x^2 = 0$ *(a strong form of market efficiency), using the notation* $\hat{P}_B(s_1, s_2, \mu_x) = P_B(s_1, s_2, \mu_x) + \frac{a\mu_x}{h + h_1 + h_2}$*, the unique linear equilibrium satisfies:*

(i) $\operatorname{cov}[\tilde{v} - \hat{P}_B(\tilde{s}_1, \tilde{s}_2, \mu_x), \tilde{s}_1] = 0$ *;*

(ii) $\operatorname{cov}[\tilde{v} - \hat{P}_B(\tilde{s}_1, \tilde{s}_2, \mu_x), \tilde{s}_2] = 0$ *; and*

(iii) $\operatorname{cov}[\tilde{v} - \hat{P}_B(\tilde{s}_1, \tilde{s}_2, \mu_x), \hat{P}_B(\tilde{s}_1, \tilde{s}_2, \mu_x) - \mu] = 0$.

It follows from Observation 2 that, when the market is efficient in the strong sense, the risk-adjusted return $\hat{P}_B(\tilde{s}_1, \tilde{s}_2, \mu_x) - \mu$, which the risky asset yields immediately after the arrival of information in the market, is uncorrelated with the subsequently earned risk-adjusted return $\tilde{v} - \hat{P}_B(\tilde{s}_1, \tilde{s}_2, \mu_x)$. Observation 2 thus confirms the common perception that autocorrelation in risk-adjusted stock returns cannot appear in a strong efficient market. The strong form of the market efficiency hypothesis, which implies fully revealing prices, is however conventionally considered as being not very descriptive of the realm of financial markets. Fully revealing equilibrium prices may seem appealing at first glance, as they portray the market prices as being the invisible hand that somehow disperses the available information in the market among all its participants. Nevertheless, the full revelation result appears to be too strong on both empirical and theoretical grounds. It stands in contrast with the common view that information asymmetries do exist in capital markets and even provide the primary motivation for the trade in these markets. It is also inconsistent with costly acquisition of private information. Fully revealing equilibrium prices remove the incentives for the market participants to spend resources on collecting costly information, because the exact same information can be fully and freely obtained from observing the price. Yet, the price can be so informative only if some information is collected by the market participants. This paradox is the seminal impossibility result of Grossman and Stiglitz (1980), which implies that it is logically impossible for financial markets to be efficient in the strong sense. The analysis thus continues by considering the more descriptive case of $\sigma_x^2 > 0$, where the uncertainty of the traders about the per-capita supply for the risky asset masks some of the private information held by informed traders and prevents its full revelation through the price. Under the widely employed assumption that $\sigma_x^2 > 0$, and consistent with the earlier literature, the model depicts a semi-strong efficient market, where the price of the risky asset fully reflects the publicly available information but only partially conveys the privately held information. Observation 3 formally indicates the existence and uniqueness of equilibrium with a linear pricing rule in the case of $\sigma_x^2 > 0$ and characterizes the form of the equilibrium pricing rule.

Observation 3. *In the case of* $\sigma_x^2 > 0$ *(a semi-strong form of market efficiency), the model yields a unique equilibrium with a linear pricing rule. The equilibrium pricing rule* $P:\Re^3 \to \Re$ *takes the form* $P(s_1,s_2,x) = \frac{\varphi_1}{\varphi_1+\varphi_2} E\left[\tilde{v}\,\middle|\,\tilde{s}_1 = s_1, \tilde{s}_2 = s_2\right] + \frac{\varphi_2}{\varphi_1+\varphi_2} E\left[\tilde{v}\,\middle|\,\tilde{s} = s_1, \tilde{s}_2 - \frac{a}{\lambda h_2}(\tilde{x}-\mu_x) = s_2 - \frac{a}{\lambda h_2}(x-\mu_x)\right] - \frac{ax}{\varphi_1+\varphi_2}$

for any $s_1, s_2, x \in \Re$*, where* $\varphi_1 = \lambda \operatorname{var}\left[\tilde{v}\,\middle|\,\tilde{s}_1, \tilde{s}_2\right]^{-1}$ *and* $\varphi_2 = (1-\lambda)\operatorname{var}\left[\tilde{v}\,\middle|\,\tilde{s}_1, \tilde{s}_2 - \frac{a}{\lambda h_2}(\tilde{x}-\mu_x)\right]^{-1}$*. The equilibrium pricing rule* $P:\Re^3 \to \Re$ *can be also represented as follows for any* $s_1, s_2, x \in \Re$*:*

$$P(s_1,s_2,x) = \frac{h\mu + h_1 s_1 + \lambda h_2 s_2 + (1-\lambda) q_2 \left(s_2 - \frac{a}{\lambda h_2}(x-\mu_x)\right) - ax}{h + h_1 + \lambda h_2 + (1-\lambda) q_2}, \quad \text{where } q_2 = h_2\left(1 + \frac{a^2\sigma_x^2}{\lambda^2 h_2}\right)^{-1}.$$

As has been well established in the literature, Observation 3 indicates that, although the equilibrium price $P(s_1,s_2,x)$ of the risky asset reflects both the content s_1 of the public signal $\tilde{s}_1$

and the content s_2 of the private signal $\tilde{s}_2$ possessed by the informed traders, it nevertheless deviates from the benchmark fully revealing price $P_B(s_1, s_2, x)$ given in Observation 1. The equilibrium price places more weight on the public signal and less weight on the private signal relative to the benchmark price. This is because the equilibrium price now only imperfectly conveys the private signal $\tilde{s}_2$ from the informed traders to the uninformed traders. To capture the information embedded in the equilibrium price of the risky asset, it should be noted that the price $P(s_1, s_2, x) = \dfrac{h\mu + h_1 s_1 + \lambda h_2 s_2 + (1-\lambda) q_2 \left(s_2 - \dfrac{a}{\lambda h_2}(x - \mu_x) \right) - ax}{h + h_1 + \lambda h_2 + (1-\lambda) q_2}$ of the risky asset, as presented in Observation 3, can also be represented (after simple algebraic rearrangements) as a linear function of s_1 and $s_2 - \dfrac{a}{\lambda h_2}(x - \mu_x)$. Therefore, the uninformed traders, who face uncertainty about the realizations of both $\tilde{s}_2$ and $\tilde{x}$, cannot perfectly decipher the content of the private signal $\tilde{s}_2$ from the price. They cannot unequivocally determine whether a high (low) market price of the risky asset stems from a favorable (unfavorable) content of the privately held signal $\tilde{s}_2$ or is alternatively caused by a low (high) realization of the random per-capita supply $\tilde{x}$ for the risky asset. To the uninformed traders, the price conveys only the realization of the random variable $\tilde{s}_2 - \dfrac{a}{\lambda h_2}(\tilde{x} - \mu_x)$, which is a noisy estimator of the signal $\tilde{s}_2$. The precision of the noisy signal $\tilde{s}_2 - \dfrac{a}{\lambda h_2}(\tilde{x} - \mu_x)$ in estimating the value $\tilde{v}$ of the risky asset is $q_2 = h_2 \left(1 + \dfrac{a^2 \sigma_x^2}{\lambda^2 h_2} \right)^{-1}$, which is lower than the precision h_2 of the private signal $\tilde{s}_2$ in evaluating the risky asset. Corollary 4 clarifies that, in equilibrium, the publicly available information set at the trading time is equivalent to the information content of $\tilde{s}_1$ and $\tilde{s}_2 - \frac{a}{\lambda h_2}(\tilde{x} - \mu_x)$.

Corollary 4. *In the case of* $\sigma_x^2 > 0$ *(a semi-strong form of market efficiency), the information content conveyed in the public signal* $\tilde{s}_1$ *and in the equilibrium price* $P(\tilde{s}_1, \tilde{s}_2, \tilde{x})$ *of the risky asset is equivalent to that of* $\tilde{s}_1$ *and* $\tilde{s}_2 - \frac{a}{\lambda h_2}(\tilde{x} - \mu_x)$.

The results presented in Observation 3 and Corollary 4, which are all well known in the literature, are now utilized in order to demonstrate that continuation in stock returns can appear in a semi-strong efficient market after the announcement of public information. To do so, it should be noted that, in accordance with Corollary 4, the equilibrium market price of the risky asset, as presented in Observation 3, is a weighted average of the expectations $E\left[\tilde{v} \middle| \tilde{s}_1 = s_1, \tilde{s}_2 = s_2\right]$ of the informed traders about the uncertain value $\tilde{v}$ of the risky asset and the different expectations $E\left[\tilde{v} \middle| \tilde{s}_1 = s_1, \tilde{s}_2 - \frac{a}{\lambda h_2}(\tilde{x} - \mu_x) = s_2 - \frac{a}{\lambda h_2}(x - \mu_x)\right]$ of the uninformed traders about $\tilde{v}$, minus a risk premium of $\frac{ax}{\varphi_1 + \varphi_2}$, which equals $ax\left(\lambda \operatorname{var}\left[\tilde{v} \middle| \tilde{s}_1, \tilde{s}_2\right]^{-1} + (1-\lambda)\operatorname{var}\left[\tilde{v} \middle| \tilde{s}_1, \tilde{s}_2 - \frac{a}{\lambda h_2}(\tilde{x} - \mu_x)\right]^{-1}\right)^{-1}$ or $\frac{ax}{h + h_1 + \lambda h_2 + (1-\lambda) q_2}$. As in the benchmark case, it is again useful to use the notation $\hat{P}(s_1, s_2, x)$ to denote the equilibrium market price of the risky asset after neutralizing the component $\frac{ax}{h + h_1 + \lambda h_2 + (1-\lambda) q_2}$ in the price that captures the risk-premium associated with holding the risky asset. Accordingly, $\hat{P}(s_1, s_2, x)$ is defined as $P(s_1, s_2, x) + \dfrac{ax}{h + h_1 + \lambda h_2 + (1-\lambda)q_2}$. That is, the random variable $\hat{P}(\tilde{s}_1, \tilde{s}_2, \tilde{x}) - \mu$ represents the risk-adjusted return that the risky asset yields immediately after the arrival of information in the market, whereas the subsequently earned risk-adjusted return is represented by the random variable $\tilde{v} - \hat{P}(\tilde{s}_1, \tilde{s}_2, \tilde{x})$.[5] Using this notation, Proposition 5 presents the main results of this study.

Proposition 5. *In the case of* $\sigma_x^2 > 0$ *(a semi-strong form of market efficiency), using the notation* $\hat{P}(s_1, s_2, x) = P(s_1, s_2, x) + \dfrac{ax}{h + h_1 + \lambda h_2 + (1-\lambda)q_2}$ *where* $q_2 = h_2\left(1 + \dfrac{a^2\sigma_x^2}{\lambda^2 h_2}\right)^{-1}$*, the unique linear equilibrium satisfies:*

(i) $\mathrm{cov}[\tilde{v} - \hat{P}(\tilde{s}_1, \tilde{s}_2, \tilde{x}), \tilde{s}_1] = 0$*;*

(ii) $\mathrm{cov}[\tilde{v} - \hat{P}(\tilde{s}_1, \tilde{s}_2, \tilde{x}), \tilde{s}_2 - \dfrac{a}{\lambda h_2}(\tilde{x} - \mu_x)] = 0$*;*

(iii) $\mathrm{cov}[\tilde{v} - \hat{P}(\tilde{s}_1, \tilde{s}_2, \tilde{x}), \tilde{s}_2] = \dfrac{(1-\lambda)a^2\sigma_x^2}{a^2\sigma_x^2(h + h_1 + \lambda h_2) + \lambda^2 h_2 (h + h_1 + h_2)} > 0$*; and*

(iv) $\mathrm{cov}[\tilde{v} - \hat{P}(\tilde{s}_1, \tilde{s}_2, \tilde{x}), \hat{P}(\tilde{s}_1, \tilde{s}_2, \tilde{x}) - \mu] = \dfrac{(1-\lambda)\lambda h_2 \left(\lambda^2 h_2 + a^2\sigma_x^2\right) a^2\sigma_x^2}{\left(a^2\sigma_x^2\left(h + h_1 + \lambda h_2\right) + \lambda^2 h_2\left(h + h_1 + h_2\right)\right)^2} > 0$.

---

[5] The reliance of the analysis on a model with a single round of trading significantly simplifies the exposition of the results, without detracting from their generality. This is because the price of the risky asset prior to the information arrival is anyway independent of the random variables $\tilde{s}_1$, $\tilde{s}_2$ and $\tilde{x}$, so it can be considered as an exogenously given constant, whereas the price of the risky asset at the liquidation time must be the realized value of $\tilde{v}$. Indeed, the results of the analysis qualitatively prevail when an overlapping-generation model with three rounds of trading is considered, in which public information and private information arrive in the market before the second round of trading and all the privately held information becomes commonly known before the third round of trading.

It follows from Proposition 5 that the risk-adjusted return $\tilde{v} - \hat{P}(\tilde{s}_1, \tilde{s}_2, \tilde{x})$ on the risky asset continues to be uncorrelated with the public signal $\tilde{s}_1$ when $\sigma_x^2 > 0$, as in the benchmark case of $\sigma_x^2 = 0$. Proposition 5 further demonstrates that, in the case of $\sigma_x^2 > 0$, the risk-adjusted return $\tilde{v} - \hat{P}(\tilde{s}_1, \tilde{s}_2, \tilde{x})$ on the risky asset is also uncorrelated with the noisy information $\tilde{s}_2 - \frac{a}{\lambda h_2}(\tilde{x} - \mu_x)$ that the uninformed traders can extract from the market price of the risky asset about the unobservable private signal $\tilde{s}_2$. Recalling that the equilibrium market price $P(s_1, s_2, x)$ of the risky asset is a linear function of s_1 and $s_2 - \frac{a}{\lambda h_2}(x - \mu_x)$, it immediately follows from the equalities $\mathrm{cov}[\tilde{v} - \hat{P}(\tilde{s}_1, \tilde{s}_2, \tilde{x}), \tilde{s}_1] = 0$ and $\mathrm{cov}[\tilde{v} - \hat{P}(\tilde{s}_1, \tilde{s}_2, \tilde{x}), \tilde{s}_2 - \frac{a}{\lambda h_2}(\tilde{x} - \mu_x)] = 0$ given in Proposition 5 that $\mathrm{cov}[\tilde{v} - \hat{P}(\tilde{s}_1, \tilde{s}_2, \tilde{x}), P(\tilde{s}_1, \tilde{s}_2, \tilde{x})] = 0$. Therefore, the information set publicly available at the trading time, which includes the realization of the public signal $\tilde{s}_1$ and the information content that the market price $P(\tilde{s}_1, \tilde{s}_2, \tilde{x})$ of the risky asset conveys, is not useful at all in predicting the subsequent risk-adjusted return $\tilde{v} - \hat{P}(\tilde{s}_1, \tilde{s}_2, \tilde{x})$ on the risky asset.

However, Proposition 5 further shows that, unlike the benchmark case, the risk-adjusted return $\tilde{v} - \hat{P}(\tilde{s}_1, \tilde{s}_2, \tilde{x})$ on the risky asset is positively correlated with the private signal $\tilde{s}_2$ when $\sigma_x^2 > 0$. Intuitively, when the per-capita supply for the risk asset is random, the equilibrium price of the risky asset fully reflects the realization of the public signal $\tilde{s}_1$, but only partially conveys the realization of the private signal $\tilde{s}_2$. Therefore, as time goes by and new public information about the value of the risky asset arrives in the market, subsuming the private signal $\tilde{s}_2$, the risky asset yields a risk-adjusted return that is positively correlated with the previously held private signal $\tilde{s}_2$, although it is uncorrelated with the previously available public signal $\tilde{s}_1$.[6] The positive correlation between the risk-adjusted return $\tilde{v} - \hat{P}(\tilde{s}_1, \tilde{s}_2, \tilde{x})$ earned on the risky asset and the previously held private signal $\tilde{s}_2$ implies a positive autocorrelation in the asset risk-adjusted returns following the arrival of information in the market. Hence, as formally stated in Proposition 5, when $\sigma_x^2 > 0$, the risk-adjusted return $\hat{P}(\tilde{s}_1, \tilde{s}_2, \tilde{x}) - \mu$, which is earned on the risky asset immediately after the public announcement of information, appears to be positively correlated with the subsequently earned risk-adjusted return $\tilde{v} - \hat{P}(\tilde{s}_1, \tilde{s}_2, \tilde{x})$. The inequality $\mathrm{cov}[\tilde{v} - \hat{P}(\tilde{s}_1, \tilde{s}_2, \tilde{x}), \hat{P}(\tilde{s}_1, \tilde{s}_2, \tilde{x}) - \mu] > 0$ given in Proposition 5 is the most important result of this study, as it provides a theoretical foundation for the empirical evidence of a positive autocorrelation in stock returns following earnings announcements.[7]

The equilibrium exhibits a positive autocorrelation of the risk-adjusted returns earned on the risky asset after the information arrival in spite of the fact that all the traders in the market behave in a perfectly rational manner, utilizing correctly and instantly their entire set of information.

---

[6] In the model, the arrival of new public information in the market entirely resolves the uncertainty about the value $\tilde{v}$ of the risky asset. This is a simplifying modeling assumption that makes it possible to draw the results from the analysis of a model with a single trading round. However, the results continue to hold when a model with more than one round of trading is analyzed, assuming that the content of the signal $\tilde{s}_2$ is privately observed only by informed traders at one round of trading and subsequently becomes commonly known at the following round of trading.

[7] While it follows from Proposition 5 that the risk-adjusted returns on the risky asset exhibit a positive autocorrelation (i.e., $\mathrm{cov}[\tilde{v} - \hat{P}(\tilde{s}_1, \tilde{s}_2, \tilde{x}), \hat{P}(\tilde{s}_1, \tilde{s}_2, \tilde{x}) - \mu] > 0$), it appears that the raw returns (before adjusting for the risk premium) on the risky asset are negatively autocorrelated (i.e., $\mathrm{cov}[\tilde{v} - P(\tilde{s}_1, \tilde{s}_2, \tilde{x}), P(\tilde{s}_1, \tilde{s}_2, \tilde{x}) - \mu] < 0$). This observation (whose proof appears as a supplement to the proof of Proposition 5) implies that the sign of any empirically documented autocorrelation in risk-adjusted stock returns is highly sensitive to the empirical estimation of the risk premium. It thus perhaps provides a clue that might guide researchers in reconciling the empirical evidence of a short-term positive autocorrelation in stock returns (e.g., Jegadeesh and Titman, 1993) with the empirical evidence of a long-term negative autocorrelation in stock returns (e.g., DeBondt and Thaler, 1985).

Following the public announcement of the signal $\tilde{s}_1$ and the production of the private signal $\tilde{s}_2$ by privately informed traders, the change in the expectations of each individual trader about the value of the risky asset is indeed uncorrelated over time. That is, the initial change $E[\tilde{v}|\tilde{s}_1, \tilde{s}_2] - \mu$ in the expectations of the informed traders about the value $\tilde{v}$ of the risky asset is followed by the uncorrelated change $\tilde{v} - E[\tilde{v}|\tilde{s}_1, \tilde{s}_2]$ in their expectations. Similarly, the initial change $E\left[\tilde{v}\middle|\tilde{s}_1, \tilde{s}_2 - \frac{a}{\lambda h_2}(\tilde{x} - \mu_x)\right] - \mu$ in the expectations of the uninformed traders about the value $\tilde{v}$ of the risky asset is followed by the uncorrelated change $\tilde{v} - E\left[\tilde{v}\middle|\tilde{s}_1, \tilde{s}_2 - \frac{a}{\lambda h_2}(\tilde{x} - \mu_x)\right]$ in their expectations. Yet, a positive autocorrelation of the asset risk-adjusted returns emerges in equilibrium when the heterogeneous expectations of all traders are combined together into the price formation process. This is because the initial risk-adjusted return $\hat{P}(\tilde{s}_1, \tilde{s}_2, \tilde{x}) - \mu$ on the risky asset is a weighted average of $E[\tilde{v}|\tilde{s}_1, \tilde{s}_2] - \mu$ and $E\left[\tilde{v}\middle|\tilde{s}_1, \tilde{s}_2 - \frac{a}{\lambda h_2}(\tilde{x} - \mu_x)\right] - \mu$, whereas the subsequent risk-adjusted return $\tilde{v} - \hat{P}(\tilde{s}_1, \tilde{s}_2, \tilde{x})$ on the risky asset is a weighted average of $\tilde{v} - E[\tilde{v}|\tilde{s}_1, \tilde{s}_2]$ and $\tilde{v} - E\left[\tilde{v}\middle|\tilde{s}_1, \tilde{s}_2 - \frac{a}{\lambda h_2}(\tilde{x} - \mu_x)\right]$. So, the correlation between the sequential risk-adjusted returns $\hat{P}(\tilde{s}_1, \tilde{s}_2, \tilde{x}) - \mu$ and $\tilde{v} - \hat{P}(\tilde{s}_1, \tilde{s}_2, \tilde{x})$ reflects not only the correlation between the sequential changes in the expectations of each particular type of traders. It is also affected by the correlation between the initial change in the expectations of one type of traders with the subsequent change in the expectations of another type of traders. The sequential changes in the expectations of each particular type of traders about the value $\tilde{v}$ of the risky asset are uncorrelated. Also, the initial change $E\left[\tilde{v}\middle|\tilde{s}_1,\tilde{s}_2-\frac{a}{\lambda h_2}(\tilde{x}-\mu_x)\right]-\mu$ in the expectations of the uninformed traders about $\tilde{v}$ is uncorrelated with the subsequent change $\tilde{v}-E[\tilde{v}|\tilde{s}_1,\tilde{s}_2]$ in the expectations of the informed traders about $\tilde{v}$. However, the initial change $E[\tilde{v}|\tilde{s}_1,\tilde{s}_2]-\mu$ in the expectations of the informed traders about $\tilde{v}$ is positively correlated with the subsequent change $\tilde{v}-E\left[\tilde{v}\middle|\tilde{s}_1,\tilde{s}_2-\frac{a}{\lambda h_2}(\tilde{x}-\mu_x)\right]$ in the expectations of the uninformed traders about $\tilde{v}$. This triggers the positive autocorrelation in the sequential risk-adjusted returns $\hat{P}(\tilde{s}_1,\tilde{s}_2,\tilde{x})-\mu$ and $\tilde{v}-\hat{P}(\tilde{s}_1,\tilde{s}_2,\tilde{x})$ earned on the risky asset after the arrival of information in the market.

Contrary to the conventional perception, the continuation in the asset risk-adjusted returns that arises in equilibrium after the arrival of information in the market (formally depicted in the model by $\mathrm{cov}[\tilde{v}-\hat{P}(\tilde{s}_1,\tilde{s}_2,\tilde{x}),\hat{P}(\tilde{s}_1,\tilde{s}_2,\tilde{x})-\mu]>0$) does not represent a slow adjustment of the asset price to previously available public information (as indicated by $\mathrm{cov}[\tilde{v}-\hat{P}(\tilde{s}_1,\tilde{s}_2,\tilde{x}),\tilde{s}_1]=0$ and $\mathrm{cov}[\tilde{v}-\hat{P}(\tilde{s}_1,\tilde{s}_2,\tilde{x}),\tilde{s}_2-\frac{a}{\lambda h_2}(\tilde{x}-\mu_x)]=0$). It rather reflects a slow adjustment of the asset price to previously held private information (as indicated by $\mathrm{cov}[\tilde{v}-\hat{P}(\tilde{s}_1,\tilde{s}_2,\tilde{x}),\tilde{s}_2]>0$). Proposition 5 thus leads to the surprising conclusion that the observed continuation of stock returns following earnings announcements does not contradict the semi-strong form of the market efficiency hypothesis. It should be noted, in particular, that in spite of the positive autocorrelation of the risk-adjusted returns earned on the risky asset after the information arrival, the asset price nevertheless follows a martingale process, and historical price data are thus not useful at all in predicting the future return on the risky asset. This is because the same noise that prevents the full revelation of private information via the price of the risky asset also prevents the full revelation of the risk premium hidden in the price. More explicitly, due to the market uncertainty about the realization x of the random per-capita supply $\tilde{x}$ for the risky asset, uninformed traders cannot precisely detect the risk premium $\frac{ax}{h+h_1+\lambda h_2+(1-\lambda)q_2}$ embedded in the price of the risky asset. They thus cannot perfectly deduce the historical risk-adjusted return $\hat{P}(\tilde{s}_1,\tilde{s}_2,\tilde{x})-\mu$ on the risky asset just from observing its past price behavior, and they are incapable of doing so even when considering the additional public signal $\tilde{s}_1$ available to them. So, uninformed traders cannot take advantage of the positive autocorrelation of the risk-adjusted returns earned on the risky asset after the information arrival. At the trading time, the expected value of the forthcoming risk-adjusted return $\tilde{v}-\hat{P}(\tilde{s}_1,\tilde{s}_2,\tilde{x})$ on the risky asset, conditional on all the publicly available information, is zero. Only from the viewpoint of privately informed traders, who additionally conditioned their expectations on the content of their private signal $\tilde{s}_2$, the expected value of the forthcoming risk-adjusted return $\tilde{v}-\hat{P}(\tilde{s}_1,\tilde{s}_2,\tilde{x})$ on the risky asset is not necessarily zero.

Taken together, the results presented in Proposition 5 suggest an alternative interpretation of the empirical findings regarding the post-earnings announcement drift, which are conventionally viewed as indicating a slow adjustment of stock prices to public information. The results of Proposition 5 imply that the seemingly slow adjustment of stock prices to observable public information, like earnings announcements, could in fact be driven by the slow adjustment of prices to correlated, unobservable, private information. This alternative interpretation cannot be directly tested on an empirical ground, because empirical tests can only pertain to observable public information but obviously cannot incorporate unobservable privately held information. Moreover, due the inevitable omission of the privately held information from the empirical tests, any empirical estimate of the power of currently available public information in predicting future stock returns is likely to be a biased estimate. The conclusion that the apparently slow market reaction to observable public information could actually be a slow reaction to unobservable private information is, however, indirectly reinforced by empirical findings indicating that the post-earnings announcement drift is more prominently observed in environments with significant asymmetry in information (e.g., Vega, 2005; Sadka, 2006; Francis, Lafond, Olsson and Schipper, 2007). It is also indirectly supported by empirical findings of intensive activities of private information production by equity market participants following public earnings announcements (e.g., Barron, Byard and Kim, 2002).

4. Concluding Remarks

A large body of empirical research documents the tendency of cumulative abnormal stock returns to drift in the direction of the earnings surprise for several weeks or even months following an earnings announcement. This pricing pattern, known as the post-earnings announcement drift, is considered in the literature as one of the strongest and most puzzling stock pricing anomalies. Researchers have long been struggling to reconcile it with the market efficiency hypothesis. Thus far, the intensive research endeavor to do so fails not only to reconcile the observed continuation of stock returns following earnings announcements with the strong form of the market efficiency hypothesis, which claims that all information, even privately held information, is fully reflected in stock prices. Researchers also have trouble in reconciling the post-earnings announcement drift with the less restrictive semi-strong form of the market efficiency hypothesis, which is commonly perceived as being much more descriptive of the realm of financial markets, because it only asserts that stock prices fully reflect the publicly available information. The post-earnings announcement drift is therefore conventionally attributed to market inefficiency and widely viewed as being caused by irrational under reaction of traders to publicly available information.

Taking a different tack, this study argues that the post-earnings announcement drift indeed violates the strong form of the market efficiency hypothesis, but it can interestingly appear in a semi-strong efficient market as the rational consequence of the existence of private information in the possession of some traders. Using a traditional rational expectations model of trading, the study counter-intuitively demonstrates that the arrival of public information in the market can be followed by positively auto-correlated stock returns in equilibrium, after adjusting for risk compensation, even if the market is efficient in the semi-strong sense and stock prices thus instantly and correctly adjust to the available public information. Contrary to common wisdom, the continuation of the risk-adjusted stock returns that emerges in equilibrium following the arrival of information in the market does not represent a slow adjustment of the stock price to previously available public information. It rather reflects a slow price adjustment to previously held, correlated, private information.

This study is rooted in the well established notion of Grossman and Stiglitz (1980) that some noise must exist in a semi-strong efficient market that prevents the full revelation of privately held information via the stock price. The analysis given in this study highlights that the same noise also prevents the full revelation of the risk premium embedded in the stock price, implying that not all traders are capable of exploiting trends in risk-adjusted stock returns. It thus follows from the analysis that trends in risk-adjusted stock returns do not necessarily stand in contrast with the semi-strong form of the market efficiency hypothesis, as conventionally presumed in the literature. This observation, which lies in the basis of the study, is useful in explaining not only the drift in stock returns documented following earnings announcements, but also similar drifts in stock returns empirically observed after other major public events, like dividend payments, spinoffs, stock splits, tender offers and open-market share repurchases (e.g., Lakonishock and Vermaelen, 1990; Cusatis, Miles and Woolridge, 1993; Ikenberry, Lakonishok and Vermaelen, 1995; Michaely, Thaler and Womack, 1995; Ikenberry, Rankine and Stice, 1996; Desai and Jain, 1997). The same observation might also be useful in explaining the momentum phenomenon identified by Jegadeesh and Titman (1993), which appears to be closely related to the post-earnings announcement drift (e.g., Chan, 2003). It perhaps might additionally suggest a starting point for future research attempts to explore the rationale behind a variety of other empirically observed trends in risk-adjusted stock returns.

Appendix – Proofs

Proof of Observation 1. In the benchmark case of $\sigma_x^2 = 0$, the demand for the risky asset of an informed trader is $D_B^I(s_1, s_2, p) \in \arg\max_d E[-e^{-ad(\tilde{v}-p)} | \tilde{s}_1 = s_1, \tilde{s}_2 = s_2]$. As $\tilde{v}$ is normally distributed,

$E[-e^{-ad(\tilde{v}-p)} | \tilde{s}_1 = s_1, \tilde{s}_2 = s_2] = -e^{-ad(E[\tilde{v}|\tilde{s}_1 = s_1, \tilde{s}_2 = s_2] - p) + 0.5a^2 d^2 \operatorname{var}[\tilde{v}|\tilde{s}_1, \tilde{s}_2]}$, and thus

$D_B^I(s_1, s_2, p) \in \arg\max_d ad(E[\tilde{v}|\tilde{s}_1 = s_1, \tilde{s}_2 = s_2] - p) - 0.5a^2 d^2 \operatorname{var}[\tilde{v}|\tilde{s}_1, \tilde{s}_2]$. The first-order condition is $a(E[\tilde{v}|\tilde{s}_1 = s_1, \tilde{s}_2 = s_2] - p) - a^2 d \operatorname{var}[\tilde{v}|\tilde{s}_1, \tilde{s}_2] = 0$ and the second-order condition is

$-a^2 \operatorname{var}[\tilde{v}|\tilde{s}_1, \tilde{s}_2] < 0$. Hence, $D_B^I(s_1, s_2, p) = a^{-1} \dfrac{E[\tilde{v}|\tilde{s}_1 = s_1, \tilde{s}_2 = s_2] - p}{\operatorname{var}[\tilde{v}|\tilde{s}_1, \tilde{s}_2]}$.

Looking for an equilibrium with a linear pricing rule, it is assumed that there exist scalars α, β, γ and δ such that $P_B(s_1, s_2, x) = \alpha + \beta s_1 + \gamma s_2 + \delta x$. So, the demand for the risky asset of an uninformed trader is $D_B^U(s_1, p) \in \arg\max_d E\left[-e^{-ad(\tilde{v}-p)} \middle| \tilde{s}_1 = s_1, \tilde{s}_2 = \dfrac{p - \alpha - \beta s_1 - \delta x}{\gamma}\right]$. As $\tilde{v}$ is normally distributed,

$$E\left[-e^{-ad(\tilde{v}-p)} \middle| \tilde{s}_1 = s_1, \tilde{s}_2 = \frac{p - \alpha - \beta s_1 - \delta x}{\gamma}\right] = -e^{-ad\left(E\left[\tilde{v} \middle| \tilde{s}_1 = s_1, \tilde{s}_2 = \frac{p - \alpha - \beta s_1 - \delta x}{\gamma}\right] - p\right) + 0.5a^2 d^2 \operatorname{var}[\tilde{v}|\tilde{s}_1, \tilde{s}_2]}$$, and thus

$D_B^U(s_1, p) \in \arg\max_d ad\left(E\left[\tilde{v} \middle| \tilde{s}_1 = s_1, \tilde{s}_2 = \dfrac{p - \alpha - \beta s_1 - \delta x}{\gamma}\right] - p\right) - 0.5a^2 d^2 \operatorname{var}[\tilde{v}|\tilde{s}_1, \tilde{s}_2]$. The first-order condition is $a\left(E\left[\tilde{v} \middle| \tilde{s}_1 = s_1, \tilde{s}_2 = \dfrac{p - \alpha - \beta s_1 - \delta x}{\gamma}\right] - p\right) - a^2 d \operatorname{var}[\tilde{v}|\tilde{s}_1, \tilde{s}_2] = 0$ and the second-order condition is $-a^2 \operatorname{var}[\tilde{v}|\tilde{s}_1, \tilde{s}_2] < 0$. So, $D_B^U(s_1, p) = a^{-1} \dfrac{E\left[\tilde{v} \middle| \tilde{s}_1 = s_1, \tilde{s}_2 = \dfrac{p - \alpha - \beta s_1 - \delta x}{\gamma}\right] - p}{\operatorname{var}[\tilde{v}|\tilde{s}_1, \tilde{s}_2]}$.

The market clearing price condition is thus

$$\lambda a^{-1}\frac{E[\tilde{v}|\tilde{s}_1=s_1,\tilde{s}_2=s_2]-P_B(s_1,s_2,x)}{\mathrm{var}[\tilde{v}|\tilde{s}_1,\tilde{s}_2]}+(1-\lambda)a^{-1}\frac{E\left[\tilde{v}\middle|\tilde{s}_1=s_1,\tilde{s}_2=\frac{P_B(s_1,s_2,x)-\alpha-\beta s_1-\delta x}{\gamma}\right]-P_B(s_1,s_2,x)}{\mathrm{var}[\tilde{v}|\tilde{s}_1,\tilde{s}_2]}=x\,.$$

As the equilibrium market price of the risky asset equals $P_B(s_1,s_2,x)=\alpha+\beta s_1+\gamma s_2+\delta x$ and the realization $x=\mu_x$ of $\tilde{x}$ is commonly known, it follows that $\frac{P_B(s_1,s_2,x)-\alpha-\beta s_1-\delta x}{\gamma}=s_2$. So, the market clearing condition is reduced to $a^{-1}\frac{E[\tilde{v}|\tilde{s}_1=s_1,\tilde{s}_2=s_2]-P(s_1,s_2,\mu_x)}{\mathrm{var}[\tilde{v}|\tilde{s}_1,\tilde{s}_2]}=\mu_x$. The equilibrium pricing rule is, therefore, $P_B(s_1,s_2,\mu_x)=E[\tilde{v}|\tilde{s}_1=s_1,\tilde{s}_2=s_2]-a\,\mathrm{var}[\tilde{v}|\tilde{s}_1,s_2]\mu_x$. Using the properties of the normal distribution, $E[\tilde{v}|\tilde{s}_1=s_1,\tilde{s}_2=s_2]=\frac{h\mu+h_1s_1+h_2s_2}{h+h_1+h_2}$ and $\mathrm{var}[\tilde{v}|\tilde{s}_1,\tilde{s}_2]=\frac{1}{h+h_1+h_2}$. It thus follows that the equilibrium price of the risky asset is $P_B(s_1,s_2,\mu_x)=\frac{h\mu+h_1s_1+h_2s_2-a\mu_x}{h+h_1+h_2}$. □

Proof of Observation 2. Since $\mathrm{cov}[\tilde{v},\tilde{s}_1]=\mathrm{cov}[\tilde{v},\tilde{s}_2]=\mathrm{cov}[\tilde{s}_1,\tilde{s}_2]=\frac{1}{h}$, $\mathrm{var}[\tilde{s}_1]=\frac{1}{h}+\frac{1}{h_1}$ and $\mathrm{var}[\tilde{s}_2]=\frac{1}{h}+\frac{1}{h_2}$, it follows that $\mathrm{cov}[\tilde{v}-\hat{P}_B(\tilde{s}_1,\tilde{s}_2,\mu_x),\tilde{s}_1]=\mathrm{cov}\left[\tilde{v}-\frac{h\mu+h_1\tilde{s}_1+h_2\tilde{s}_2}{h+h_1+h_2},\tilde{s}_1\right]$ equals

$$\mathrm{cov}[\tilde{v},\tilde{s}_1]-\frac{h_1}{h+h_1+h_2}\mathrm{var}[\tilde{s}_1]-\frac{h_2}{h+h_1+h_2}\mathrm{cov}[\tilde{s}_1,\tilde{s}_2]\ \text{ or }\ \frac{1}{h}-\frac{h_1}{h+h_1+h_2}\left(\frac{1}{h}+\frac{1}{h_1}\right)-\frac{h_2}{h+h_1+h_2}\cdot\frac{1}{h},$$

and is thus zero. Also, $\mathrm{cov}[\tilde{v}-\hat{P}_B(\tilde{s}_1,\tilde{s}_2,\mu_x),\tilde{s}_2]=\mathrm{cov}\left[\tilde{v}-\frac{h\mu+h_1\tilde{s}_1+h_2\tilde{s}_2}{h+h_1+h_2},\tilde{s}_2\right]$ equals

$$\mathrm{cov}[\tilde{v},\tilde{s}_2]-\frac{h_1}{h+h_1+h_2}\mathrm{cov}[\tilde{s}_1,\tilde{s}_2]-\frac{h_2}{h+h_1+h_2}\mathrm{var}[\tilde{s}_2]\ \text{ or }\ \frac{1}{h}-\frac{h_1}{h+h_1+h_2}\cdot\frac{1}{h}-\frac{h_2}{h+h_1+h_2}\left(\frac{1}{h}+\frac{1}{h_2}\right),$$

and is thus zero. Now,

$\text{cov}[\tilde{v}-\hat{P}_B(\tilde{s}_1,\tilde{s}_2,\mu_x),\hat{P}_B(\tilde{s}_1,\tilde{s}_2,\mu_x)-\mu]=\text{cov}\left[\tilde{v}-\hat{P}_B(\tilde{s}_1,\tilde{s}_2,\mu_x),\frac{h\mu+h_1\tilde{s}_1+h_2\tilde{s}_2}{h+h_1+h_2}-\mu\right]$ equals

$\frac{h_1}{h+h_1+h_2}\text{cov}[\tilde{v}-\hat{P}_B(\tilde{s}_1,\tilde{s}_2,\mu_x),\tilde{s}_1]+\frac{h_2}{h+h_1+h_2}\text{cov}[\tilde{v}-\hat{P}_B(\tilde{s}_1,\tilde{s}_2,\mu_x),\tilde{s}_2]$. Since

$\text{cov}[\tilde{v}-\hat{P}_B(\tilde{s}_1,\tilde{s}_2,\mu_x),\tilde{s}_1]=0$ and $\text{cov}[\tilde{v}-\hat{P}_B(\tilde{s}_1,\tilde{s}_2,\mu_x),\tilde{s}_2]=0$, it follows that

$\text{cov}[\tilde{v}-\hat{P}_B(\tilde{s}_1,\tilde{s}_2,\mu_x),\hat{P}_B(\tilde{s}_1,\tilde{s}_2,\mu_x)-\mu]=0$. □

Proof of Observation 3. In the case of $\sigma_x^2>0$, the demand for the risky asset of an informed trader is $D^I(s_1,s_2,p)\in\arg\max_d E[-e^{-ad(\tilde{v}-p)}|\tilde{s}_1=s_1,\tilde{s}_2=s_2]$. As $\tilde{v}$ is normally distributed,

$E[-e^{-ad(\tilde{v}-p)}|\tilde{s}_1=s_1,\tilde{s}_2=s_2]=-e^{-ad(E[\tilde{v}|\tilde{s}_1=s_1,\tilde{s}_2=s_2]-p)+0.5a^2d^2\,\text{var}[\tilde{v}|\tilde{s}_1,\tilde{s}_2]}$, and thus

$D^I(s_1,s_2,p)\in\arg\max_d ad(E[\tilde{v}|\tilde{s}_1=s_1,\tilde{s}_2=s_2]-p)-0.5a^2d^2\,\text{var}[\tilde{v}|\tilde{s}_1,\tilde{s}_2]$. The first-order condition is $a(E[\tilde{v}|\tilde{s}_1=s_1,\tilde{s}_2=s_2]-p)-a^2d\,\text{var}[\tilde{v}|\tilde{s}_1,\tilde{s}_2]=0$ and the second-order condition is $-a^2\,\text{var}[\tilde{v}|\tilde{s}_1,\tilde{s}_2]<0$. Hence, $D^I(s_1,s_2,p)=a^{-1}\frac{E[\tilde{v}|\tilde{s}_1=s_1,\tilde{s}_2=s_2]-p}{\text{var}[\tilde{v}|\tilde{s}_1,\tilde{s}_2]}$.

Looking for an equilibrium with a linear pricing rule, it is assumed that there exist scalars α, β, γ and δ such that $P(s_1,s_2,x)=\alpha+\beta s_1+\gamma s_2+\delta x$. So, the demand for the risky asset of an uninformed trader is $D^U(s_1,p)\in\arg\max_d E\left[-e^{-ad(\tilde{v}-p)}\middle|\tilde{s}_1=s_1,\tilde{s}_2+\frac{\delta}{\gamma}\tilde{x}=\frac{p-\alpha-\beta s_1}{\gamma}\right]$. As $\tilde{v}$ is normally distributed,

$E\left[-e^{-ad(\tilde{v}-p)}\middle|\tilde{s}_1=s_1,\tilde{s}_2+\frac{\delta}{\gamma}\tilde{x}=\frac{p-\alpha-\beta s_1}{\gamma}\right]=-e^{-ad\left(E\left[\tilde{v}\middle|\tilde{s}_1=s_1,\tilde{s}_2+\frac{\delta}{\gamma}\tilde{x}=\frac{p-\alpha-\beta s_1}{\gamma}\right]-p\right)+0.5a^2d^2\,\text{var}\left[\tilde{v}\middle|\tilde{s}_1,\tilde{s}_2+\frac{\delta}{\gamma}\tilde{x}\right]}$, and thus

$D^U(s_1,p) \in \arg\max_d ad\left(E\left[\tilde{v} \middle| \tilde{s}_1 = s_1, \tilde{s}_2 + \frac{\delta}{\gamma}\tilde{x} = \frac{p-\alpha-\beta s_1}{\gamma} \right] - p \right) - 0.5a^2 d^2 \operatorname{var}\left[\tilde{v} \middle| \tilde{s}_1, \tilde{s}_2 + \frac{\delta}{\gamma}\tilde{x} \right]$. The first-order condition is $a\left(E\left[\tilde{v} \middle| \tilde{s}_1 = s_1, \tilde{s}_2 + \frac{\delta}{\gamma}\tilde{x} = \frac{p-\alpha-\beta s_1}{\gamma} \right] - p \right) - a^2 d \operatorname{var}\left[\tilde{v} \middle| \tilde{s}_1, \tilde{s}_2 + \frac{\delta}{\gamma}\tilde{x} \right] = 0$ and the second-order condition is $-a^2 \operatorname{var}\left[\tilde{v} \middle| \tilde{s}_1, \tilde{s}_2 + \frac{\delta}{\gamma}\tilde{x} \right] < 0$. Hence,

$$D^U(s_1,p) = a^{-1} \frac{E\left[\tilde{v} \middle| \tilde{s}_1 = s_1 \tilde{s}_2 + \frac{\delta}{\gamma}\tilde{x} = \frac{p-\alpha-\beta s_1}{\gamma} \right] - p}{\operatorname{var}[\tilde{v} | \tilde{s}_1, \tilde{s}_2 + \frac{\delta}{\gamma}\tilde{x}]}.$$

The market clearing price condition is thus

$$\lambda a^{-1} \frac{E[\tilde{v} | \tilde{s}_1 = s_1, \tilde{s}_2 = s_2] - P(s_1,s_2,x)}{\operatorname{var}[\tilde{v} | \tilde{s}_1, \tilde{s}_2]} + (1-\lambda) a^{-1} \frac{E\left[\tilde{v} \middle| \tilde{s}_1 = s_1, \tilde{s}_2 + \frac{\delta}{\gamma}\tilde{x} = \frac{P(s_1,s_2,x)-\alpha-\beta s_1}{\gamma} \right] - P(s_1,s_2,x)}{\operatorname{var}\left[\tilde{v} \middle| \tilde{s}_1, \tilde{s}_2 + \frac{\delta}{\gamma}\tilde{x} \right]} = x.$$

As the equilibrium market price of the risky asset equals $P(s_1,s_2,x) = \alpha + \beta s_1 + \gamma s_2 + \delta x$, it follows that $\frac{P(s_1,s_2,x)-\alpha-\beta s_1}{\gamma} = s_2 + \frac{\delta}{\gamma}x$. So, the market clearing condition is

$$\lambda a^{-1} \frac{E[\tilde{v} | \tilde{s}_1 = s_1, \tilde{s}_2 = s_2] - P(s_1,s_2,x)}{\operatorname{var}[\tilde{v} | \tilde{s}_1, \tilde{s}_2]} + (1-\lambda) a^{-1} \frac{E\left[\tilde{v} \middle| \tilde{s}_1 = s_1, \tilde{s}_2 + \frac{\delta}{\gamma}\tilde{x} = s_2 + \frac{\delta}{\gamma}x \right] - P(s_1,s_2,x)}{\operatorname{var}\left[\tilde{v} \middle| \tilde{s}_1, \tilde{s}_2 + \frac{\delta}{\gamma}\tilde{x} \right]} = x.$$

The equilibrium pricing rule is, therefore,

$$P(s_1,s_2,x) = \frac{\varphi_1 E[\tilde{v} | \tilde{s}_1 = s_1, \tilde{s}_2 = s_2] + \varphi_2 E\left[\tilde{v} \middle| \tilde{s}_1 = s_1, \tilde{s}_2 + \frac{\delta}{\gamma}\tilde{x} = s_2 + \frac{\delta}{\gamma}x \right] - ax}{\varphi_1 + \varphi_2}, \text{ where}$$

$\varphi_1 = \lambda \operatorname{var}\left[\tilde{v}\middle|\tilde{s}_1, \tilde{s}_2\right]^{-1}$, $\varphi_2 = (1-\lambda)\operatorname{var}\left[\tilde{v}\middle|\tilde{s}_1, \tilde{s}_2 + \frac{\delta}{\gamma}\tilde{x}\right]^{-1}$. Using the properties of the normal distribution, $E[\tilde{v}|\tilde{s}_1 = s_1, \tilde{s}_2 = s_2] = \frac{h\mu + h_1 s_1 + h_2 s_2}{h + h_1 + h_2}$, $\operatorname{var}[\tilde{v}|\tilde{s}_1, \tilde{s}_2] = \frac{1}{h + h_1 + h_2}$,

$E\left[\tilde{v}\middle|\tilde{s}_1 = s_1, \tilde{s}_2 + \frac{\delta}{\gamma}\tilde{x} = s_2 + \frac{\delta}{\gamma}x\right] = \frac{h\mu + h_1 s_1 + q_2\left(s_2 + \frac{\delta}{\gamma}x - \frac{\delta}{\gamma}\mu_x\right)}{h + h_1 + q_2}$ and

$\operatorname{var}\left[\tilde{v}\middle|\tilde{s}_1, \tilde{s}_2 + \frac{\delta}{\gamma}\tilde{x}\right] = \frac{1}{h + h_1 + q_2}$, where $\frac{1}{q_2} = \frac{1}{h_2} + \left(\frac{\delta}{\gamma}\right)^2 \sigma_x^2$. So, the equilibrium price is

$P(s_1, s_2, x) = \frac{h\mu + h_1 s_1 + \lambda h_2 s_2 + (1-\lambda) q_2\left(s_2 + \frac{\delta}{\gamma}(x - \mu_x)\right) - ax}{h + h_1 + \lambda h_2 + (1-\lambda) q_2}$ and the price coefficients are

$\alpha = \frac{h\mu - (1-\lambda) q_2 \frac{\delta}{\gamma}\mu_x}{h + h_1 + \lambda h_2 + (1-\lambda) q_2}$, $\beta = \frac{h_1}{h + h_1 + \lambda h_2 + (1-\lambda) q_2}$, $\gamma = \frac{\lambda h_2 + (1-\lambda) q_2}{h + h_1 + \lambda h_2 + (1-\lambda) q_2}$ and

$\delta = \frac{(1-\lambda) q_2 \frac{\delta}{\gamma} - a}{h + h_1 + \lambda h_2 + (1-\lambda) q_2}$. So $\frac{\delta}{\gamma} = \frac{(1-\lambda) q_2 \frac{\delta}{\gamma} - a}{\lambda h_2 + (1-\lambda) q_2}$, implying $\left(\lambda h_2 + (1-\lambda) q_2\right)\frac{\delta}{\gamma} = (1-\lambda) q_2 \frac{\delta}{\gamma} - a$ or $\frac{\delta}{\gamma} = -\frac{a}{\lambda h_2}$. Now, substituting $\frac{\delta}{\gamma} = -\frac{a}{\lambda h_2}$, the equilibrium price of the risky asset becomes

$$P(s_1, s_2, x) = \frac{\varphi_1}{\varphi_1 + \varphi_2} E\left[\tilde{v}\middle|\tilde{s}_1 = s_1, \tilde{s}_2 = s_2\right] + \frac{\varphi_2}{\varphi_1 + \varphi_2} E\left[\tilde{v}\middle|\tilde{s} = s_1, \tilde{s}_2 - \frac{a}{\lambda h_2}(\tilde{x} - \mu_x) = s_2 - \frac{a}{\lambda h_2}(x - \mu_x)\right] - \frac{ax}{\varphi_1 + \varphi_2},$$

where $\varphi_1 = \lambda \operatorname{var}\left[\tilde{v}\middle|\tilde{s}_1, \tilde{s}_2\right]^{-1}$ and $\varphi_2 = (1-\lambda)\operatorname{var}\left[\tilde{v}\middle|\tilde{s}_1, \tilde{s}_2 - \frac{a}{\lambda h_2}(\tilde{x} - \mu_x)\right]^{-1}$. Alternatively, the price can be represented as $P(s_1, s_2, x) = \dfrac{h\mu + h_1 s_1 + \lambda h_2 s_2 + (1-\lambda) q_2 \left(s_2 - \dfrac{a}{\lambda h_2}(x - \mu_x) \right) - ax}{h + h_1 + \lambda h_2 + (1-\lambda) q_2}$, where

$$\frac{1}{q_2} = \frac{1}{h_2} + \left(\frac{a}{\lambda h_2} \right)^2 \sigma_x^2 = \frac{1}{h_2}\left(1 + \frac{a^2 \sigma_x^2}{\lambda^2 h_2} \right) \text{ or } q_2 = h_2 \left(1 + \frac{a^2 \sigma_x^2}{\lambda^2 h_2} \right)^{-1}. \square$$

Proof of Corollary 4. By Observation 3, the equilibrium market price of the risky asset takes the form $P(s_1, s_2, x) = \dfrac{h\mu + h_1 s_1 + \lambda h_2 s_2 + (1-\lambda) q_2 \left(s_2 - \dfrac{a}{\lambda h_2}(x - \mu_x) \right) - ax}{h + h_1 + \lambda h_2 + (1-\lambda) q_2}$. Applying simple algebraic rearrangements, the price equals $P(s_1, s_2, x) = \dfrac{h\mu + h_1 s_1 + \left(\lambda h_2 + (1-\lambda) q_2 \right) \left(s_2 - \dfrac{a}{\lambda h_2}(x - \mu_x) \right) - a\mu_x}{h + h_1 + \lambda h_2 + (1-\lambda) q_2}$,

and it thus a linear function of s_1 and $s_2 - \dfrac{a}{\lambda h_2}(x - \mu_x)$. $\square$

Proof of Proposition 5.

Proof of (i): The covariance $\mathrm{cov}[\tilde{v} - \hat{P}(\tilde{s}_1, \tilde{s}_2, \tilde{x}), \tilde{s}_1]$ equals

$$\mathrm{cov}\left[\tilde{v} - \frac{h\mu + h_1 \tilde{s}_1 + \lambda h_2 \tilde{s}_2 + (1-\lambda) q_2 \left(\tilde{s}_2 - \dfrac{a}{\lambda h_2}(\tilde{x} - \mu_x) \right)}{h + h_1 + \lambda h_2 + (1-\lambda) q_2}, \tilde{s}_1 \right] \text{ or}$$

$$\mathrm{cov}[\tilde{v}, \tilde{s}_1] - \frac{h_1}{h + h_1 + \lambda h_2 + (1-\lambda) q_2} \mathrm{var}[\tilde{s}_1] - \frac{\lambda h_2 + (1-\lambda) q_2}{h + h_1 + \lambda h_2 + (1-\lambda) q_2} \mathrm{cov}[\tilde{s}_1, \tilde{s}_2] \text{ or}$$

$$\frac{1}{h} - \frac{h_1}{h + h_1 + \lambda h_2 + (1-\lambda) q_2} \left(\frac{1}{h} + \frac{1}{h_1} \right) - \frac{\lambda h_2 + (1-\lambda) q_2}{h + h_1 + \lambda h_2 + (1-\lambda) q_2} \cdot \frac{1}{h},$$ and is thus zero.

Proof of (ii): The covariance $\operatorname{cov}\left[\tilde{v}-\hat{P}(\tilde{s}_1,\tilde{s}_2,\tilde{x}),\tilde{s}_2-\frac{a}{\lambda h_2}(\tilde{x}-\mu_x)\right]$ equals

$$\operatorname{cov}\left[\tilde{v}-\frac{h\mu+h_1\tilde{s}_1+\lambda h_2\tilde{s}_2+(1-\lambda)q_2\left(\tilde{s}_2-\frac{a}{\lambda h_2}(\tilde{x}-\mu_x)\right)}{h+h_1+\lambda h_2+(1-\lambda)q_2},\tilde{s}_2-\frac{a}{\lambda h_2}(\tilde{x}-\mu_x)\right] \text{ or}$$

$$\frac{1}{h}-\frac{h_1}{h+h_1+\lambda h_2+(1-\lambda)q_2}\cdot\frac{1}{h}-\frac{\lambda h_2+(1-\lambda)q_2}{h+h_1+\lambda h_2+(1-\lambda)q_2}\left(\frac{1}{h}+\frac{1}{h_2}\right)-\frac{(1-\lambda)q_2}{h+h_1+\lambda h_2+(1-\lambda)q_2}\cdot\frac{a^2}{\lambda^2 h_2^2}\sigma_x^2 .$$

Note that $q_2=h_2\left(1+\frac{a^2\sigma_x^2}{\lambda^2 h_2}\right)^{-1}$ implies $\frac{1}{q_2}-\frac{1}{h_2}=\frac{a^2\sigma_x^2}{\lambda^2 h_2^2}$. So, $\operatorname{cov}\left[\tilde{v}-\hat{P}(\tilde{s}_1,\tilde{s}_2,\tilde{x}),\tilde{s}_2-\frac{a}{\lambda h_2}(\tilde{x}-\mu_x)\right]$

equals $\frac{1}{h}-\frac{h_1}{h+h_1+\lambda h_2+(1-\lambda)q_2}\cdot\frac{1}{h}-\frac{\lambda h_2+(1-\lambda)q_2}{h+h_1+\lambda h_2+(1-\lambda)q_2}\left(\frac{1}{h}+\frac{1}{h_2}\right)-\frac{(1-\lambda)q_2}{h+h_1+\lambda h_2+(1-\lambda)q_2}\left(\frac{1}{q_2}-\frac{1}{h_2}\right)$,

and is thus zero.

Proof of (iii): The covariance $\operatorname{cov}[\tilde{v}-\hat{P}(\tilde{s}_1,\tilde{s}_2,\tilde{x}),\tilde{s}_2]$ equals

$$\operatorname{cov}\left[\tilde{v}-\frac{h\mu+h_1\tilde{s}_1+\lambda h_2\tilde{s}_2+(1-\lambda)q_2\left(\tilde{s}_2-\frac{a}{\lambda h_2}(\tilde{x}-\mu_x)\right)}{h+h_1+\lambda h_2+(1-\lambda)q_2},\tilde{s}_2\right] \text{ or}$$

$$\operatorname{cov}[\tilde{v},\tilde{s}_2]-\frac{h_1}{h+h_1+\lambda h_2+(1-\lambda)q_2}\operatorname{cov}[\tilde{s}_1,\tilde{s}_2]-\frac{\lambda h_2+(1-\lambda)q_2}{h+h_1+\lambda h_2+(1-\lambda)q_2}\operatorname{var}[\tilde{s}_2] \text{ or}$$

$\frac{1}{h}-\frac{h_1}{h+h_1+\lambda h_2+(1-\lambda)q_2}\cdot\frac{1}{h}-\frac{\lambda h_2+(1-\lambda)q_2}{h+h_1+\lambda h_2+(1-\lambda)q_2}\left(\frac{1}{h}+\frac{1}{h_2}\right)$, and can be thus reduced to

$\frac{1-\lambda}{h+h_1+\lambda h_2+(1-\lambda)q_2}\left(1-\frac{q_2}{h_2}\right)$. Substituting $q_2 = h_2\left(1+\frac{a^2\sigma_x^2}{\lambda^2 h_2}\right)^{-1}$ and rearranging, it appears that

$\text{cov}[\tilde{v}-\hat{P}(\tilde{s}_1,\tilde{s}_2,\tilde{x}),\tilde{s}_2]$ equals $\frac{(1-\lambda)a^2\sigma_x^2}{a^2\sigma_x^2(h+h_1+\lambda h_2)+\lambda^2 h_2(h+h_1+h_2)}$ and is strictly positive.

Proof of (iv): The covariance $\text{cov}[\tilde{v}-\hat{P}(\tilde{s}_1,\tilde{s}_2,\tilde{x}),\hat{P}(\tilde{s}_1,\tilde{s}_2,\tilde{x})-\mu]$ equals

$$\text{cov}\left[\tilde{v}-\hat{P}(\tilde{s}_1,\tilde{s}_2,\tilde{x}),\frac{h\mu+h_1\tilde{s}_1+\lambda h_2\tilde{s}_2+(1-\lambda)q_2\left(\tilde{s}_2-\frac{a}{\lambda h_2}(\tilde{x}-\mu_x)\right)}{h+h_1+\lambda h_2+(1-\lambda)q_2}-\mu\right]$$. Since

$\text{cov}[\tilde{v}-\hat{P}(\tilde{s}_1,\tilde{s}_2,\tilde{x}),\tilde{s}_1]=0$, $\text{cov}[\tilde{v}-\hat{P}(\tilde{s}_1,\tilde{s}_2,\tilde{x}),\tilde{s}_2]=\frac{1-\lambda}{h+h_1+\lambda h_2+(1-\lambda)q_2}\left(1-\frac{q_2}{h_2}\right)$ and

$\text{cov}[\tilde{v}-\hat{P}(\tilde{s}_1,\tilde{s}_2,\tilde{x}),\tilde{x}]=\frac{\frac{a(1-\lambda)q_2}{\lambda h_2}}{h+h_1+\lambda h_2+(1-\lambda)q_2}\sigma_x^2$, it follows that

$\text{cov}[\tilde{v}-\hat{P}(\tilde{s}_1,\tilde{s}_2,\tilde{x}),\hat{P}(\tilde{s}_1,\tilde{s}_2,\tilde{x})-\mu]$ equals

$$\frac{\lambda h_2+(1-\lambda)q_2}{h+h_1+\lambda h_2+(1-\lambda)q_2}\cdot\frac{1-\lambda}{h+h_1+\lambda h_2+(1-\lambda)q_2}\left(1-\frac{q_2}{h_2}\right)-\frac{\frac{a(1-\lambda)q_2}{\lambda h_2}}{h+h_1+\lambda h_2+(1-\lambda)q_2}\cdot\frac{\frac{a(1-\lambda)q_2}{\lambda h_2}}{h+h_1+\lambda h_2+(1-\lambda)q_2}\sigma_x^2$$ or

$\frac{(1-\lambda)q_2(\lambda h_2+(1-\lambda)q_2)}{(h+h_1+\lambda h_2+(1-\lambda)q_2)^2}\left(\left(\frac{1}{q_2}-\frac{1}{h_2}\right)-\frac{(1-\lambda)q_2a^2\sigma_x^2}{\lambda^2h_2^2(\lambda h_2+(1-\lambda)q_2)}\right)$. Note now that $q_2 = h_2\left(1+\frac{a^2\sigma_x^2}{\lambda^2 h_2}\right)^{-1}$

implies $\frac{1}{q_2}-\frac{1}{h_2}=\frac{a^2\sigma_x^2}{\lambda^2h_2^2}$. So, $\text{cov}[\tilde{v}-\hat{P}(\tilde{s}_1,\tilde{s}_2,\tilde{x}),\hat{P}(\tilde{s}_1,\tilde{s}_2,\tilde{x})-\mu]$ equals

$\frac{(1-\lambda)q_2(\lambda h_2+(1-\lambda)q_2)}{(h+h_1+\lambda h_2+(1-\lambda)q_2)^2}\cdot\frac{a^2\sigma_x^2}{\lambda^2h_2^2}\left(1-\frac{(1-\lambda)q_2}{\lambda h_2+(1-\lambda)q_2}\right)$ or $\frac{(1-\lambda)q_2a^2\sigma_x^2}{\lambda h_2(h+h_1+\lambda h_2+(1-\lambda)q_2)^2}$. Substituting

$q_2 = h_2\left(1+\frac{a^2\sigma_x^2}{\lambda^2 h_2}\right)^{-1}$ and rearranging, it appears that $\text{cov}[\tilde{v}-\hat{P}(\tilde{s}_1,\tilde{s}_2,\tilde{x}),\hat{P}(\tilde{s}_1,\tilde{s}_2,\tilde{x})-\mu]$ equals

$\frac{(1-\lambda)\lambda h_2\left(\lambda^2 h_2 + a^2\sigma_x^2\right)a^2\sigma_x^2}{\left(a^2\sigma_x^2(h+h_1+\lambda h_2)+\lambda^2 h_2(h+h_1+h_2)\right)^2}$ and is strictly positive.

As a supplement to the proof, it is also shown that $\text{cov}[\tilde{v}-P(\tilde{s}_1,\tilde{s}_2,\tilde{x}),P(\tilde{s}_1,\tilde{s}_2,\tilde{x})-\mu]<0$, as argued in footnote 7. To proof this argument, it should be noted that

$\text{cov}[\tilde{v}-P(\tilde{s}_1,\tilde{s}_2,\tilde{x}),\tilde{s}_1]=\text{cov}[\tilde{v}-\hat{P}(\tilde{s}_1,\tilde{s}_2,\tilde{x}),\tilde{s}_1]=0$,

$\text{cov}[\tilde{v}-P(\tilde{s}_1,\tilde{s}_2,\tilde{x}),\tilde{s}_2]=\text{cov}[\tilde{v}-\hat{P}(\tilde{s}_1,\tilde{s}_2,\tilde{x}),\tilde{s}_2]=\frac{1-\lambda}{h+h_1+\lambda h_2+(1-\lambda)q_2}\left(1-\frac{q_2}{h_2}\right)$ and

$\text{cov}[\tilde{v}-P(\tilde{s}_1,\tilde{s}_2,\tilde{x}),\tilde{x}]=\frac{a\frac{\lambda h_2+(1-\lambda)q_2}{\lambda h_2}}{h+h_1+\lambda h_2+(1-\lambda)q_2}\sigma_x^2$. It thus follows that

$\text{cov}[\tilde{v}-P(\tilde{s}_1,\tilde{s}_2,\tilde{x}),P(\tilde{s}_1,\tilde{s}_2,\tilde{x})-\mu]$ equals

$$\frac{\lambda h_2+(1-\lambda)q_2}{h+h_1+\lambda h_2+(1-\lambda)q_2}\cdot\frac{1-\lambda}{h+h_1+\lambda h_2+(1-\lambda)q_2}\left(1-\frac{q_2}{h_2}\right)-\frac{a\frac{\lambda h_2+(1-\lambda)q_2}{\lambda h_2}}{h+h_1+\lambda h_2+(1-\lambda)q_2}\cdot\frac{a\frac{\lambda h_2+(1-\lambda)q_2}{\lambda h_2}}{h+h_1+\lambda h_2+(1-\lambda)q_2}\sigma_x^2$$ or

$\frac{(1-\lambda)q_2(\lambda h_2+(1-\lambda)q_2)}{(h+h_1+\lambda h_2+(1-\lambda)q_2)^2}\left(\left(\frac{1}{q_2}-\frac{1}{h_2}\right)-\frac{(\lambda h_2+(1-\lambda)q_2)a^2\sigma_x^2}{\lambda^2 h_2^2(1-\lambda)q_2}\right)$. Note now that $q_2 = h_2\left(1+\frac{a^2\sigma_x^2}{\lambda^2 h_2}\right)^{-1}$

implies $\frac{1}{q_2}-\frac{1}{h_2}=\frac{a^2\sigma_x^2}{\lambda^2 h_2^2}$. So, $\text{cov}[\tilde{v}-P(\tilde{s}_1,\tilde{s}_2,\tilde{x}),P(\tilde{s}_1,\tilde{s}_2,\tilde{x})-\mu]$ equals

$\frac{(1-\lambda)q_2(\lambda h_2+(1-\lambda)q_2)}{(h+h_1+\lambda h_2+(1-\lambda)q_2)^2}\cdot\frac{a^2\sigma_x^2}{\lambda^2 h_2^2}\left(1-\frac{\lambda h_2+(1-\lambda)q_2}{(1-\lambda)q_2}\right)$ or $-\frac{(\lambda h_2+(1-\lambda)q_2)a^2\sigma_x^2}{\lambda h_2(h+h_1+\lambda h_2+(1-\lambda)q_2)^2}$, and is thus strictly negative. □

References

Ball, R., Brown, P., 1968. An empirical evaluation of accounting income numbers. *Journal of Accounting Research* 6, 159–177.

Barberis, N., Shleifer, A., Vishny, R., 1998. A model of investor sentiment. *Journal of Financial Economics* 49, 307–343.

Barron, O.E., Byard, D., Kim, O.I., 2002. Changes in analysts' information around earnings announcements. *The Accounting Review* 77, 821-846.

Bernard,V. L., Thomas, J. K., 1989. Post-earnings-announcement drift: delayed price response or risk premium? *Journal of Accounting Research* 27, 1–36.

Bernard,V. L., Thomas, J. K., 1990. Evidence that stock prices do not fully reflect the implications of current earnings for future earnings. *Journal of Accounting and Economics* 13, 305–340.

Bhushan, R., 1994. An informational efficiency perspective on the post-earnings announcement drift. *Journal of Accounting and Economics* 18, 45-65.

Campbell, J.Y., Cochrane, J.H., 1999. By force of habit: a consumption-based explanation of aggregate stock market behavior. *Journal of Political Economy* 107, 205–251.

Cao, S.C., Narayanamoorthy, G.S., 2012. Earnings volatility, post-earnings announcement drift, and trading frictions. *Journal of Accounting Research* 50, 41–74.

Chan, W.S., 2003. Stock price reaction to news and no-news: drift and reversal after headlines. *Journal of Financial Economics* 70, 223–260.

Chan, L.K.C., Jegadeesh, N., and Lakonishok, J., 1996. Momentum strategies. *Journal of Finance* 51, 1681-1713.

Cusatis, P., Miles, J., Woolridge, J., 1993. Restructuring through spinoffs. *Journal of Financial Economics* 33, 293–311.

Daniel, K.D., Hirshleifer, D.A., Subramanyam, A., 1998. Investor psychology and security market under- and over-reactions. *Journal of Finance* 53, 1839–1885.

DeBondt, W., Thaler, R., 1985. Does the stock market overreact? *Journal of Finance* 40, 793–805.

Desai, H., Jain, P., 1997. Long-run common stock returns following splits and reverse splits. *Journal of Business* 70, 409–433.

Dontoh, A., Ronen J., and Sarath, B., 2003. On the rationality of the post-earnings announcement drift. *Review of Accounting Studies* 8, 69-104.

Fama, E.F., 1998. Market efficiency, long-term returns, and behavioral finance. *Journal of Financial Economics* 25, 383–417.

Feldman, R., Govindaraj, S., Livnat, J., and Segal, B., 2010. Management's tone change, post earnings announcement drift and accruals. *Review of Accounting Studies* 15, 915–953.

Foster, G., Olsen, C., Shevlin, T, 1984. Earnings releases, anomalies and the behavior of security returns. *The Accounting Review* 59, 574–603.

Francis, J., Lafond, R., Olsson, P., Schipper, K., 2007. Information uncertainty and post-earnings announcement drift. *Journal of Business, Finance and Accounting* 34, 403-433.

Grossman, S.J., Stiglitz, J.E., 1980. On the impossibility of informationally efficient markets. *American Economic Review* 70, 393–408.

Ikenberry, D., Lakonishok, J., Vermaelen, T., 1995. Market underreaction to open market share repurchases. *Journal of Financial Economics* 39, 181–208.

Ikenberry, D., Rankine, G., Stice, E., 1996. What do stock splits really signal? *Journal of Financial and Quantitative Analysis* 31, 357–377.

Jegadeesh, N., and Livnat, J., 2006. Revenue surprises and stock returns. *Journal of Accounting and Economics* 41, 147–171.

Jegadeesh, N., and Titman, S., 1993. Returns to buying winners and selling losers: implications for stock market efficiency. *Journal of Finance* 48, 65–91.

Jones, C. P., Litzenberger, R. H., 1970. Quarterly earnings reports and intermediate stock price trends. *Journal of Finance* 25, 143–148.

Joy, O. M., Litzenberger, R. H., McEnally, R. W., 1977. The adjustment of stock prices to announcements of unanticipated changes in quarterly earnings. *Journal of Accounting Research* 15, 207–225.

Lakonishok, J., Vermaelen, T., 1990. Anomalous price behavior around repurchase tender offers. *Journal of Finance* 45, 455–477.

Latané, H. A., Jones, C. P., 1979. Standardized unexpected earnings—1971-1977. *Journal of Finance* 34, 717–724.

Latané, H. A., Joy, O. M. Jones, C. P., 1970. Quarterly data, sort-rank routines, and security evaluation. *Journal of Business* 43, 427–438.

Lewellen, J., and Shanken, J., 2002. Learning, asset-pricing tests, and market efficiency. *Journal of Finance* 57, 1113-1145.

Liang, L., 2003. Post-earnings announcement drift and market participants' information processing biases. *Review of Accounting Studies* 8, 321–345.

Livnat, J., Mendenhall, R.R., 2005. Comparing the post-earnings announcement drift for surprises calculated from analyst and time series forecast. *Journal of Accounting Research* 44, 177–205.

Mendenhall, R. R., 2004. Arbitrage risk and post-earnings-announcement drift. *Journal of Business* 77, 875–894.

Michaely, R., Thaler, R., Womack, K., 1995. Price reactions to dividend initiations and omissions. *Journal of Finance* 50, 573–608.

Narayanamoorthy, G.S., 2006. Conservatism and cross-sectional variation in the post-earnings announcement drift. *Journal of Accounting Research* 44, 763–789.

Rendleman, R. J., Jones, C. P., Latané, H. A., 1982. Empirical anomalies based on unexpected earnings and the importance of risk adjustments. *Journal of Financial Economics* 10, 269–287.

Sadka, R., 2006. Momentum and post-earnings-announcement drift anomalies. *Journal of Financial Economics* 80, 309–349.

Vega, C., 2006. Stock price reaction to public and private information. *Journal of Financial Economics* 82, 103–133.

Watts, R.L., 1978. Systematic 'abnormal' returns after quarterly earnings announcements, *Journal of Financial Economics* 6, 127-150.