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Partial cross ownership and tacit collusion

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We examine the effects that passive investments in rival firms have on the incentives of firms to engage in tacit collusion. In general, these incentives depend in a complex way on the entire partial cross ownership (PCO) structure in the industry. We establish necessary and sufficient conditions for PCO arrangements to facilitate tacit collusion and also examine how tacit collusion is affected when firms' controllers make direct passive investments in rival firms.

1. Introduction

■ There are many cases in which firms acquire their rivals' stock as passive investments that give them a share in the rivals' profits but not in the rivals' decision making. For example, Microsoft acquired in August 1997 approximately 7% of the nonvoting stock of Apple, its historic rival in the PC market, and in June 1999 it took a 10% stake in Inprise/Borland Corp., which is one of its main competitors in the software applications market.[1] Gillette, the international and U.S. leader in the wet shaving razor blade market, acquired 22.9% of the nonvoting stock and approximately 13.6% of the debt of Wilkinson Sword, one of its largest rivals.[2] Investments in rivals are often multilateral; examples of industries that feature complex webs of partial cross ownerships include the Japanese and the U.S. automobile industries (Alley, 1997), the global airline industry (Airline Business, 1998), the Dutch Financial Sector (Dietzenbacher, Smid, and Volkerink, 2000), the Nordic power market (Amundsen and Bergman, 2002), and the global steel industry (Gilo and Spiegel, 2003). There are also many cases in which a controller (majority or dominant shareholder) makes a passive investment in rivals. For instance, during the first half of the 1990s, National

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[1] See "Microsoft Investments Draw Federal Scrutiny," *Pittsburgh Post-Gazette*, August 10, 1997, p. B-11, and "Corel Again Buys a 'Victim' of Microsoft Juggernaut," *Ottawa Citizen*, February 8, 2000, p. C1.

[2] *United States* v. *Gillette Co*., 55 FR 28312 (1990).

Car Rental's controller, GM, passively held a 25% stake in Avis, National's rival in the car rental industry, while Hertz's controller, Ford, had acquired 100% of the preferred nonvoting stock of Budget Rent a Car (Purohit and Staelin, 1994; Talley, 1990).[3]

While horizontal mergers are subject to substantial antitrust scrutiny and are often opposed by antitrust authorities, passive investments in rivals were either granted a de facto exemption from antitrust liability or have gone unchallenged by antitrust agencies in recent cases (Gilo, 2000). This lenient approach toward passive investment in rivals stems from the courts' interpretation of the exemption for stock acquisitions "solely for investment" included in Section 7 of the Clayton Act.

In this article we wish to examine whether this lenient approach of courts and antitrust agencies toward passive investments in rivals is justified. Like other horizontal practices (e.g., horizontal mergers), (passive) partial cross ownership (PCO) arrangements raise two main antitrust concerns: concerns about unilateral competitive effects and concerns about coordinated competitive effects. We focus on the latter and study the effect of PCO on the ability of firms to engage in tacit collusion. To this end, we consider an infinitely repeated Bertrand oligopoly model in which firms and/or their controllers acquire some of their rivals' (nonvoting) shares. This setting allows us to deal with the complexity generated by the chain effects of multilateral PCO. This complexity arises since, in general, the profit of each firm, both under collusion as well as under deviation from collusion, depends on the whole set of PCO in the industry and not only on the firm's own stake in rivals. Another advantage of this model is that PCO does not affect the equilibrium in the one-shot case. Consequently, the competitive effect of PCO comes only from its effect on the incentive of firms to engage in tacit collusion. We say that PCO arrangements facilitate tacit collusion if they expand the range of discount factors for which tacit collusion can be sustained.

It might be thought that since PCO allows firms to internalize part of the harm they impose on rivals when deviating from a collusive scheme, any increase in the level of PCO in the industry will necessarily facilitate tacit collusion. This intuition, however, ignores the fact that PCO arrangements create an infinite recursion between the profits of firms that hold each other's shares, both under collusion and following a deviation from collusion. Consequently, PCO arrangements affect the incentive of each firm to collude in a complex and subtle way.

Despite this complexity, we are able to prove that an increase in the stake of firm r in a rival firm s never hinders collusion. Moreover, we show that such an increase will surely facilitate collusion provided that (i) each firm in the industry holds a stake in at least one rival, (ii) an *industry maverick* firm (a firm with the strongest incentive to deviate from a collusive agreement)[4] has a direct or an indirect stake in firm r,[5] and (iii) firm s is not an industry maverick. If either one of these conditions fails, the increased stake of firm r in firm s will not affect tacit collusion. In addition, we show that a controlling shareholder (whether a person or a parent corporation) can facilitate tacit collusion further by making a direct passive investment in rival firms. Such investment particularly facilitates collusion if the controller has a relatively small stake in his own firm.

The unilateral competitive effects of PCO have already been studied in the context of static oligopoly models by Reynolds and Snapp (1986), Bolle and Güth (1992), Flath (1991, 1992), Reitman (1994), and Dietzenbacher, Smid, and Volkerink (2000).[6] Our article, by contrast, focuses

[3] See also "Will Ford Become The New Repo Man? Financial Powerhouse Takes Aim at Bad Credit Risks," *New York Times*, December 15, 1996, p. C1. For additional examples of investments by firms and their controllers in rivals, see Gilo (2000).

[4] The Horizontal Merger Guidelines of the U.S. Department of Justice and Federal Trade Commission (FTC) define maverick firms as "firms that have a greater economic incentive to deviate from the terms of coordination than do most of their rivals"; see DoJ (1997). For an excellent discussion of the role that the concept of maverick firms plays in the analysis of coordinated competitive effects, see Baker (2002).

[5] Firm i has an indirect stake in firm r if it either has a stake in a firm that has a stake in firm r, or has a stake in a firm that has a stake in a firm that has a stake in firm r, and so on.

[6] See also Bresnahan and Salop (1986) and Kwoka (1992) for a related analysis of static models of horizontal joint ventures. Alley (1997) and Parker and Röller (1997) provide empirical evidence on the competitive effect of PCO.

on the coordinated competitive effects of PCO and examines a repeated Bertrand model. The distinction between the unilateral and coordinated competitive effects of PCO is important. In particular, PCO arrangements that may be unprofitable in static oligopoly models are shown to be profitable in our model once their coordinated effects are taken into account. For example, given that in a perfectly competitive capital market the price of the rival's shares reflects their post-acquisition value, an investing firm can gain only if its own shares increase in value. As Flath (1991) shows, this is the case only when product market competition involves strategic complements.[7] By contrast, our results show that once repeated interaction is taken into account, firms may benefit from investing in rivals even if such investments have no effect in one-shot interactions. Reitman (1994) shows that symmetric firms may not wish to invest in rivals because such investments benefit noninvesting firms more than they benefit the investing firms. In our model, there is no such free-rider problem, since when firms are symmetric, all of them need to invest in rivals to sustain tacit collusion (i.e., each firm is "pivotal").

We are aware of only one other article, Malueg (1992), that studies the coordinated effects of PCO. His work differs from ours in at least three important ways. First, Malueg considers a repeated Cournot game and finds that in general, PCO has an ambiguous effect on collusion. The ambiguity arises because in the Cournot model, PCO has two conflicting effects. On the one hand, PCO implies that firms internalize part of the losses that they inflict on rivals when they deviate. On the other hand, PCO also softens product market competition following a breakdown of the collusive scheme and hence strengthens the incentives of firms to deviate. We believe that in practice, the first effect is likely to dominate the second, otherwise firms would have no incentive to invest in rivals. The Bertrand framework that we use allows us to neutralize the negative effect of PCO on collusion and focus attention on the first positive effect. Second, Malueg considers a symmetric duopoly in which the firms hold identical stakes in one another, while we consider an n-firm oligopoly in which firms need not have similar stakes in one another. Third, Malueg effectively considers passive investments in rivals by controllers rather than by firms; consequently, his analysis does not feature the complex chain-effect interaction between the profits of rival firms that is a main focus of our article.

The rest of the article is organized as follows: In Section 2 we examine the effect of PCO on the ability of firms to achieve the fully collusive outcome in the context of an infinitely repeated Bertrand model with symmetric firms. Section 3 shows that PCO by firms' controllers may further facilitate collusion. We conclude in Section 4. All proofs are in the Appendix.

2. Partial cross ownership (PCO) by firms

■ In this section we examine the coordinated competitive effects of PCO in the context of the familiar infinitely repeated Bertrand oligopoly model with $n \geq 2$ identical firms that produce a homogeneous product at a constant marginal cost c. In every period, the n firms simultaneously choose prices and the lowest-price firm captures the entire market. In case of a tie, the set of lowest-price firms gets equal shares of the total sales. Using $Q(p)$ to denote the demand function, the monopoly price is defined by

$$p^m \equiv \underset{p}{\operatorname{argmax}}\, Q(p)(p-c),$$

and the monopoly profit is

$$\pi^m \equiv Q(p^m)(p^m - c).$$

Alley (1997) finds that failure to account for PCO leads to misleading estimates of the price-cost margins in the Japanese and U.S. automobile industries. Parker and Röller (1997) find that cellular telephone companies in the United States tend to collude more in one market if they have a joint venture in another market.

[7] Charléty, Fagart, and Souam (2002) study a related model but consider PCO by controllers rather than by firms. They show that although a controller's investments in rivals lower the profit of the controller's firm, they may increase the rival's profit by a larger amount and thereby benefit the controller at the expense of the minority shareholders in his own firm.

As is well known (e.g., Tirole, 1988, Ch. 6.3.2.1), the fully collusive outcome in which all firms charge p^m and each firm gets an equal share in the monopoly profit, π^m, can be sustained as a subgame-perfect equilibrium of the infinitely repeated game provided that the intertemporal discount factor, δ, is sufficiently high:

$$\delta \geq \widehat{\delta} = 1 - \frac{1}{n}. \tag{1}$$

Taking condition (1) as a benchmark, we shall examine the competitive effects of PCO by looking at its effect on the critical discount factor, $\widehat{\delta}$, above which the fully collusive outcome can be sustained. In other words, $\widehat{\delta}$ will be our measure of the ease of collusion.[8] We will say that PCO arrangements facilitate tacit collusion if they lower $\widehat{\delta}$ and thereby widen the set of discount factors for which the fully collusive scheme can be sustained. Conversely, we will say that PCO arrangements hinder tacit collusion if they raise $\widehat{\delta}$.

□ **Accounting profits under PCO.** Let α_{ij} be firm i's ownership stake in firm j. We assume that the pricing decisions of each firm are effectively made by its controller (i.e., a controlling shareholder). Now, suppose that all controllers adopt the same trigger strategy whereby each firm charges the monopoly price, p^m, in every period unless at least one firm has charged a different price in any previous period; from that point onward, all firms use marginal cost pricing and make zero profits in every period.[9] To write the condition that ensures that this trigger strategy can support the fully collusive scheme as a subgame-perfect equilibrium, we first need to express the profit of each firm under collusion and following a deviation from the fully collusive scheme.

If all firms charge the monopoly price, then each firm earns π^m/n directly.[10] In addition, each firm gets a share in its rivals' profits due to its ownership stake in these firms. The profit of firm i is therefore $\pi_i = \pi^m/n + \sum_{k \neq i} \alpha_{ik}\pi_k$. The vector of collusive profits in the industry, $\pi = (\pi_1, \pi_2, \ldots, \pi_n)'$, is therefore given by the solution of the equation

$$\pi = \widehat{\pi} + A\pi, \tag{2}$$

where $\widehat{\pi} = (\pi^m/n, \ldots, \pi^m/n)'$ is an n-dimensional vector and

$$A = \begin{pmatrix} 0 & \alpha_{12} & \cdots & \alpha_{1n} \\ \alpha_{21} & 0 & \cdots & \alpha_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_{n1} & \alpha_{n2} & \cdots & 0 \end{pmatrix}$$

is an $n \times n$ PCO matrix whose ith row specifies firm i's ownership stakes in its $n - 1$ rivals (the diagonal terms in A are all zero because firms do not hold direct stakes in themselves).

However, if firm i deviates from the fully collusive scheme and slightly undercuts the monopoly price, then the direct profit of all firms but i (excluding their share in the rivals' profits) is zero, while firm i's direct profit is arbitrarily close to π^m; to simplify matters, we write it as π^m. After taking into account the shares that firms have in their rivals' profits, the profit of the deviant firm i is $\pi_i = \pi^m + \sum_{k \neq i} \alpha_{ik}\pi_k$ and the profit of each nondeviant firm j is $\pi_j = \sum_{k \neq j} \alpha_{jk}\pi_k$. Consequently, the vector of firms' profits in the period in which firm i's

[8] Of course, the repeated game admits multiple equilibria. We focus on the fully collusive outcome and on $\widehat{\delta}$ because this is a standard way to measure the notion of "ease of collusion."

[9] Since each firm can guarantee itself a payoff of at least zero in each period (say by setting a high-enough price to ensure that it makes no sales), the Nash reversion is the most severe punishment that firms can impose on each other.

[10] We study here "pure" price fixing: firms fix a price and let consumers randomize their purchases between the n firms. There could be more-elaborate collusive schemes in which firms also divide the market (not necessarily in equal shares) among themselves. Such schemes, however, will require some firms to ration their sales and will therefore be harder for the firms to enforce and easier for antitrust authorities to detect.

controller deviates, $\pi^{d_i} = (\pi_1^{d_i}, \pi_2^{d_i}, \ldots, \pi_n^{d_i})'$, is given by the solution of the equation

$$\pi^{d_i} = \widehat{\pi}^{d_i} + A\pi^{d_i}, \tag{3}$$

where $\widehat{\pi}^{d_i} = (0, \ldots, 0, \pi^m, 0, \ldots, 0)'$ is an n-dimentional vector with π^m in the ith entry and zeros in all other entries. In all subsequent periods following a deviation from the fully collusive scheme, all firms use marginal cost pricing and make zero profits.

Equations (2) and (3) reveal that in general, the profit of each firm depends on the profits of *all* other firms and on the structure of PCO in the industry. For instance, firm 1 may get a share α_{12} of firm 2's profit, which may reflect firm 2's share, α_{25}, in the profit of firm 5, which in turn may reflect firm 5's share, α_{51}, in the profit of firm 1. Notice that in this example, firm 1 has a direct stake in firm 2 but only an indirect stake in firm 5 due to its stake in firm 2. Likewise, firm 2 has a direct stake in firm 5 but only an indirect stake in firm 1, while firm 5 has a direct stake in firm 1 but only an indirect stake in firm 2. The fact that each firm's profit depends on the whole PCO matrix is striking. It implies, for instance, that a firm's profit and incentive to collude may be affected by a change in PCO levels among rivals even if this change does not affect the firm directly (i.e., even if the firm's PCO levels in rivals or the rivals' PCO in that firm remain unchanged).

To solve (2) and (3), note that the PCO matrix, A, is nonnegative and the sum of each of its columns is strictly less than one (the sum of column i represents the aggregate stake of rival firms in firm i). Consequently, (2) and (3) are Leontief systems and have unique solutions $\pi(A) \geq 0$ and $\pi^{d_i}(A) \geq 0$ (see Berck and Sydsæter, 1993) defined by

$$\pi(A) = B\widehat{\pi}, \qquad \pi^{d_i}(A) = B\widehat{\pi}^{d_i}, \tag{4}$$

where $B \equiv (I - A)^{-1}$ is the inverse Leontief matrix. We will use b_{ij} to denote the entry in the ith row and the jth column in B. The matrix B specifies the effective stake that "real" equityholders (i.e., controllers and outside equityholders, but not rival firms) have in the profits of the n firms. For instance, b_{ij} is the effective stake in firm j's profits that a "real" equityholder with a 1% direct stake in firm i receives.

Equation (4) implies that the accounting collusive profit of firm i is $\pi_i(A) = (\sum_{k=1}^{n} b_{ik}/n)\pi^m$. This expression represents the average effective stake that firm i's "real" equityholders have in the n firms times the industry profit, π^m. However, if firm i deviates from the fully collusive scheme, then its one-time profit is $\pi_i^{d_i}(A) = b_{ii}\pi^m$, where b_{ii} is the effective stake that firm i's "real" equityholders have in firm i's profit. Also, if firm j deviates from the fully collusive scheme, then firm i's one-time profit is $\pi_i^{d_j}(A) = b_{ij}\pi^m$.

Given the key role that the matrix B plays in what follows, we now study its properties in Lemma 1. Its proof appears in the Appendix along with all other proofs.

Lemma 1. The inverse Leontief matrix B has the following properties.

(i) $b_{ii} \geq 1$ for all i, and $0 \leq b_{ij} < b_{ii}$ for all i and all $j \neq i$.
(ii) Let i and j be two distinct firms. Then $b_{ij} = 0$ if and only if firm i does not have a direct or an indirect stake in firm j.[11]
(iii) $b_{ii} > 1$ if and only if there exists a firm $j \neq i$ such that firm j has a direct or an indirect stake in firm i (i.e., $b_{ji} > 0$) and firm i has a direct or an indirect stake in firm j (i.e., $b_{ij} > 0$).
(iv) $\widehat{b}_i \equiv \sum_{j=1}^{n}(1 - \sum_{k \neq j} \alpha_{kj})b_{ji} = 1$ for all i.

Part (i) of Lemma 1 shows that a 1% stake in each firm i may give the "real" equityholders of firm i more than a 1% share in the firm's profit. Intuitively, a "real" equityholder of firm i is

[11] We will say that firm i has no indirect stake in firm j, and has no stake in a firm that has a stake in firm j if it has no stake in a firm that has a stake in a firm that has a stake in firm j and so on.

entitled to a fraction of firm i's profit in direct proportion to his equity stake in the firm. Indeed, absent PCO, $B = I$, so $b_{ii} = 1$: the equityholder's share in firm i's profit is equal to his equity stake in the firm. Things are different, however, when firm i has a stake in rival firms which in turn have direct or indirect stakes in firm i. In that case, part of firm i's profit flows back to the firm. As part (iii) of the lemma shows, the "real" equityholder of firm i captures in this case an additional fraction of firm i's profit, so his total share in firm i's profit exceeds his equity stake in the firm, i.e., $b_{ii} > 1$.

Part (ii) of Lemma 1 implies that a "real" equityholder of firm i will receive a share in firm j's profit, unless firm i has no direct or indirect stake in firm j. Recalling that $\pi_i^{d_j}(A) = b_{ij}\pi^m$, this implies in turn that, unlike the traditional repeated Bertrand model without PCO, here firm i may still earn a positive profit in the period in which a rival firm j deviates from the fully collusive scheme. In fact, this profit may exceed firm i's profit under collusion if $b_{ij} > \sum_{k=1}^n b_{ik}/n$. For instance, if there are n firms in the industry and only firm i has a stake α_{ij} in firm j while all other firms have no stakes in each other, then the collusive profit of firm i is $(1 + \alpha_{ij})(\pi^m/n)$, while its profit when firm j deviates is $\alpha_{ij}\pi^m$; the latter exceeds the former whenever $\alpha_{ij} > 1/(n-1)$. Nonetheless, since part (i) of Lemma 1 shows that $b_{ii} > b_{ij}$ for all i and all $j \neq i$, the profit of each firm i when it deviates from the fully collusive scheme, $\pi_i^{d_i}(A) = b_{ii}\pi^m$, exceeds its collusive profit, $\pi_i(A) = (\sum_{k=1}^n b_{ik}/n)\pi^m$, and its profit when firm j deviates, $\pi_i^{d_j}(A) = b_{ij}\pi^m$.

It is important to note that since $b_{ii} \geq 1$, in general $\sum_{k=1}^n \pi_k(A) > \pi^m$ and $\sum_{k=1}^n \pi_k^{d_j}(A) > \pi^m$, so the aggregate accounting profits under collusion, and following a deviation by some firm j, overstate the firms' cash flows.[12] Part (iv) of the lemma ensures, however, that the aggregate payoffs of "real" equityholders are not overstated and do sum up to π^m. To see why, notice that $1 - \sum_{k\neq j}\alpha_{kj}$ is the aggregate stake of "real" equityholders in each firm j, and $(1 - \sum_{k\neq j}\alpha_{kj})b_{ji}$ is their aggregate share in the profits of firm i. Part (iv) of Lemma 1 shows that the aggregate shares of "real" equityholders (from all firms) in each firm i's profit, $\widehat{b_i}$, sum up to one. This ensures in turn that the aggregate payoffs of "real" equityholders sum up to the industry profit, π^m. Indeed, if we premultiply both sides of (2) by the summation $1 \times n$ vector $(1, \ldots, 1)$ and rearrange terms, we get

$$\sum_{j=1}^{n}\left(1 - \sum_{k\neq j}\alpha_{kj}\right)\pi_j = \pi^m, \tag{5}$$

where the left-hand side of the equation is the aggregate payoffs of "real" equityholders. A similar computation shows that this is also the case following a deviation by some firm j from the fully collusive scheme.

To illustrate the point that the aggregate payoffs of "real" equityholders sum up to the industry profit, π^m, suppose that there are only two firms that hold 25% stakes in each other; the rest of the 75% ownership stakes in firms 1 and 2 are held by controllers 1 and 2, respectively. Assuming further that $\pi^m = 100$, the collusive profits are $\pi_1 = (100/2) + .25\pi_2$ and $\pi_2 = (100/2) + .25\pi_1$. Solving this system, we get $\pi_1 = \pi_2 = 66.66$, implying that the collusive payoff of each controller is $66.66 \times .75 = 50$. Consequently, the controllers' payoffs sum up to 100 (the real cash flow) despite the fact that the accounting profits sum up to 133.33. If firm 1's controller, say, deviates, the profits become $\pi_1 = 100 + .25\pi_2$ and $\pi_2 = 0 + .25\pi_1$, so $\pi_1 = 106.66$ and $\pi_2 = 26.66$. Now the controllers' payoffs are 80 and 20, respectively. Again, these payoffs sum up to 100 despite the fact that the firms' profits sum up to 133.33. It is worth noting that due to the fact that firm 1 receives part of its cash flow back from firm 2, controller 1 captures 80% of the industry profits despite the fact that he holds only a 75% stake in firm 1.

□ **Collusion with PCO.** Given the profits of the n firms under collusion and following a deviation from the fully collusive scheme, the condition that ensures that the fully collusive

[12] See Dietzenbacher, Smid, and Volkerink (2000) and Ritzberger and Shorish (2003) for additional discussion of this effect of PCO.

outcome can be sustained as a subgame-perfect equilibrium is

$$\frac{\gamma_{ii}\pi_i(A)}{1-\delta} \geq \gamma_{ii}\pi_i^{d_i}(A), \qquad i = 1, \ldots, n, \tag{6}$$

where γ_{ii} is the ownership stake of firm i's controller. When (6) holds, the infinite discounted payoff of each controller under collusion exceeds his one-time gain when his firm deviates from the collusive scheme. Consequently, no controller wishes to unilaterally deviate from the fully collusive scheme.

Recalling that $\pi_i(A) = (\sum_{k=1}^n b_{ik}/n)\pi^m$ and $\pi_i^{d_i}(A) = b_{ii}\pi^m$, condition (6) immediately yields the following result.

Lemma 2. With PCO, the fully collusive outcome can be sustained as a subgame-perfect equilibrium of the infinitely repeated game if and only if

$$\delta \geq \widehat{\delta}^{po}(A) \equiv \max\{\widehat{\delta}_1(A), \ldots, \widehat{\delta}_n(A)\}, \tag{7}$$

where

$$\widehat{\delta}_i(A) \equiv 1 - \frac{\pi_i(A)}{\pi_i^{d_i}(A)} = 1 - \frac{\sum_{k=1}^n b_{ik}/n}{b_{ii}}. \tag{8}$$

The intuition for Lemma 2 is as follows. Although the n firms produce a homogeneous product and have the same marginal cost, their incentives to collude are not necessarily identical due to their possibly different ownership stakes in rivals. Lemma 2 shows that whether the fully collusive scheme can be sustained or not depends entirely on the firm (or firms) with the minimal ratio between the collusive profit, $\pi_i(A)$, and the profit from deviation, $\pi_i^{d_i}(A)$. We shall refer to this firm as an *industry maverick* (there may be more than one industry maverick if several firms are tied for the minimal ratio between $\pi_i(A)$ and $\pi_i^{d_i}(A)$).

Since part (i) of Lemma 1 implies that $b_{ij} \geq 0$ for all i and all j, it follows immediately from equation (8) that $\widehat{\delta}_i(A) \leq \widehat{\delta} \equiv 1 - (1/n)$ for all i: in the presence of PCO, firms have either the same or stronger incentives to collude than they have absent PCO. Moreover, if firm i does not invest in any rival, then $b_{ij} = 0$ for all $j \neq i$, so firm i is necessarily an industry maverick and $\widehat{\delta}_i(A) = \widehat{\delta} \equiv 1 - (1/n)$.

The question however, is whether starting from a given PCO structure, an increase in one firm's stake in a rival firm facilitates or hinders collusion. Addressing this question is a formidable task, since in general, even a single change in the PCO matrix, A, will affect all entries in the inverse Leontief matrix, B. From an economic standpoint, that means that an increase in, say, firm r's stake in rival firm s may affect the incentives of all firms to collude by affecting their profits both under the fully collusive scheme and following a deviation from that scheme. From a purely mathematical standpoint, things are complicated because we are not simply interested in the comparative statics properties of the matrix B. Rather, we wish to know how the lowest ratio between the average value of the entries in row i of B, $\sum_{k=1}^n b_{ik}/n$, and the diagonal term in that row, b_{ii}, changes following a change in the PCO matrix A. Nonetheless, in Theorem 1 below, we are able to show that an increase in firm r's stake in rival firm s never hinders tacit collusion, and moreover we establish the precise conditions under which such an increase will surely facilitate tacit collusion. For the purpose of this result, it does not matter whether firm r increases its stake in firm s at the expense of outside shareholders or at the expense of firm s's controller (as long as the controller retains control).

Theorem 1. Starting with a PCO matrix A, suppose that firm r increases its stake in firm s by some $\omega > 0$, so that the new PCO matrix A' differs from A only with respect to the rsth entry which is increased by ω. Then,

$$\widehat{\delta}_i(A') \leq \widehat{\delta}_i(A), \qquad i = 1, \ldots, n,$$

with equality holding if and only if $b_{ir} = 0$ or $i = s$.

Theorem 1 may be of independent interest for those interested in the comparative static properties of Leontief systems (these systems play an important role in many areas in economics, e.g., input-output analysis). In our context, Theorem 1 has the following important implication.

Corollary 1. An increase in firm r's stake in firm s never hinders tacit collusion.

Corollary 1 follows immediately from the fact that for each firm i, $\widehat{\delta}_i(A') \leq \widehat{\delta}_i(A)$. Given that PCO never hinders tacit collusion, one may wonder when it will surely facilitate tacit collusion. In the next corollary of Theorem 1, we address this question.

Corollary 2. An increase in firm r's stake in firm s facilitates tacit collusion if and only if (i) each industry maverick has a direct or an indirect stake in firm r, and (ii) firm s is not an industry maverick.

Recalling that a firm that does not invest in rivals is an industry maverick, an important implication of Corollary 2 is that PCO can facilitate tacit collusion only if *every* firm in the industry has a stake in at least one rival. So long as at least one firm does not invest in rivals, this firm is an industry maverick, and by part (i) of the corollary, all other PCO in the industry will have no effect on tacit collusion. From a policy perspective, this implies that in industries with similar firms, antitrust authorities should not be too concerned with unilateral PCO, since only multilateral PCO arrangements can facilitate tacit collusion.[13]

However, in the presence of multilateral PCO arrangements, Corollary 2 implies that in general, an increase in firm r's stake in a rival firm s will have coordinated anticompetitive effects and should therefore raise antitrust concerns. The only two exceptions to this conclusion are cases in which an industry maverick has no direct or indirect stake in the investing firm r, or the rival firm s is itself an industry maverick.

To illustrate Corollary 2, suppose that there are 10 firms in the industry. Firms 5–10 have direct or indirect stakes in all rivals, while firms 1–4 invest only in each other and do not have direct or indirect stakes in firms 5–10. Then, any increase in the stakes that firms 5–10 hold in rivals, including their stakes in firms 1–4, will surely facilitate tacit collusion unless (i) the industry maverick is either firm 1, 2, 3, or 4, or (ii) the increased ownership stake is in a maverick firm. When either (i) or (ii) hold, the increased stake of firms 5–10 in rivals will not affect tacit collusion and will therefore justify a lenient treatment by antitrust authorities.

Condition (ii) in Corollary 2 implies that investment in a maverick firm has no effect on tacit collusion. This result is striking because the Horizontal Merger Guidelines of the U.S. Department of Justice and the FTC state that the "acquisition of a maverick firm is one way in which a merger may make coordinated interaction more likely."[14] This concern indicates that there is a fundamental difference between horizontal mergers in which firms obtain control over their rivals and passive investments in rivals that we study here. In particular, while gaining control over a maverick firm via a horizontal merger especially raises concerns about coordinated anticompetitive effects, Corollary 2 shows that a mere passive investment in a maverick firm should not raise any such concerns.

□ **The symmetric PCO case.** To obtain further insights about the effect of PCO on tacit collusion, we now consider the symmetric case in which all firms hold exactly the same ownership stake, $\overline{\alpha}$, in each other. Since some of the shares of each firm are held by its controller and potentially other outside shareholders, it must be the case that the aggregate stake of rivals in each firm i, $(n-1)\overline{\alpha}$, is less than one.

Proposition 1. Consider the symmetric case in which $\alpha_{ij} = \overline{\alpha} < 1/(n-1)$ for all i and all $j \neq i$. Then, as n increases, tacit collusion is hindered if the aggregate stake of rivals in each firm is small, i.e., $(n-1)\overline{\alpha} < 1/2$, and is facilitated otherwise.

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[13] In Gilo and Spiegel (2003), we showed that when firms are not similar, even a unilateral investment by the most efficient firm in its rivals can facilitate tacit collusion.

[14] See DoJ (1997).

As equation (1) shows, absent PCO, an increase in the number of firms hinders collusion. Proposition 1 shows that in the presence of PCO, this is no longer necessarily true: when the aggregate stake of rivals in each firm exceeds 50%, an increase in the number of firms facilitates collusion rather than hinders it.[15] The reason for this surprising result is that, holding $\overline{\alpha}$ fixed, an increase in n implies that each firm receives a larger fraction of its profits from rivals. Hence, deviation from the fully collusive scheme that hurts rivals may become unattractive. To illustrate, suppose that each firm holds a passive stake of 10% in rivals. Then, moving from six to seven firms will facilitate collusion, whereas moving from four to five firms will hinder it.

Next we ask how a deviation from the symmetric stakes case considered in Proposition 1 affects tacit collusion. To this end, suppose that one firm, say firm 1, changes its *aggregate* stake in rivals by ω so that $\sum_{k\neq 1} \alpha_{1k} = (n-1)\overline{\alpha} + \omega$. To ensure that the aggregate stake that rivals hold in each firm j is less than one, we will assume that $\omega < 1 - (n-1)\overline{\alpha}$. All firms other than firm 1 continue to hold an ownership stake $\overline{\alpha}$ in each of their rivals.

Proposition 2. Starting from the symmetric case in which $\alpha_{ij} = \overline{\alpha} < 1/(n-1)$ for all i and all $j \neq i$, suppose that firm 1 changes its aggregate stake in rivals by $\omega < 1 - (n-1)\overline{\alpha}$.

(i) If $\omega > 0$, then tacit collusion is facilitated, i.e., $\widehat{\delta}^{po} < \widehat{\delta}$, provided that ω is spread over at least two of firm 1's rivals. The incentives to collude are strongest when ω is spread evenly among all of firm 1's rivals.

(ii) If $\omega < 0$, then tacit collusion is hindered, i.e., $\widehat{\delta}^{po} > \widehat{\delta}$. Moreover, only the aggregate change in firm 1's stake in rivals matters and not how it is spread among firm 1's rivals.

Proposition 2 indicates that if we start from a symmetric PCO configuration, a unilateral increase in PCO by one firm raises more antitrust concerns the more evenly it is spread among the rival firms. Intuitively, the firm in which firm 1 has invested the most becomes the industry maverick, since its controller gains the most from deviation as a larger fraction of its profit from deviation flows back to the firm via its stake in firm 1. Obviously, an even spread of ω among all rivals minimizes firm 1's stake in the industry maverick and therefore minimizes the incentive of the maverick's controller to deviate from the fully collusive scheme.[16]

Proposition 2 assumes implicitly that when firm 1 increases its stake in rivals, it buys additional shares from outside investors or from controllers. The next proposition considers a transfer of ownership from one rival firm to another. A recent example of such a transfer occurred in the steel industry, where Luxembourg-based Arcelor, the world's largest steelmaker at the time, increased its stake in Brazilian CST, one of the world's largest steelmakers, from 18.6% to 27.95% by buying shares from Acesita, another Brazilian steelmaker.[17]

Proposition 3. Starting from the symmetric case in which $\alpha_{ij} = \overline{\alpha} < 1/(n-1)$ for all i and all $j \neq i$, suppose that firm 1 buys a stake $\omega \leq \overline{\alpha}$ in firm 3 from firm 2, so after the transaction, firm 1's stake in firm 3 increases to $\overline{\alpha} + \omega$, while firm 2's stake in firm 3 falls to $\overline{\alpha} - \omega$. This change in the PCO configuration hinders tacit collusion and more so when ω increases.

Proposition 3 differs from Proposition 2 in that the increase in firm 1's ownership stake comes at the expense of firm 2's stake. Hence, the aggregate amount of shares held by rival firms in each other does not increase as in Proposition 2. While before the transfer of ownership all firms were mavericks, following the transfer of ownership firm 2 becomes the only industry maverick, since

[15] Note that since we consider passive investments in rivals, the fact that rival firms have a combined share of more than 50% in the profits of each firm does not prevent the firm's controller from controlling more than 50% of the voting rights.

[16] One can show that if we start from an asymmetric PCO configuration, then it is no longer necessarily true that an even spread of ω leads to a more collusive outcome than an uneven spread of ω.

[17] Prior to the sale, Acesita held a 18.7% stake in CST but sold its entire stake in CST to Arcelor and to CVRD, which is a large Brazilian miner of iron and ore. In addition to its stake in CST, Arcelor also owns stakes in Acesita and in Belgo-Mineira, which is another Brazilian steelmaker (see "CVRD, Arcelor Team Up for CST," *Daily Deal*, December 28, 2002, M&A; "Minister: Steel Duties Still Under Study—Brazil," *Business News Americas*, April 8, 2002.)

it now has the smallest stake in rivals. Consequently, firm 2 becomes more eager than before to deviate from collusion, and this hinders tacit collusion. Together, Propositions 2 and 3 suggest that with identical firms, symmetric PCO configurations are the most conducive to tacit collusion and should therefore raise particular anticompetitive concerns.

3. PCO by controllers

■ In this section we consider the possibility that controllers will directly acquire (passive) ownership stakes in rival firms. As mentioned in the Introduction, a case in point is the car rental industry in the first half of the 1990s, where National Car Rental's controller, GM, passively held a 25% stake in Avis, National's rival, while Hertz's controller, Ford, had acquired 100% of the preferred nonvoting stock of Budget Rent a Car. The question that we address in this section is what effect, if any, such investments have on tacit collusion, above and beyond the effect that we have already identified in the previous section.

To this end, let γ_{ij} be the stake that firm i's controller holds in firm $j \neq i$, in addition to his controlling stake in firm i, γ_{ii}. Of course, if firm i's controller does not hold a stake in rival firm j, then $\gamma_{ij} = 0$. To avoid triviality, we assume that γ_{ij} represents a completely passive investment (e.g., nonvoting shares) that gives the controller a share γ_{ij} of firm j's profit but no control over its actions. Moreover, we assume that γ_{ii} is sufficiently large relative to γ_{ij} for all i and all j so that the controller of each firm i is better off maximizing firm i's profit than sacrificing firm i's profit in order to boost the profits of rival firms in which the controller has stakes.[18] Then, the condition that ensures that collusion can be sustained becomes

$$\frac{\sum_{j=1}^{n} \gamma_{ij}\pi_j(A)}{1-\delta} \geq \sum_{j=1}^{n} \gamma_{ij}\pi_j^{d_i}(A), \qquad i = 1, \ldots, n. \tag{9}$$

Condition (9) generalizes condition (6) to the case where controllers hold direct stakes in rival firms. The left-hand side of (9) is the infinite discounted payoff of firm i's controller (who may now get a share in the profits of all firms). The right-hand side of (9) is the controller's one-time payoff when the firm he controls, firm i, deviates from the fully collusive scheme (recall that Lemma 1 implies that $\pi_j^{d_i}(A) > 0$ if and only if firm j has a direct or an indirect stake in firm i.)

Using (9) and recalling that $\pi_j(A) = (\sum_{k=1}^{n} b_{jk}/n)\pi^m$ and $\pi_j^{d_i}(A) = b_{ji}\pi^m$, it follows that with PCO by controllers, the fully collusive scheme can be sustained as a subgame-perfect equilibrium of the infinitely repeated game provided that

$$\delta \geq \widehat{\delta}^c(A) \equiv \max\{\widehat{\delta}_1^c(A), \ldots, \widehat{\delta}_n^c(A)\}, \tag{10}$$

where

$$\widehat{\delta}_i^c(A) = 1 - \frac{\sum_{j=1}^{n}(\gamma_{ij}(\sum_{k=1}^{n} b_{jk}/n))}{\sum_{j=1}^{n} \gamma_{ij} b_{ji}}. \tag{11}$$

Without PCO by firms (i.e., when $A = 0$), $B = I$ so $b_{ii} = 1$ and $b_{ij} = 0$ for all i and all $j \neq i$. Hence, (11) implies that PCO by controllers facilitates collusion as $\widehat{\delta}_i^c(0) = 1-(\sum_{j=1}^{n} \gamma_{ij}/n)/\gamma_{ii} \leq 1-(1/n)$. The following theorem proves that this continues to be the case even when $A \neq 0$.

Theorem 2. PCO by controllers facilitates tacit collusion in the sense that $\widehat{\delta}_i^c(A) \leq \widehat{\delta}_i(A)$ for all i, with strict inequality holding whenever $\gamma_{ij} > 0$ for some $j \neq i$. Moreover, $\widehat{\delta}_i(A) - \widehat{\delta}_i^c(A)$ increases

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[18] Formally, note that firm j's profit when all n firms charge the monopoly price is $(\sum_{k=1}^{n} b_{jk}/n)\pi^m$. If firm i's controller charges above the monopoly price, then firm i's profit is zero, while the profit of each firm $j \neq i$ is $(\sum_{k=1}^{n} b_{jk}/(n-1))\pi^m$. Hence, firm i's controller would prefer to set firm i's price equal to the monopoly price rather than a higher price provided that $\sum_{j=1}^{n}(\gamma_{ij}(\sum_{k=1}^{n} b_{jk}/n))\pi^m \geq \sum_{j\neq i}(\gamma_{ij}(\sum_{k=1}^{n} b_{jk}/(n-1)))\pi^m$. This condition is equivalent to $\gamma_{ii} \geq \sum_{j\neq i}^{n}[\gamma_{ij}(\sum_{k=1}^{n} b_{jk}/(n-1))]/\sum_{k=1}^{n} b_{ik}$ and it holds provided that γ_{ii} is sufficiently large.

as y_{ii} falls; hence, PCO by firm i's controller is more effective in strengthening the controller's incentive to collude the smaller is the controller's stake in his own firm.

Theorem 2 shows that when firm i's controller invests in at least one rival firm, the controller is willing to participate in the fully collusive scheme for a wider set of discount factors. Moreover, this set becomes even wider as the controller's stake in the firm he controls, i.e., firm i, becomes smaller. This implies in turn that firm i's controller can lower $\widehat{\delta_i^c}(A)$ either by raising his stake in rival firms or by diluting his stake in firm i (subject, of course, to retaining control over the firm's actions). Such dilution effectively raises the weight that the controller assigns to rivals' profits and therefore weakens the controller's incentive to deviate from the collusive scheme. This implies in turn that even relatively small direct passive investments by controllers in rival firms can raise considerable antitrust concern. It should also be noted that $\widehat{\delta_i^c}(A)$ depends only on the stakes that firm i's controller has in rival firms but is completely independent of the stakes that other controllers have in rival firms.

An important implication of Theorem 2 that, to the best of our knowledge, has been overlooked in antitrust cases involving PCO by controllers is that antitrust agencies need to be concerned not only with a controller's stakes in rival firms, but also with the controller's stake (current or future) in his own firm. This suggests in turn that consent decrees approving passive investments by controllers should stipulate that the controllers will abstain from further diluting their stakes in their own firms.[19] For example, shortly after it acquired a passive stake in Budget, Ford diluted its controlling stake in Hertz from 55% to 49% by selling shares to Volvo.[20] Theorem 2 suggests that such dilution by Ford may have promoted collusion in the car rental industry. Similarly, the FTC has approved TCI's passive 9% stake in Time Warner and even allowed this stake to increase to 14.99% in the future despite the fact that TCI controlled movie networks Starz and Encore (with an 80% stake) while Time Warner wholly owned rival movie networks HBO and Cinemax.[21] Theorem 2 suggests that the FTC should have been concerned not only with TCI's stake in Time Warner, but also with its stake in the movie networks Starz and Encore. In particular, it suggests that the consent decree approving TCI's stake in Time Warner should have stipulated that TCI refrain from diluting its stake in Starz and Encore in the future, since such dilution might facilitate tacit collusion in markets in which these movie networks compete against each other.[22]

Theorem 2 also has implications for the recent decision of the Brazilian antitrust authorities to allow Telecom Italia (TI) to raise its stake in Telecom Brazil (TB) from 19% to 37.3% provided that TI is a passive investor as far as TB's cellular and long distance operations are concerned. TI holds a 56% controlling stake in Telecom Italia Mobile (TIM), Brazil's second-largest cellular provider, while TB has acquired a cellular license and will be competing with TIM in Brazilian cellular markets.[23] Theorem 2 suggests that stipulating that TI be a passive investor in TB was not enough to alleviate anticompetitive concerns in the Brazilian cellular market, and moreover it implies that the fact that TI's controlling stake in TIM is merely 56% (rather than 100%) only exacerbates these concerns.

Interestingly, the ability of firms to collude is greatly diminished when a firm's controller internalizes the interests of the minority shareholders and acts to maximize total firm value rather than only the value of his own stake. This is because such behavior has the opposite effect of dilution of the controller's stake: a controller who acts to maximize total firm value acts as if $y_{ii} = 1$, in which case $\widehat{\delta_i^c}(A)$ is maximized. In this sense, minority shareholders would prefer the

[19] In firms that are controlled by managers, compensation that is linked to the profits of rivals may play the same role as investments in rivals. This suggests that executive compensation should receive antitrust scrutiny similar to that of the investments of controllers in rival firms.

[20] See "Chrysler Buying Thrifty Rent-A-Car," *St. Louis Post-Dispatch*, May 19, 1989, p. 8C.

[21] See Proposed Consent Agreement, Time Warner, Inc., 61 Fed. Reg. 50,301 (September 25, 1996) for the FTC's decision and Waterman and Weiss (1997) for details about the cable TV industry.

[22] See Gilo (2000) for more details on these and similar examples.

[23] See "Spying Becomes an Issue in Brazilian Phone Dispute," *New York Times*, July 23, 2004, www.company.tim.it/investor/cp_dettaglio/0,,29_94_97,00.html; "Telecom Italia Refocuses in Brazil," *Financial Times*, August 29, 2002, p. 26; "2 From Europe Make Brazil Phone Deal," *New York Times*, January 17, 2003, p. C13.

controller to disregard their interests when choosing the firm's pricing decisions. Thus, contrary to conventional wisdom that sees the disregard of minority shareholders as a value-decreasing "agency cost," here such disregard is actually beneficial to all shareholders.

One may wonder whether Theorem 1 continues to hold when controllers hold stakes directly in rival firms. That is, is it still true that any increase in one firm's stake in a rival firm will never hinder collusion? The following example shows that the answer is no.

Example (an increase in a firm's stake in rivals may hinder collusion). Consider an industry with two firms and let α_{12} be firm 1's stake in firm 2 and α_{21} be firm 2's stake in firm 1. Moreover, suppose that the controller of firm $i = 1, 2$ has a stake of γ_{i1} in firm 1 and γ_{i2} in firm 2. It is straightforward to verify that the inverse Leontief matrix is such that $b_{11} = b_{22} = 1/(1 - \alpha_{12}\alpha_{21})$, $b_{12} = \alpha_{12}/(1 - \alpha_{12}\alpha_{21})$, and $b_{21} = \alpha_{21}/(1 - \alpha_{12}\alpha_{21})$. Using equation (11) we get

$$\begin{aligned}\widehat{\delta}_1^c(A) &= 1 - \frac{\gamma_{11}(1+\alpha_{12}) + \gamma_{12}(1+\alpha_{21})}{2(\gamma_{11} + \gamma_{12}\alpha_{21})} \\ &= \frac{1}{2} - \frac{\gamma_{11}\alpha_{12} + \gamma_{12}}{2(\gamma_{11} + \gamma_{12}\alpha_{21})}.\end{aligned}$$

It is easy to see that $\widehat{\delta}_1^c(A)$ decreases with α_{12}: an increase in firm 1's stake in firm 2 strengthens the incentive of firm 1's controller to collude. However, so long as $\gamma_{12} > 0$, $\widehat{\delta}_1^c(A)$ increases with α_{21}, implying that an increase in firm 2's stake in firm 1 weakens the incentive of firm 1's controller to collude. Consequently, whenever $\widehat{\delta}_1^c(A) > \widehat{\delta}_2^c(A)$ (firm 1 is the industry maverick) and $\gamma_{12} > 0$ (firm 1's controller holds a stake in firm 2), an increase in firm 2's stake in firm 1 will hinder collusion rather than facilitate it. Moreover, this effect becomes stronger as the stake that firm 1's controller holds in firm 2, γ_{12}, increases. Hence, Theorem 1 is no longer true in this case.

Finally, Corollary 2 above implies that absent PCO by controllers, an increase in firm r's stake in firm s affects neither firm s's incentive to collude nor the incentive of each firm i for which $b_{ir} = 0$, i.e., each firm i that does not have a direct or an indirect stake in firm r. The following result shows that this is no longer true in the presence of PCO by controllers.

Proposition 4. Starting with a PCO matrix A, suppose that firm r increases its stake in firm s by some $\omega > 0$.

(i) The change weakens firm s's incentive to collude if firm s's controller has a direct or an indirect stake in firm r, i.e., $\sum_{k=1}^{n} \gamma_{sk} b_{kr} > 0$, but leaves firm s's incentives unchanged otherwise.

(ii) The change does not affect firm i's incentive to collude if firm i's controller does not have a direct or an indirect stake in firm r, i.e., $b_{ir} = 0$ and $\sum_{k \neq i} \gamma_{ik} b_{kr} = 0$.

To illustrate Corollary 4, consider once again an industry with 10 firms such that firms 1–4 invest only in each other, while firms 5–10 have direct or indirect stakes in all rivals. Now, suppose that firm 5 increases its stakes in firm 4. Part (i) of Proposition 4 shows that the incentive of firm 4's controller to collude will remain unchanged if he has no stake in other firms or has stakes only in firms 1–4. If firm 4's controller has a stake in at least one of firms 5–10, then his incentive to collude would be weakened. Part (ii) of Proposition 4 shows that the increase in firm 5's stake in firm 4 will not affect the incentives of firms 1–3 to collude, provided that their controllers do not have stakes in firms 5–10.

In the context of the car rental industry case mentioned above, Proposition 4 implies that had Budget made a passive investment in Hertz, Hertz's incentive to engage in tacit collusion would have become weaker given that Hertz's controller, Ford, already held a passive stake in Budget. Similarly, a passive investment by Avis in National would have weakened National's incentive to engage in tacit collusion given that its controller, GM, also held a passive stake in Avis. This suggests in turn that firms have no incentive to acquire stakes in rivals when some of their own shares are held by the controllers of these rivals. Indeed, in the cases involving PCO by controllers discussed here and in Gilo (2000), PCO by controllers in rivals was never accompanied by PCO by the firms themselves in rivals.

4. Conclusion

■ Acquisitions of one firm's stock by a rival firm have been traditionally treated under Section 7 of the Clayton Act, which condemns such acquisitions when their effect "may be substantially to lessen competition." However, the third paragraph of Section 7 effectively exempts investments made "solely for investment." As argued in Gilo (2000), antitrust agencies and courts, when applying this exemption, did not conduct full-blown examinations as to whether such passive investments among rivals might substantially lessen competition.[24]

In this article we have shown that an across-the-board lenient attitude toward passive investments in rivals may be misguided. These investments may facilitate tacit collusion, especially when they are multilateral, are in firms that are not industry mavericks, and are by firms in which mavericks hold either direct or indirect stakes. In addition, we showed that direct investments by firms' controllers in rivals may either substitute investments by the firms themselves or facilitate collusion further, especially when the controllers have small stakes in their own firms. On the other hand, if a firm's controller holds a stake in a rival firm, passive investment by this rival in the controller's firm warrants a lenient antitrust approach. We believe that antitrust courts and agencies should take account of these factors when considering cases involving passive investments among rivals.

Throughout the article we have focused exclusively on the effect of PCO on the ability of firms to engage in (tacit) price fixing. However, if in addition to price fixing firms can also divide the market among themselves, then they would clearly be able to sustain collusion for a larger set of discount factors, since they would have more instruments (the collusive price and the market shares). In particular, it would be possible to relax the incentive constraints of maverick firms by increasing their market shares at the expense of firms with nonbinding incentive constraints. This suggests in turn that in the presence of market-sharing schemes, firms may have an incentive to become industry mavericks in order to receive a larger share of the market. As our analysis shows, one way to become an industry maverick is to avoid investing in rivals.[25] Interestingly, this implies that beside the fact that market-sharing schemes are harder to enforce (firms need to commit to ration their sales) and are more susceptible to antitrust scrutiny, they have another drawback in that they discourage PCO.

Finally, throughout the article we made two simplifying assumptions. The first was that firms produce a homogeneous product and have the same cost functions. In Gilo and Spiegel (2003) we began looking at the case where firms have asymmetric costs. We showed that even unilateral PCO by the most efficient firm in its rivals may facilitate tacit collusion, and the resulting collusive price is higher than it would be absent PCO. Moreover, we showed that the most efficient firm prefers to first invest in its most efficient rival both because this is the most effective way to promote tacit collusion and because such investment leads to a collusive price that is closer to the most efficient firm's monopoly price. The second simplifying assumption we made in this article was that the level of PCO in the industry is exogenously given. In a sense, then, our analysis is done from the perspective of antitrust authorities: When can you allow a firm to acquire a passive stake in a rival firm, and when should you disallow such acquisition? In future research we wish to look also at PCO from the perspective of firms: that is, we wish to endogenize the configuration of PCO in the industry and examine when a firm should try to acquire a passive stake in rivals and when it should not.

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[24] We are aware of only two cases in which the ability of passive investments to lessen competition was acknowledged: the FTC's decision in *Golden Grain Macaroni Co*. (78 F.T.C. 63, 1971), and the consent decree reached with the U.S. Department of Justice regarding US West's acquisition of Continental Cablevision (this decree was approved by the district court in *United States* v. *US West Inc*., 1997-1 Trade cases (CCH),¶71,767, D.C., 1997).

[25] Indeed, in a previous version of the article we showed that under market-sharing schemes and cost asymmetries, only the most efficient firm in the industry has an incentive to invest in rivals to sustain collusion, while all other firms find it optimal not to invest in rivals.

Appendix

■ Proofs of Lemma 1, Theorems 1 and 2, Corollary 2, and Propositions 1–4 follow.

Proof of Lemma 1. (i) Since A is a Leontief matrix, $B = (I - A)^{-1} = I + A + A^2 + \ldots$ (see Berck and Sydsæter, 1993). Hence, $b_{ij} \geq 0$ for all i and all j and $b_{ii} \geq 1$ for all i.

To prove that $b_{ij} < b_{ii}$, let C_k and e_k, respectively, be the kth columns of B and I. Since $(I - A)B = I$, we have $(I - A)(C_i - C_j) = e_i - e_j$. Moreover, since the ith coordinate of $C_i - C_j$ is $b_{ii} - b_{ij}$, Cramer's rule implies that $b_{ii} - b_{ij} = \det(I - A^{ij})/\det(I - A)$, where the matrix A^{ij} is obtained by replacing the ith column of A by e_j. To establish that $b_{ii} - b_{ij} > 0$, we will next show that $\det(I - A) > 0$ and $\det(I - A^{ij}) > 0$.

Since A is a Leontief matrix, so is ωA for every $\omega \in [0, 1]$. Hence, $I - \omega A$ is invertible, and $\det(I - \omega A)$, which is a continuous function of ω, is different from zero for all $\omega \in [0, 1]$. This implies that $\det(I - \omega A)$ never changes sign, so $\det(I - A)$ and $\det(I)$ must have the same sign. Since $\det(I) > 0$, we obtain that $\det(I - A) > 0$.

Next, note that A^{ij} is not a Leontief matrix since, by construction, its ith column is e_j, so the sum of the ith column is one (recall that in a Leontief matrix, the sum of each column must be strictly less than one). However, for every $\omega \in [0, 1)$, ωA^{ij} is a Leontief matrix. Hence, arguments similar to those above establish that $\det(I - \omega A^{ij}) > 0$ for every $\omega \in [0, 1)$, and $\det(I - A^{ij}) \geq 0$. To complete the proof we must show that $\det(I - A^{ij}) \neq 0$.

To this end, we begin by showing that $(A^{ij})^2$ is a Leontief matrix. Let v_k denote the kth column in A^{ij}. By the construction of A^{ij}, it follows that for each $k \neq i$, v_k is the kth column of A, while $v_i = e_j$. Hence, for each $k \neq i$, the kth column of $(A^{ij})^2$ equals $\sum_{r=1}^{n} \alpha_{rk} v_r$. Since $\sum_{r=1}^{n} \alpha_{rk} < 1$ (the sum of the ownership stakes of rival firms in each firm k is less than one) and since the sum of each v_r is less or equal to one, we conclude that the sum of the kth column of $(A^{ij})^2$ is strictly less than one. Moreover, the ith column of $(A^{ij})^2$ equals v_j, so its sum is also strictly less than one. Consequently, $(A^{ij})^2$ is a Leontief matrix.

Since $(A^{ij})^2$ is a Leontief matrix, then $I - (A^{ij})^2$ is invertible; hence, $\det(I - (A^{ij})^2) \neq 0$. However,

$$I - (A^{ij})^2 = (I - A^{ij})(I + A^{ij}),$$

so $\det(I - (A^{ij})^2) = \det(I - A^{ij})\det(I + A^{ij}) \neq 0$. This implies in turn that $\det(I - A^{ij}) \neq 0$, as required.

(ii) To prove the result, note that firm i does not have a direct or an indirect stake in firm j if and only if there is a partition (X, Y) of the set of firms $\{1, 2, \ldots, n\}$ (i.e., $X \cap Y = \emptyset$, $X \cup Y = \{1, 2, \ldots, n\}$, $X, Y \neq \emptyset$) such that $i \in X$, $j \in Y$, and $\alpha_{rk} = 0$ for each $r \in X$, $k \in Y$. That is, no firm in the subset X has a stake in a firm that belongs to Y. However, the existence of such partition is equivalent to the property that the ijth entry in A^{ℓ} is zero for each ℓ (see Frobenius (1912) and Jones, Klin, and Moshe (2002)). The proof is completed by noting that $b_{ij} = 0$ if and only if the ijth entry of A^{ℓ} is zero for each ℓ.

(iii) Let α_{ij}^{ℓ} denote the ijth entry of A^{ℓ}.

"If" part. Suppose that there exists a firm $j \neq i$ such that firm j has a direct or an indirect stake in firm i and firm i has a direct or an indirect stake in firm j. By part (ii) of the lemma, $b_{ji} > 0$ and $b_{ij} > 0$. Since $B = I + A + A^2 + \cdots$, then $\alpha_{ij}^{\ell_1} > 0$, $\alpha_{ji}^{\ell_2} > 0$ for some $\ell_1, \ell_2 \geq 1$. Given that $A^{\ell_1+\ell_2} = A^{\ell_1}A^{\ell_2}$, it follows that $\alpha_{ii}^{\ell_1+\ell_2} = \sum_{k=1}^{n} \alpha_{ik}^{\ell_1}\alpha_{ki}^{\ell_2}$, so $\alpha_{ii}^{\ell_1+\ell_2} > 0$. Since $B = I + A + A^2 + \cdots$, we conclude that $b_{ii} > 1$.

"Only if" part. Suppose that $b_{ii} > 1$. Since $B = I + A + A^2 + \cdots$, then $\alpha_{ii}^{\ell} > 0$ for some $\ell \geq 1$. But since $\alpha_{ii}^{\ell} = \sum_{k=1}^{n} \alpha_{ik}^{\ell-1}\alpha_{ki}$, there must exist a firm $j \neq i$ such that $\alpha_{ij}^{\ell-1} > 0$ and $\alpha_{ji} > 0$. Since $B = I + A + A^2 + \cdots$, we conclude that $b_{ij} > 0$ and $b_{ji} > 0$.

(iv) Recalling that α_{kj} is firm k's stake in firm j, the aggregate stake of "real equityholders" (i.e., controllers and outside equityholders) in each firm j is $1 - \sum_{k \neq j} \alpha_{kj}$. Since firm j's direct and indirect stake in each firm i is b_{ji}, the aggregate stake that real equityholders of firm j have in firm i is $(1 - \sum_{k \neq j} \alpha_{kj})b_{ji}$. Summing over all j, the aggregate share of real equityholders (of all firms) have in each firm i is $\widehat{b_i} \equiv \sum_{j=1}^{n}(1 - \sum_{k \neq j} \alpha_{kj})b_{ji}$. To prove the result, we need to show that $\widehat{b_i} = 1$ for all i.

To this end, note that the vector $(\widehat{b}_1, \ldots, \widehat{b}_n)$ can also be written as $(1, \ldots, 1)(I - A)B$, where $(1, \ldots, 1)$ is a $1 \times n$ summation vector. But since by definition, $(I - A)B = I$,

$$\left(\widehat{b}_1, \ldots, \widehat{b}_n\right) = (1, \ldots, 1)\,(I - A)\,B = (1, \cdots 1)\,.$$

Consequently, $\widehat{b_i} = 1$ for all i as required. *Q.E.D.*

Let B_i and I_i, respectively, denote the ith rows of the inverse Leontief Matrix B and the identity matrix I, and let $S(B_i) \equiv \sum_{k=1}^{n} b_{ik}$ be the sum of entries in B_i. In order to prove Theorem 1, we begin with the following three lemmas.

Lemma A1. Let A and A' be two PCO matrices such that A' is generated from A by adding some constant $\omega > 0$ to the rsth entry of A. Let B and B', respectively, be the inverse matrices of $I - A$ and $I - A'$. Then, $\omega b_{sr} < 1$ and the ith row of B' is given by $B'_i = B_i + \varepsilon_i B_s$, where $\varepsilon_i = \omega b_{ir}/(1 - \omega b_{sr}) \geq 0$.

Proof. B is an invertible matrix and therefore $B_1, \ldots, B_n$ is a basis of $\mathbb{R}^n$. Thus we may write $B'_i = \sum_{k=1}^{n} \rho_k B_k$ for some

$\rho_1, \ldots, \rho_n \in \mathbb{R}$. Note that $B_i(I - A) = B'_i(I - A') = I_i$ and $B_i(A' - A) = \omega b_{ir} I_s$. Thus,

$$\begin{aligned} I_i &= \left(\sum_{k=1}^{n} \rho_k B_k\right)(I - A') \\ &= \left(\sum_{k=1}^{n} \rho_k B_k\right)(I - A) + \left(\sum_{k=1}^{n} \rho_k B_k\right)(A - A') \\ &= \sum_{k=1}^{n} \rho_k I_k - \left(\sum_{k=1}^{n} \omega \rho_k b_{kr}\right) I_s. \end{aligned}$$

Since $I_1, \ldots, I_n$ are independent, we get $\rho_k = 0$ for each $k \neq i, s$. If $i \neq s$ then $\rho_i = 1$ and $\rho_s = \omega b_{ir}/(1 - \omega b_{sr})$, and if $i = s$, then $\rho_i = 1/(1 - \omega b_{sr})$. Thus, $B'_i = B_i + [\omega b_{ir}/(1 - \omega b_{sr})]B_s$ and in particular, $\omega b_{sr} \neq 1$. The same reasoning shows that $\omega' b_{sr} \neq 1$ for each $\omega' \leq \omega$. Thus we must have $\omega b_{sr} < 1$. *Q.E.D.*

Lemma A2. Let A, A', B be as in Lemma A1. Then for every i,

$$\hat{\delta}_i(A) - \hat{\delta}_i(A') = \frac{\varepsilon_i}{n(b_{ii} + \varepsilon_i b_{si})}\left(S(B_s) - \frac{b_{si}}{b_{ii}} S(B_i)\right). \tag{A1}$$

Proof. By Lemma A1, the ith row of B' is $B_i + \varepsilon_i B_s$. Thus,

$$\begin{aligned} \hat{\delta}_i(A) - \hat{\delta}_i(A') &= \frac{\frac{1}{n}(S(B_i) + \varepsilon_i S(B_s))}{b_{ii} + \varepsilon_i b_{si}} - \frac{\frac{1}{n} S(B_i)}{b_{ii}} \\ &= \frac{\varepsilon_i b_{ii} S(B_s) - \varepsilon_i b_{si} S(B_i)}{n(b_{ii} + \varepsilon_i b_{si}) b_{ii}} \\ &= \frac{\varepsilon_i}{n(b_{ii} + \varepsilon_i b_{si})}\left(S(B_s) - \frac{b_{si}}{b_{ii}} S(B_i)\right). \end{aligned}$$

Q.E.D.

Lemma A3. For every distinct pair of firms, i and s, we have

$$S(B_s) - \frac{b_{si}}{b_{ii}} S(B_i) \geq 1.$$

Proof. Let $M \equiv BE$, where E is a diagonal $n \times n$ matrix with $1-(S(B_i)/b_{ii})$ in the iith entry and 1 in all other entries along the diagonal. That is, M is the matrix obtained from B by multiplying the ith column of B by $1-(S(B_i)/b_{ii})$. Let M_k denote the kth row in M and let $S(M_k)$ be the sum of entries in M_k. Now, consider the column vector $m = (S(M_1), \ldots, S(M_n))'$. The definition of M implies that $S(M_s) = S(B_s) - (b_{si}/b_{ii})S(B_i)$. We need to prove that $S(M_s) \geq 1$. Note that

$$u \equiv (I - A)m = (I - A)BE \begin{pmatrix} 1 \\ 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix} = E \begin{pmatrix} 1 \\ 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix} = \begin{matrix} 1 \\ \vdots \\ 1 - \dfrac{S(B_i)}{b_{ii}} \\ \vdots \\ 1 \end{matrix},$$

where $1 - (S(B_i)/b_{ii})$ is the ith entry of u (and each other entry of u equals 1).

Let $\overline{A}$, $\overline{I - A}$ denote the $(n-1) \times (n-1)$ matrices obtained from A, $I - A$ by omitting the ith row and ith column, and let $\overline{m}, \overline{u}$ be the column vectors obtained from m, u by omitting the ith entry (note that $\overline{u}$ is an $n - 1$ unit vector). Since $u = (I - A)m$ and $S(M_i) = S(B_i) - (b_{ii}/b_{ii})S(B_i) = 0$, we obtain

$$\overline{u} = (\overline{I - A})\overline{m}. \tag{A2}$$

Denote $\overline{B} = (\overline{I - A})^{-1}$ and observe that $\overline{B} = I + \overline{A} + \overline{A}^2 + \ldots \geq I$. Multiplying (A2) by $\overline{B}$, we obtain $\overline{B}\overline{u} = \overline{m}$. Thus, $\overline{m} \geq \overline{u}$. Recalling that $\overline{u}$ is a unit vector, we obtain $S(M_s) \geq 1$. *Q.E.D.*

Proof of Theorem 1. First, note that if $b_{ir} = 0$, then $\varepsilon_i = \omega b_{ir}/(1 - \omega b_{sr}) = 0$. Second, note that if $i = s$, then $S(B_s) = (b_{si}/b_{ii})S(B_i)$. In both cases, equation (A1) above implies that $\hat{\delta}_i(A) = \hat{\delta}_i(A')$.

Next, assume that $i \neq s$ and $b_{ir} \neq 0$. In Lemma A1 we show that $\omega b_{sr} < 1$ and hence get $\varepsilon_i > 0$. By Lemma A3, $B_s - (b_{si}/b_{ii})S(B_i) > 0$, so $\hat{\delta}_i(A') < \hat{\delta}_i(A)$. *Q.E.D.*

Proof of Corollary 2. By Theorem 1, we can prove the corollary by proving that an increase in firm r's stake in firm s has no effect on tacit collusion if and only if (i) there exists an industry maverick, m, without a direct or an indirect stake in firm r, or (ii) firm s is an industry maverick.

"If" part. Let firm m be an industry maverick, i.e., $\hat{\delta}^{po}(A) = \hat{\delta}_m(A)$. If $b_{mr} = 0$ (firm m has no direct or indirect stake in firm r) or $m = s$ (firm s is an industry maverick), then Theorem 1 implies that $\widehat{\delta}_m(A') = \widehat{\delta}_m(A)$ and $\widehat{\delta}_j(A') \leq \widehat{\delta}_j(A)$ for all $j \neq m$. Hence, firm m remains an industry maverick (i.e., the firm with the highest $\widehat{\delta}$), so $\hat{\delta}^{po}(A') = \hat{\delta}_m(A')$. Altogether, then, $\hat{\delta}^{po}(A') = \hat{\delta}^{po}(A)$.

"Only if" part. Assume that $\hat{\delta}^{po}(A') = \hat{\delta}^{po}(A)$. Since by Theorem 1, $\hat{\delta}_i(A') \leq \hat{\delta}_i(A)$ for all i, we must have $\hat{\delta}_m(A') = \hat{\delta}_m(A)$ for some m with $\hat{\delta}^{po}(A) = \hat{\delta}_m(A)$. By Theorem 1, then, it must be the case that $b_{mr} = 0$ or $m = s$. *Q.E.D.*

Proof of Proposition 1. If $\alpha_{ij} = \overline{\alpha}$ for all i and all $j \neq i$, then equation (2) has a symmetric solution

$$\pi_i = \frac{\pi^m}{n\,(1-(n-1)\overline{\alpha})}, \qquad i = 1, \ldots, n. \tag{A3}$$

If firm i's controller deviates from the fully collusive scheme, then system (3) can be written as

$$\begin{aligned} \pi_i^{d_i} &= \pi^m + (n-1)\,\overline{\alpha}\pi_j^{d_i}, \\ \pi_j^{d_i} &= \overline{\alpha}\pi_i^{d_i} + (n-2)\,\overline{\alpha}\pi_j^{d_i}, \qquad j = 1, \ldots, n, \quad j \neq i. \end{aligned}$$

Solving this system for $\pi_i^{d_i}$ yields

$$\pi_i^{d_i} = \frac{(1-(n-2)\overline{\alpha})\,\pi^m}{(1-(n-1)\overline{\alpha})\,(1+\overline{\alpha})}. \tag{A4}$$

Substituting from (A3) and (A4) into equation (8) reveals that

$$\widehat{\delta}_i = 1 - \frac{1+\overline{\alpha}}{n\,(1-(n-2)\overline{\alpha})}, \qquad i = 1, \ldots, n. \tag{A5}$$

It is straightforward to verify that this expression increases with n if $(n-1)\,\overline{\alpha} < 1/2$, and decreases with n otherwise. *Q.E.D.*

Proof of Proposition 2. Given that $\alpha_{ij} = \overline{\alpha}$ for all $i \neq 1$ and all $j \neq i$, and since by symmetry, $\pi_2 = \cdots = \pi_n$, system (2) can be written as

$$\begin{aligned} \pi_1 &= \frac{\pi^m}{n} + ((n-1)\overline{\alpha} + \omega)\,\pi_j, \\ \pi_j &= \frac{\pi^m}{n} + \overline{\alpha}\pi_1 + (n-1)\,\overline{\alpha}\pi_j, \qquad j = 2, \ldots, n. \end{aligned}$$

Solving this system yields

$$\begin{aligned} \pi_1 &= \frac{(1+\overline{\alpha}+\omega)\,\frac{\pi^m}{n}}{H - \overline{\alpha}\omega}, \\ \pi_j &= \frac{(1+\overline{\alpha})\,\frac{\pi^m}{n}}{H - \overline{\alpha}\omega}, \qquad j = 2, \ldots, n, \end{aligned} \tag{A6}$$

where $H \equiv (1-(n-1)\overline{\alpha})(1+\overline{\alpha})$.

We now need to compute the profit that each firm obtains when its controller deviates from the fully collusive scheme. If firm 1's controller deviates, then system (3) becomes

$$\begin{aligned} \pi_1^{d_1} &= \pi^m + ((n-1)\overline{\alpha} + \omega)\,\pi_j^{d_1}, \\ \pi_j^{d_1} &= \overline{\alpha}\pi_1^{d_1} + (n-1)\,\overline{\alpha}\pi_j^{d_1}, \qquad j = 2, \ldots, n. \end{aligned}$$

Solving for $\pi_1^{d_1}$ yields

$$\pi_1^{d_1} = \frac{(1-(n-2)\overline{\alpha})\,\pi^m}{H - \overline{\alpha}\omega}. \tag{A7}$$

From (A6) and (A7) it follows that

$$\widehat{\delta}_1 \equiv 1 - \frac{\pi_1}{\pi_1^{d_1}} = 1 - \frac{1 + \overline{\alpha} + \omega}{n\,(1 - (n-2)\overline{\alpha})}. \tag{A8}$$

If the controller of some firm $i \neq 1$ deviates from the fully collusive scheme, then system (3) can be written as

$$\begin{aligned}
\pi_1^{d_i} &= \alpha_{1i}\pi_i^{d_i} + (\overline{\alpha}(n-1) + \omega - \alpha_{1i})\,\pi_j^{d_i}, \\
\pi_i^{d_i} &= \pi^m + \overline{\alpha}\pi_1^{d_i} + (n-2)\,\overline{\alpha}\pi_j^{d_i}, \\
\pi_j^{d_i} &= \overline{\alpha}\pi_1^{d_i} + \overline{\alpha}\pi_i^{d_i} + (n-3)\,\overline{\alpha}\pi_j^{d_i}, \qquad j = 2, \dots, n, \quad j \neq i.
\end{aligned}$$

Solving this system for $\pi_i^{d_i}$ yields

$$\pi_i^{d_i} = \frac{(H - \overline{\alpha}\omega + \overline{\alpha}\,(1 + \alpha_{1i}))\,\pi^m}{(1 + \overline{\alpha})\,(H - \overline{\alpha}\omega)}, \qquad i \neq 1. \tag{A9}$$

From (A6) and (A9) it follows that

$$\widehat{\delta}_i \equiv 1 - \frac{\pi_i}{\pi_i^{d_i}} = 1 - \frac{(1 + \overline{\alpha})^2}{n\,(H + \overline{\alpha}\,(1 + \alpha_{1i} - \omega))}. \tag{A10}$$

To compare $\widehat{\delta}_1$ and $\widehat{\delta}_i$, note that holding ω constant, $\widehat{\delta}_i$ increases with α_{1i} and hence is minimized at $\alpha_{1i} = \overline{\alpha}$, i.e., when the increase in firm 1's PCO is in firms other than i. Now, for all $i \neq 1$,

$$\widehat{\delta}_i\big|_{\alpha_{1i} = \overline{\alpha}} - \widehat{\delta}_1 = \frac{\omega\,(H - \overline{\alpha}\omega)}{n\,(1 - (n-2)\overline{\alpha})\,(H - \overline{\alpha}\omega + \overline{\alpha}\,(1 + \overline{\alpha}))}. \tag{A11}$$

If $\omega \geq 0$, then $\widehat{\delta}_i > \widehat{\delta}_1$ for all values of α_{1i} and all $i \neq 1$. Now suppose that firm 1's largest PCO is in firm i so that $\alpha_{1i} \geq \alpha_{1j}$ for all $j \neq 1$. Since $\widehat{\delta}_i$ increases with α_{1i}, $\max\{\widehat{\delta}_2, \widehat{\delta}_3, \dots, \widehat{\delta}_n\} = \widehat{\delta}_i$. That is, firm i is the industry maverick and $\widehat{\delta}^{po} = \widehat{\delta}_i$. When either $\omega = 0$ (firm 1 does not increase its stake in rivals so that $\alpha_{1i} = \overline{\alpha}$) or $\alpha_{1i} = \overline{\alpha} + \omega$ (firm 1 increases its ownership stake only in firm j), $\widehat{\delta}_i$ coincides with the expression in equation (A5). Otherwise, since $\widehat{\delta}_i$ decreases with ω, tacit collusion is facilitated when firm 1 increases its aggregate stake in rivals. Since $\widehat{\delta}_i$ increases with α_{1i}, tacit collusion is particularly facilitated when ω is spread evenly among all of its rivals, in which case, for every ω, α_{1i} is minimal and equal to $\overline{\alpha} + [\omega/(n-1)]$.

By contrast, if $\omega < 0$, then $\widehat{\delta}_i$ is maximized at $\alpha_{1i} = \overline{\alpha}$, i.e., whenever firm 1 lowers its ownership stake in firms other than firm i. Moreover, (A11) shows that $\widehat{\delta}_i < \widehat{\delta}_1$ for all $i \neq 1$. Consequently, $\widehat{\delta}^{po} = \widehat{\delta}_1$. From (A8) it is easy to see that $\widehat{\delta}_1$ increases as ω falls, implying that tacit collusion is hindered. *Q.E.D.*

Proof of Proposition 3. Given the transfer of ownership stake in firm 3 from firm 2 to firm 1, system (2) becomes

$$\begin{aligned}
\pi_1 &= \frac{\pi^m}{n} + \overline{\alpha}\pi_2 + (\overline{\alpha} + \omega)\,\pi_3 + \cdots + \overline{\alpha}\pi_n, \\
\pi_2 &= \frac{\pi^m}{n} + \overline{\alpha}\pi_1 + (\overline{\alpha} - \omega)\,\pi_3 + \cdots + \overline{\alpha}\pi_n, \\
\pi_j &= \frac{\pi^m}{n} + \overline{\alpha}\sum_{k \neq j}\pi_k, \qquad j = 3, \dots, n.
\end{aligned} \tag{A12}$$

By symmetry, $\pi_3 = \cdots = \pi_n$; hence, the solution of the system is given by

$$\begin{aligned}
\pi_1 &= \frac{(1 + \overline{\alpha} + \omega)\,\pi^m}{nH}, \qquad \pi_2 = \frac{(1 + \overline{\alpha} - \omega)\,\pi^m}{nH}, \\
\pi_i &= \frac{\pi^m}{n\,(1 - (n-1)\overline{\alpha})}, \qquad i = 3, \dots, n.
\end{aligned} \tag{A13}$$

If the controller of firm 1 deviates from the fully collusive scheme, then system (A12) needs to be modified by replacing π^m/n with π^m in the first line of the system and replacing π^m/n with zero in all other lines. Solving the modified system for firm 1's profit yields

$$\pi_1^{d_1} = \frac{((1 - (n-2)\,\overline{\alpha})\,(1 + \overline{\alpha}) + \overline{\alpha}\omega)\,\pi^m}{H\,(1 + \overline{\alpha})}. \tag{A14}$$

Using (A13) and (A14) yields

$$\widehat{\delta}_1(\omega) \equiv 1 - \frac{\pi_1}{\pi_1^{d_1}} = 1 - \frac{(1+\overline{\alpha})(1+\overline{\alpha}+\omega)}{n((1-(n-2)\overline{\alpha})(1+\overline{\alpha})+\overline{\alpha}\omega)}.$$

Likewise, if firm 2's controller deviates, the solution to the modified system (A12) is such that

$$\pi_2^{d_2} = \frac{((1-(n-2)\overline{\alpha})(1+\overline{\alpha})-\overline{\alpha}\omega)\pi^m}{H(1+\overline{\alpha})}. \tag{A15}$$

Using (A13) and (A15) yields

$$\widehat{\delta}_2(\omega) \equiv 1 - \frac{\pi_2}{\pi_2^{d_2}} = 1 - \frac{(1+\overline{\alpha})(1+\overline{\alpha}-\omega)}{n((1-(n-2)\overline{\alpha})(1+\overline{\alpha})-\overline{\alpha}\omega)}.$$

If the controller of some firm $i = 3, \ldots, n$ deviates, the solution to the modified system (2) shows that its profit, $\pi_i^{d_i}$, is equal to the right-hand side of (A4). Since the collusive profit of firm $i = 3, \ldots, n$ in (A13) is equal to the right-hand side of (A3), it follows that $\widehat{\delta}_i(\omega) = \widehat{\delta}^{po}$ for all $i = 3, \ldots, n$, where $\widehat{\delta}^{po}$ is given by the right-hand side of (A5).

Now note that (i) $\widehat{\delta}_1(\omega) = \widehat{\delta}_2(-\omega)$, (ii) $\widehat{\delta}_1(0) = \widehat{\delta}_i(\omega)$, and (iii) $\widehat{\delta}_1'(\omega) < 0$. Since $\omega > 0$, it follows that $\widehat{\delta}_2(\omega) > \widehat{\delta}_i(\omega) > \widehat{\delta}_1(\omega)$. Hence, the critical discount factor above which the fully collusive outcome can be sustained as a subgame-perfect equilibrium of the infinitely repeated game is $\widehat{\delta}_2(\omega)$. Since $\widehat{\delta}_2(\omega) > \widehat{\delta}_i(\omega) = \widehat{\delta}^{po}$, it follows that tacit collusion is hindered. *Q.E.D.*

Proof of Theorem 2. Using equations (11) and (8) and recalling that $S(B_i) \equiv \sum_{k=1}^n b_{ik}$,

$$\begin{aligned}\widehat{\delta}_i(A) - \widehat{\delta}_i^c(A) &= \frac{\frac{1}{n}\sum_{j=1}^n \gamma_{ij} S(B_j) - \frac{1}{n}\sum_{j=1}^n \gamma_{ij} b_{ji}\frac{S(B_i)}{b_{ii}}}{\sum_{j=1}^n \gamma_{ij} b_{ji}} \\ &= \frac{\frac{1}{n}\sum_{j=1}^n \gamma_{ij}\left(S(B_j) - \frac{b_{ji}}{b_{ii}}S(B_i)\right)}{\sum_{j=1}^n \gamma_{ij} b_{ji}}.\end{aligned} \tag{A16}$$

By Lemma A3, $S(B_j) - (b_{ji}/b_{ii})S(B_i) > 0$ for every distinct pair of firms, i, j. Hence, $\widehat{\delta}_i^c(A) < \widehat{\delta}_i(A)$ if $\gamma_{ij} > 0$ for some $j \neq i$ and $\widehat{\delta}_i^c(A) = \widehat{\delta}_i(A)$ otherwise. Finally, note that $(\partial/\partial\gamma_i)(\widehat{\delta}_i(A) - \widehat{\delta}_i^c(A)) < 0$. *Q.E.D.*

Proof of Proposition 4. Let A' be the new PCO matrix that differs from A only with respect to the rsth entry that is increased by ω, and let $B' = (I - A')^{-1}$. Now, suppose that $i = s$ or i is such that $b_{ir} = 0$. Using (A16) and recalling from Theorem 1 that $\hat{\delta}_i(A') = \hat{\delta}_i(A)$ for all i such that $b_{ir} = 0$ or $i = s$ yields

$$\begin{aligned}\hat{\delta}_i^c(A') - \hat{\delta}_i^c(A) &= \left(\widehat{\delta}_i(A) - \widehat{\delta}_i^c(A)\right) - \left(\widehat{\delta}_i(A') - \widehat{\delta}_i^c(A')\right) \\ &= \frac{\frac{1}{n}\sum_{j\neq i}\gamma_{ij}\left(S(B_j) - \frac{b_{ji}}{b_{ii}}S(B_i)\right)}{\sum_{j=1}^n \gamma_{ij} b_{ji}} - \frac{\frac{1}{n}\sum_{j\neq i}\gamma_{ij}\left(S(B_j') - \frac{b_{ji}'}{b_{ii}'}S(B_i')\right)}{\sum_{j=1}^n \gamma_{ij} b_{ji}'}.\end{aligned} \tag{A17}$$

Notice that since $\hat{\delta}_i(A') = \hat{\delta}_i(A)$, equation (8) implies that $S(B_i')/b_{ii}' = S(B_i)/b_{ii}$ (recall that $S(B_i) \equiv \sum_{k=1}^n b_{ik}$). Hence,

$$\begin{aligned}S(B_j') - \frac{b_{ji}'}{b_{ii}'}S(B_i') &= S(B_j') - \frac{b_{ji}'}{b_{ii}}S(B_i) \\ &= S(B_j) + \varepsilon_j S(B_s) - \frac{b_{ji} + \varepsilon_j b_{si}}{b_{ii}}S(B_i) \\ &= S(B_j) - \frac{b_{ji}}{b_{ii}}S(B_i) + \frac{\omega b_{jr}}{1 - \omega b_{sr}}\left(S(B_s) - \frac{b_{si}}{b_{ii}}S(B_i)\right),\end{aligned} \tag{A18}$$

where the second equality follows since by Lemma A1 in the Appendix, $b_{ji}' = b_{ji} + \varepsilon_j b_{si}$, and the third equality follows since $\varepsilon_j = \omega b_{jr}/(1 - \omega b_{sr})$. Similarly,

$$\sum_{j=1}^n \gamma_{ij} b_{ji}' = \sum_{j=1}^n \gamma_{ij}\left(b_{ji} + \frac{\omega b_{jr}}{1 - \omega b_{sr}} b_{si}\right) = \sum_{j=1}^n \gamma_{ij} b_{ji} + \frac{\omega b_{si}}{1 - \omega b_{sr}}\sum_{j=1}^n \gamma_{ij} b_{jr}. \tag{A19}$$

To prove part (i) of the proposition, suppose that $i = s$. Then (A18) implies that $S(B_j') - (b_{js}'/b_{ss}')S(B_s') = S(B_j) - (b_{js}/b_{ss})S(B_s)$, while (A19) implies that $\sum_{j=1}^n \gamma_{sj} b_{js}' \geq \sum_{j=1}^n \gamma_{sj} b_{js}$, with strict inequality whenever $\sum_{j=1}^n \gamma_{sj} b_{jr} > 0$.

Therefore, it follows from (A17) that $\hat{\delta}_s^c(A') \geq \hat{\delta}_s^c(A)$. Clearly, if $\gamma_{sj} = 0$ for all $j \neq s$ (firm s's controller does not invest in any of firm s's rivals), then by (A17), $\widehat{\delta}_i^c(A') = \widehat{\delta}_i^c(A)$. If $\gamma_{sj} > 0$ for some $j \neq s$, then the inequality is strict unless $\sum_{j=1}^n \gamma_{sj} b_{jr} = 0$.

To prove part (ii) of the proposition, suppose that $i \neq s$ but i is such that $b_{ir} = 0$. If in addition $\sum_{j \neq i} \gamma_{ij} b_{jr} = 0$, then by (A19), $\sum_{j=1}^n \gamma_{ij} b'_{ji} = \sum_{j=1}^n \gamma_{ij} b_{ji}$. Moreover, using (A18),

$$\sum_{j \neq i} \gamma_{ij} \left(S(B'_j) - \frac{b'_{ji}}{b'_{ii}} S(B'_i) \right) = \sum_{j \neq i} \gamma_{ij} \left(S(B_j) - \frac{b_{ji}}{b_{ii}} S(B_i) \right) + \frac{\omega}{1 - \omega b_{sr}} \sum_{j \neq i}^{n} \gamma_{ij} b_{jr} \left(S(B_s) - \frac{b_{si}}{b_{ii}} S(B_i) \right)$$
$$= \sum_{j \neq i} \gamma_{ij} \left(S(B_j) - \frac{b_{ji}}{b_{ii}} S(B_i) \right).$$

Hence, it follows from (A17) that $\hat{\delta}_i^c(A') = \hat{\delta}_i^c(A)$. *Q.E.D.*

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