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Horizontal subcontracting

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Horizontal subcontracting agreements between rival firms, each of which is capable of producing and marketing its products independently, are common. This article explains this practice and evaluates its welfare implications. The analysis shows that firms with asymmetric convex costs can use horizontal subcontracting to allocate production more efficiently between them and consequently generate a mutually beneficial surplus. For a wide range of parameters, this increase in production efficiency leads to an increase in industry output. The counterintuitive result is that welfare is thereby enhanced. In fact, even when industry output falls, welfare can still increase if production costs are sufficiently lowered.

1. Introduction

■ Subcontracting production is commonly employed in many industries. In many cases, it is done between two firms operating at the same horizontal stage of the vertical chain of production. These firms can be either immediate rivals or potential rivals, i.e., not currently operating in the same market but with the technological and financial ability to enter. Horizontal subcontracting is especially prevalent in such major industries as automobiles, computers, and aircraft. For example, Mazda Motor Co. produces the Ford Probe, which competes against its own MX-6 in the sports car market;[1] Zenith Electronics agreed to build laptop PCs for Hewlett-Packard;[2] and Lockheed, which operated in the commercial aircraft market until 1981, signed a contract to produce parts for Boeing's commercial aircraft.[3]

Each of these examples involves firms with the technological know-how to produce their products in-house and are either established in the downstream market or have the financial and technological ability to be. Nevertheless, these firms are engaged in a subcontracting arrangement with a rival firm. This raises two important questions. First, what are the underlying characteristics of the technology and market that support horizontal subcontracting? Second, what are the welfare implications of this practice? The importance of these questions was highlighted by the recent debate in Congress on the desirability of

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[1] *Los Angeles Times*, May 19, 1988, sec. 4, p. 1.

[2] *The Wall Street Journal*, May 13, 1988, p. 8.

[3] In addition, Lockheed is also considering the production of a long version of the McDonnell-Douglas MD-11 and the Fokker F-100. For details, see *The Wall Street Journal*, May 10, 1989, p. B7.

production joint ventures. Many of these joint ventures do not involve an investment in new production facilities and are in essence horizontal subcontracting agreements.

Like other forms of cooperative agreements between otherwise rival firms (e.g., cartels, mergers, and joint ventures), horizontal subcontracting presents a potential tradeoff. On the one hand, it may enable firms to better allocate production among themselves, thereby promoting production efficiency. On the other hand, it may also facilitate collusion because the agreement affects the firms' cost structures and consequently their outputs. Based on other forms of cooperative agreements, one might suspect that typically the second effect dominates and that horizontal subcontracting thus results in a collusive, less efficient outcome.[4] This article shows, however, that this intuition underestimates the efficiency rationale for horizontal subcontracting and that in fact this practice is often welfare enhancing. Moreover, for a wide range of parameters, the efficiency gains from horizontal subcontracting are so large that consumers become better off as well.

The model considers an industry consisting of two firms that can each produce a homogeneous good upstream and market it to downstream consumers. The interaction between the two firms evolves in two stages: a quantity-setting stage and a subcontracting stage. In the quantity-setting stage, the two firms engage in a Cournot quantity competition in the downstream market. In the subcontracting stage, the two firms sign a subcontracting agreement by which one firm produces some, potentially all, of its rival's output in return for a transfer payment.

Typically, contracts are treated in the industrial organization literature as long-term decisions while quantities and prices are treated as short-term decisions. Accordingly, the competitive effects of contracts are often examined in the framework of two-stage models, in which contracts are set in the first stage and competition takes place in the second.[5] This modelling approach, however, is appropriate in the context of horizontal subcontracting only to the extent that one believes that firms set the terms of their agreements before they actually determine their downstream quantities. In many cases, however, this does not seem to be the case. For example, when there is considerable uncertainty about the demand for the final good or the cost of marketing, firms may prefer to postpone their decisions about subcontracts until they actually receive orders from downstream retailers.[6]

Thus, this article analyzes two variants of the model that differ from one another only in the order in which the quantity-setting stage and the subcontracting stage are played. In the first variant, labelled *ex post* subcontracting, the two firms first commit themselves to their downstream quantities, say by signing binding supply contracts with retailers, then afterwards set a subcontracting agreement. In the second variant, labelled *ex ante* subcontracting, the firms sign a subcontracting agreement before they compete in the downstream market. For each variant the analysis predicts whether subcontracting will take place and, if it does, which firm will become the subcontractor; it examines the resulting markets

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[4] This is often the case with other forms of horizontal cooperative agreements, e.g., Jacquemin and Slade (1989).

[5] Examples for this approach include Brander and Lewis (1986) (financial contracts), Fershtman and Judd (1987) (managerial compensation), Cooper (1986) (most-favored customer contracts), and Katz and Shapiro (1985) (patent licensing).

[6] In fact, even when firms sign a subcontracting agreement before they compete in the market, they may wish to determine the actual subcontracted quantity only *ex post*. This seems to have been the case with the agreement between Mazda and Ford on Mazda's production of the Ford Probe. When the car was first introduced in March 1988, Ford could not satisfy the demand for the car, which presumably exceeded Ford's initial expectations (*Los Angeles Times*, May 19, 1988, sec. 4, p. 1). Moreover, Ford accumulated advance orders from dealers for 77,000 cars. This suggests that although the subcontracting agreement between the two firms was signed before any car was sold to consumers, the actual subcontracted quantity was set only after the quantity of the final good was determined by Ford in the market.

structure, i.e., whether the upstream and downstream markets are duopolies or monopolies; and it evaluates the welfare implications of subcontracting agreements.

Horizontal subcontracting is driven in this article by the assumption that the upstream cost functions are strictly convex. Thus, if firms produce at different marginal cost levels, horizontal subcontracting allows them to shift production from the high-marginal-cost firm to the low-marginal-cost firm, thereby creating a mutually beneficial cost savings. The analysis shows, however, that firms with identical cost structures produce the same quantities and therefore have no incentive to engage in horizontal subcontracting. Moreover, each firm produces its Cournot output, so the potential to subcontract production has no effect on firms' behavior.

Hence, if the article is to explain subcontracting, it has to consider asymmetric firms. Two types of asymmetries are considered: asymmetric upstream costs and asymmetric downstream costs. Under each type of asymmetry, absent an agreement, marginal costs of production are not equalized across firms, so horizontal subcontracting generates a surplus. Together with the fact that symmetric firms do not engage in subcontracting, this implies that firms never sign these agreements with the sole intention of supporting collusion: whenever horizontal subcontracting occurs, it necessarily enhances the efficiency of upstream production. Full efficiency is achieved, however, only when the agreement is set *ex post* or when the downstream market becomes a monopoly, but not otherwise.

Although firms may wish to use their horizontal subcontracting agreement to lower the industry output below its Cournot level, this may not be possible. To see why, note that while the agreement raises the subcontractor's costs, leading him to contract his output, it also lowers the rival's costs at the same time, leading him to expand his output. As a result, industry output may either exceed its Cournot level or fall below it. In the former case, welfare is clearly enhanced because upstream production becomes more efficient and consumers also become better off. In the latter case, welfare can still increase overall if the efficiency gains outweigh the loss to consumers. The analysis shows that for a wide range of parameters, this is indeed the case. *Ex post* subcontracting is most likely to be welfare enhancing if the subcontractor's share in the gains from the agreement is relatively small, whereas *ex ante* subcontracting is most likely to be welfare enhancing if the more efficient firm in upstream production becomes the subcontractor.

In addition, the analysis shows that in the asymmetric upstream costs case, the more efficient firm is likely to become the subcontractor, whereas in the asymmetric downstream costs case, the reverse is true. An interesting feature of the model is that, unless his downstream costs are significantly higher than those of his rival, the subcontractor not only produces for the rival, but also competes against it in the downstream market. The model can therefore explain situations such as those found in the car market (Mazda and Ford) and the PC market (Zenith and Hewlett-Packard).

Earlier literature on subcontracting includes Lewis and Sappington (1991), Kamien, Li, and Samet (1989), and Kamien and Li (1989). In Lewis and Sappington (1991), the prime contractor can either produce in-house or subcontract the production of an essential input to a more efficient firm that cannot be perfectly monitored. They find that subcontracting occurs if the subcontractor's innate cost advantage is sufficiently large to overcome the loss of control associated with subcontracting. Lewis and Sappington, however, assume that the subcontractor does not produce the final good, even though he may, because of his lower innate costs, be better off competing with the prime contractor rather than subcontracting for it.[7] In this article, firms can choose whether to operate in the downstream market, or remain in the upstream market as subcontractors, or both.

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[7] The exogenous assignment of firms to either the upstream or downstream markets is common in the vertical-relations literature, e.g., Katz (1989) and Perry (1989).

Kamien, Li, and Samet (1989) and Kamien and Li (1989) study subcontracting agreements that follow an auction for contracts, such as marine insurance contracts and architectural design contracts. They consider a two-stage model in which firms first submit bids for a contract. The firm that makes the lowest bid wins the contract and can subcontract production to the losers. They find that the potential to subcontract production alters firms' behavior in the bidding stage even when firms are symmetric and do not engage eventually in subcontracting. Specifically, they show that firms bid less aggressively when the terms of the subcontracts are set by the losers of the bidding stage. The present article, in contrast, studies subcontracting in product markets, where access to the market is obtained without winning an auction. Thus, contrary to Kamien et al. the current model gives rise to the often observed situations where subcontractors also produce for final consumers. In addition, in the current model subcontracting affects firms' behavior in the downstream market only when the firms actually engage in subcontracting.

The remainder of the article is organized as follows. The basic model is presented in Section 2. Section 3 considers *ex post* horizontal subcontracting. *Ex ante* horizontal subcontracting is analyzed in Section 4. In Section 5, the basic model is extended in order to examine the role of subcontracting in deterring entry into a downstream market. Section 6 furnishes a summary of the main results. All the proofs are in the Appendix.

2. The model

■ There are two quantity-setting firms that produce a single homogeneous good. The inverse demand for the final good is given by $P(Q)$, where P is price and $Q = Q^1 + Q^2$ is the industry output (superscripts denote firms). Production of the final good consists of an upstream process and a downstream process. The cost of production in the upstream process is $n^iC(Q/n^i)$. The parameter n can be thought of as the number of identical plants that each firm owns. To simplify, normalize n^2 to 1 and let $n^1 = n \geq 1$. The cost of production in the downstream process is m^iQ^i, $i = 1, 2$, where $m^2 \geq m^1$. For convenience, I refer to this cost as the cost of marketing the final good to consumers, although in general it may also represent the cost of manufacturing a final good from an intermediate good produced in the upstream process. The assumed properties of the demand and cost functions are as follows:

Assumption 1. P is twice continuously differentiable and downward sloping ($P' < 0$).

Assumption 2. Each firm's reaction function in the traditional Cournot model is downward sloping: $P' + P''Q^i < 0$, $i = 1, 2$.

Assumption 3. C is twice continuously differentiable and strictly convex.

Assumption 4. Production is profitable at least for firm 1: $P(0) - C'(0) > m^1$.

In what follows, I analyze two variants of the model. In each variant there is a quantity-setting stage and a subcontracting stage. In the quantity-setting stage, the two firms engage in a Cournot quantity competition in the downstream market. In the subcontracting stage, the two firms sign a subcontracting agreement, according to which firm 1 will produce q units for firm 2 and in return will receive a transfer payment t. When $q, t < 0$, firm 2 is the subcontractor. A subcontract is therefore defined by the pair (q, t). If no agreement is signed, $q = t = 0$.

Subcontracting agreements are assumed to be determined in a bargaining process that has the following two properties: First, it is efficient, in the sense that the subcontracted quantity, q, is chosen to generate the largest possible surplus. Second, the transfer payment, t, is chosen to divide this surplus between firms 1 and 2 in proportions α and $1 - \alpha$, respectively, where $\alpha \in [0, 1]$. The parameter α can be thought of as measuring firm 1's bargaining power. These properties are satisfied, for example, by the asymmetric Nash bargaining solution.

The two model variants differ in the order in which the quantity-setting stage and the subcontracting stage are played. In the first variant, labelled *ex post* subcontracting, the quantity-setting stage is played before the subcontracting stage. Thus, subcontracting occurs in this variant after the two firms have already committed themselves to their respective downstream quantities, say by signing binding supply contracts with retailers. This variant can represent situations in which there is considerable uncertainty about either the demand for the final good or the cost of marketing, and the firms can learn their realizations only when they actually set their downstream quantities. In this case, the firms may wish to postpone their decisions about subcontracting until the last stage of the game.[8]

In the second variant of the model, the firms are assumed to sign a subcontracting agreement before they compete in the downstream market. This variant is labelled *ex ante* subcontracting. It may represent situations in which the subcontracting agreement specifies a certain quantity that the prime contractor is obligated to buy from the subcontractor.

Both variants of the model are solved backwards by assuming that in each stage both firms play optimally given the history of the game.[9]

3. *Ex post* subcontracting

■ This section analyzes the first variant of the model, in which the firms engage in a Cournot quantity competition in the downstream market before they decide whether or not to subcontract production. As the model is solved backwards, the analysis begins by considering the subcontracting stage. At this stage, given their downstream quantities, (Q^1, Q^2), the two firms sign a subcontracting agreement if by so doing they can realize a mutually beneficial surplus. But, as the firms are already committed at this point to their downstream quantities, such a surplus can be generated only if upstream production is made more efficient.

Since the upstream cost functions are strictly convex, efficient production requires the same quantity to be produced in each upstream plant. As the total number of plants is $n + 1$, this implies that upstream production is efficient if and only if the production level in each plant is $Q/(n + 1)$. Therefore, so long as $Q^1/n \neq Q^2$, a positive surplus can be realized by shifting production from the high-cost producer to the low-cost one, such that eventually, the cost of upstream production is equalized across firms. Thus, the subcontracted quantity, q, is chosen to satisfy $(Q^1 + q)/n = Q^2 - q = Q/(n + 1)$, or,

$$q(Q^1, Q^2) = \frac{nQ^2 - Q^1}{n + 1}. \tag{1}$$

Note that $q(Q^1, Q^2)$ may be either positive or negative, depending on whether firm 1 is the subcontractor or not.

Using the fact that subcontracting equalizes the upstream cost of production across firms, the surplus it generates as a function of Q^1, Q^2 is

$$S(Q^1, Q^2) = nC\left(\frac{Q^1}{n}\right) + C(Q^2) - (n + 1)C\left(\frac{Q}{n + 1}\right). \tag{2}$$

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[8] In addition, this variant can also represent situations where, although subcontracts are signed before firms choose their downstream quantities, the specification of the actual subcontracted quantities is left open and is determined after firms have already signed supply contracts with retailers (see footnote 6).

[9] Note, however, that technically the equilibrium concept used in this article is not subgame perfect, since the solution for the subcontracting stage is cooperative.

This surplus represents the industry's cost savings due to the efficient allocation of upstream production between the two firms. Now, recall that t is chosen to split $S(Q^1, Q^2)$ between the firms according to their bargaining powers. Thus,

$$t(Q^1, Q^2) = (1-\alpha)n\left[C\left(\frac{Q}{n+1}\right) - C\left(\frac{Q^1}{n}\right)\right] + \alpha\left[C(Q^2) - C\left(\frac{Q}{n+1}\right)\right]. \quad (3)$$

The transfer payment $t(Q^1, Q^2)$, is therefore a weighted sum of the firms' incremental costs due to *ex post* subcontracting. Given $t(Q^1, Q^2)$, the profits of firms 1 and 2, respectively, are

$$\begin{aligned}\pi^1(Q^1, Q^2) &= (P - m^1)Q^1 + t(Q^1, Q^2) - nC\left(\frac{Q}{n+1}\right) \\ &= (P - m^1)Q^1 - nC\left(\frac{Q^1}{n}\right) + \alpha S(Q^1, Q^2) \quad (4)\end{aligned}$$

and

$$\begin{aligned}\pi^2(Q^1, Q^2) &= (P - m^2)Q^2 - t(Q^1, Q^2) - C\left(\frac{Q}{n+1}\right) \\ &= (P - m^2)Q^2 - C(Q^2) + (1-\alpha)S(Q^1, Q^2). \quad (5)\end{aligned}$$

Anticipating the outcome of the subcontracting stage, the objective of each firm in the quantity-setting stage is to maximize its profits with respect to its own quantity. The corresponding first-order conditions for Q^1 and Q^2 are

$$\pi^1_1(Q^1, Q^2) \equiv P - m^1 + P'Q^1 - (1-\alpha)C'\left(\frac{Q^1}{n}\right) - \alpha C'\left(\frac{Q}{n+1}\right) \leq 0;$$

$$\pi^1_1(Q^1, Q^2)Q^1 = 0 \quad (6)$$

and

$$\pi^2_2(Q_1, Q_2) \equiv P - m^2 + P'Q^2 - \alpha C'(Q^2) - (1-\alpha)C'\left(\frac{Q}{n+1}\right) \leq 0;$$

$$\pi^2_2(Q^1, Q^2)Q^2 = 0, \quad (7)$$

where subscripts denote partial derivatives. Assumptions 2 and 3 guarantee that (6) and (7) are necessary and sufficient for a maximum. Let (Q^{1*}, Q^{2*}) be the solution to equations (6) and (7). From Assumption 4 it follows that $Q^{1*} > 0$. By subtracting (4) from (5) and using Assumptions 1 and 3 it can be shown that $Q^{2*} > 0$ provided that m^2 is not too large. The pair (Q^{1*}, Q^{2*}) induces a subcontracting agreement (q^*, t^*), where $q^* = q(Q^{1*}, Q^{2*})$ and $t^* = t(Q^{1*}, Q^{2*})$.

Equations (6) and (7) define the reaction functions for firms 1 and 2, respectively. Denote firm i's reaction function by $R^i(Q^j, \alpha)$. An equilibrium in the overall game is the quadruple $(Q^{1*}, Q^{2*}, q^*, t^*)$, where $Q^{1*} = R^1(Q^{2*}, \alpha)$ and $Q^{2*} = R^2(Q^{1*}, \alpha)$. By Assumptions 1 through 3, this equilibrium always exists and is unique.[10] The properties of

[10] The existence of an equilibrium is ensured if each firm's profit function is concave in its own output (see Shapiro (1989)). This property is ensured by Assumptions 2 and 3. As for uniqueness, a straightforward differentiation of the profit functions of firms 1 and 2 with respect to Q^1 and Q^2, respectively, yields

$$|\pi^1_{11}(Q_1, Q_2, \alpha)| < |\pi^1_{12}(Q_1, Q_2, \alpha)|,$$

and $|\pi^2_{11}(Q_1, Q_2, \alpha)| < |\pi^2_{12}(Q_1, Q_2, \alpha)|$, where the inequalities follow from Assumptions 1 through 3. These inequalities, in turn, are sufficient to ensure uniqueness (see Shapiro (1989)).

the equilibrium depend on the assumptions about the cost structures of both firms. In what follows, three different cases are analyzed: (1) both firms have the same cost structure, i.e., $n = 1$ and $m^1 = m^2$; (2) the two firms differ only in their upstream costs; i.e., $n > 1$ and $m^1 = m^2$; and (3) the two firms differ only in their marketing costs, i.e., $n = 1$ and $m^1 < m^2$.

□ **The symmetric costs case.** In this subsection, the two firms are assumed to have the same cost structure both upstream and downstream. Given this assumption, equations (6) and (7) hold with equality, so the downstream market is a duopoly. The equilibrium in this case is characterized in Proposition 1.

Proposition 1. When firms have symmetric cost structures, they produce the same quantities and do not engage in *ex post* horizontal subcontracting. Moreover, each firm produces the quantity it would have produced in the traditional Cournot model.

Proof. See the Appendix.

The intuition behind Proposition 1 is straightforward. Since the firms are identical, horizontal subcontracting can be profitable only if one firm produces less than its rival and is therefore operating at a lower marginal cost level. But since producing directly for consumers is more profitable, each firm chooses its Cournot output and no subcontracting takes place.

□ **The asymmetric upstream costs case.** Suppose next that $n > 1$, and $m^1 = m^2$. As before, the latter assumption implies that equations (6) and (7) hold with equality, so the downstream market is a duopoly. Since the parameter n can be thought of as the number of identical plants that firm 1 owns, firm 1 is referred to as the big firm while firm 2 is the small firm.

Now, before characterizing the equilibrium, I first establish the following lemma.

Lemma 1. For every $\alpha \in [0, 1]$ and $n > 1$, $Q^{1*} > Q^{2*} > Q^{1*}/n$, i.e., the big firm sells in the downstream market a larger total quantity but a smaller average quantity per upstream plant than the small firm.

Proof. See the Appendix.

Lemma 1 implies that firm 1 is big not only because it owns more upstream plants, but also because its market share is larger. Using the result of Lemma 1, it follows from (1) that $q^* = (nQ^{2*} - Q^{1*})/(n + 1) > 0$. In addition, (1) implies that $q^* \to Q^{2*}$ as $n \to \infty$. Hence,

Proposition 2. When firms have asymmetric upstream costs, $q^* > 0$, i.e., the small firm (firm 2) subcontracts production to the big firm (firm 1). Moreover, as $n \to \infty$, $q^* \to Q^{2*}$, implying that the small firm subcontracts its entire production to the big one.

Proposition 2 is quite intuitive. As Lemma 1 shows, the small firm supplies to final consumers a larger quantity per upstream plant than does the big firm. Hence its marginal costs are higher than those of the big firm. By subcontracting production from the small firm to the big one, a mutually profitable surplus is created. Thus, when upstream production costs are asymmetric, both firms sell to consumers, but some of the small firm's quantity is actually produced by the big firm. When this asymmetry is "large" ($n \to \infty$), the big firm produces all of the small firm's quantity.

Now let us establish two important properties of the reaction functions:

Lemma 2. $0 > R^i_j \equiv \partial R^i(Q^j, \alpha)/\partial Q^j > -1$, and $R^i_\alpha \equiv \partial R^i(Q^j, \alpha)/\partial \alpha < 0$, i.e., both firms contract their output as their rival expands, but by less than the rival's expansion, and the reaction functions shift inward in the downstream quantities space as α increases.

Proof. See the Appendix.

The result that the reaction functions are downward sloping with a slope larger than minus one is a standard result in oligopoly models with Cournot competition, e.g., Farrell and Shapiro (1990). The reason why $R^i(Q^j, \alpha)$ decreases in α is the following. Differentiating $S(Q^1, Q^2)$ with respect to Q^1 and Q^2, respectively, yields

$$\partial S(Q^1, Q^2)/\partial Q^1 = C'(Q^1/n) - C'(Q/(n+1))$$

and $\partial S(Q^1, Q^2)/\partial Q^2 = C'(Q^2) - C'(Q/(n+1))$. From Lemma 1 it follows that evaluated at the equilibrium downstream quantities, $\partial S(Q^{1*}, Q^{2*})/\partial Q^1 < 0 < \partial S(Q^{1*}, Q^{2*})/\partial Q^2$. Thus, as α increases, firm 1 enjoys a larger share in $S(Q^{1*}, Q^{2*})$, and as a result, it has an incentive to increase it as much as possible by decreasing Q^1. At the same time, firm 2's share in $S(Q^{1*}, Q^2)$ becomes smaller, so firm 2 decreases Q^2 in order to counter this negative effect on its profits.

Next, the quantities Q^{1*}, Q^{2*}, and $Q^* = Q^{1*} + Q^{2*}$, supplied to the downstream market are compared to Q^{1c}, Q^{2c}, and Q^c, the corresponding quantities in the traditional Cournot model.

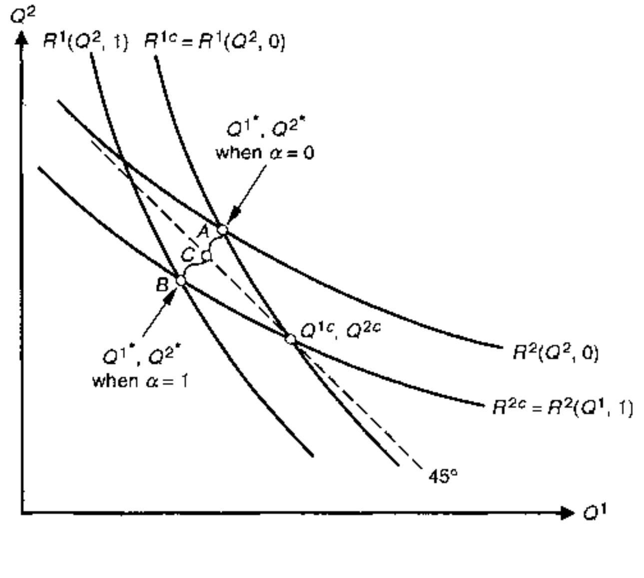
Proposition 3. For every $\alpha \in [0, 1]$, the big firm (firm 1) supplies a smaller quantity, while the small firm (firm 2) supplies a larger quantity, than they respectively supply in the traditional Cournot model. Whether the industry output is larger or smaller than it is in the traditional Cournot model, however, depends on the value of α. In particular, there exists an $\bar{\alpha} \in (0, 1)$, such that $Q^* \gtreqless Q^c$ as $\alpha \lesseqgtr \bar{\alpha}$.

Proof. See the Appendix.

Proposition 3 is illustrated in Figure 1. Given α, the equilibrium outcome in the downstream quantities space, (Q^{1*}, Q^{2*}), is given by the intersection of $R^1(Q^2, \alpha)$ and $R^2(Q^1, \alpha)$. The locus of all equilibria outcomes for $\alpha \in [0, 1]$ is the curve AB. The Cournot equilibrium outcome is given by the intersection of $R^{1c}(Q^2)$ and $R^{2c}(Q^1)$. Note that the equilibrium outcome (Q^{1*}, Q^{2*}) is achieved northwest of (Q^{1c}, Q^{2c}), implying that for

FIGURE 1

EX POST HORIZONTAL SUBCONTRACTING: THE ASYMMETRIC UPSTREAM COSTS CASE



every $\alpha \in [0, 1]$, $Q^{1*} < Q^{1c}$ and $Q^{2*} > Q^{2c}$. As for Q^*, note that since $R^i_j > -1$, the 45-degree line passing through (Q^{1c}, Q^{2c}) (which represents all the combinations of Q^1 and Q^2 for which $Q^1 + Q^2 = Q^c$), divides the curve AB into two segments: AC and CB. The segment AC, where $Q^* \geq Q^c$, represents the equilibria quantities for $\alpha \leq \bar{\alpha}$, while the segment CB, where $Q^* < Q^c$, represents the equilibria quantities for $\alpha > \bar{\alpha}$.[11]

The measure of social welfare used in this article is the sum of consumer and producer surplus or, equivalently, the difference between gross consumer benefits and production costs, given by

$$W(Q^1, Q^2) = \int_0^Q P(X)dX - m^1Q^1 - m^2Q^2 - nC\left(\frac{Q^1}{n}\right) - C(Q^2) + Is(Q^1, Q^2)$$

where $I = 1$ if *ex post* subcontracting takes place, and zero otherwise. *Ex post* horizontal subcontracting has two effects on welfare. First, it lowers upstream production costs by allocating production efficiently between the two firms. Second, it affects the industry output Q. For $\alpha < \bar{\alpha}$, both effects enhance welfare. For $\alpha > \bar{\alpha}$, however, $Q^* < Q^c$, so in general it is impossible to tell which effect dominates. This is summarized in the following proposition:

Proposition 4. When firms have asymmetric upstream production costs, *ex post* horizontal subcontracting increases welfare over its traditional Cournot level when $\alpha \leq \bar{\alpha}$, but may either increase it or decrease it otherwise.

Proof. See the Appendix.

□ **The asymmetric marketing costs case.** Now suppose that $n = 1$ and $m^2 \geq m^1$. That is, both firms are equally efficient in upstream production, but firm 1 has lower marketing costs, say because it has a better network of dealerships in the downstream market. Using the same method as in the proof of Lemma 1, one can show that $Q^{1*} > Q^{2*}$. From (1), it follows that in this case $q^* = (Q^{2*} - Q^{1*})/2 < 0$. Thus, firm 2, the less efficient in marketing, produces some (but not all) of firm 1's output. When m^2 is sufficiently large, firm 2 forgoes the downstream market altogether and operates only as a subcontractor to firm 1.

Intuitively, since firm 1 has lower marketing costs than firm 2, it sells a larger quantity to final consumers. But since the firms have the same convex upstream production cost functions, a mutually beneficial surplus can be generated if firm 1 subcontracts production to firm 2, so that eventually, each firm produces $Q^*/2$.

As before, conditions (6) and (7) define the reaction functions of firm 1 and 2, respectively. Two important properties of these functions are established in the next lemma.

Lemma 3. $0 > R^i_j > -1$, and $R^i_\alpha > 0$, i.e., both firms contract their output as their rival expands, but by less than the rival's expansion, and the reaction functions shift outward in the downstream quantities space as α increases.

Proof. See the Appendix.

Since both reaction functions shift outward in the downstream quantities space as α increases, it is clear from Figure 2 that for all α, $Q^{1*} > Q^{1c}$, $Q^{2*} < Q^{2c}$, and Q^* may be

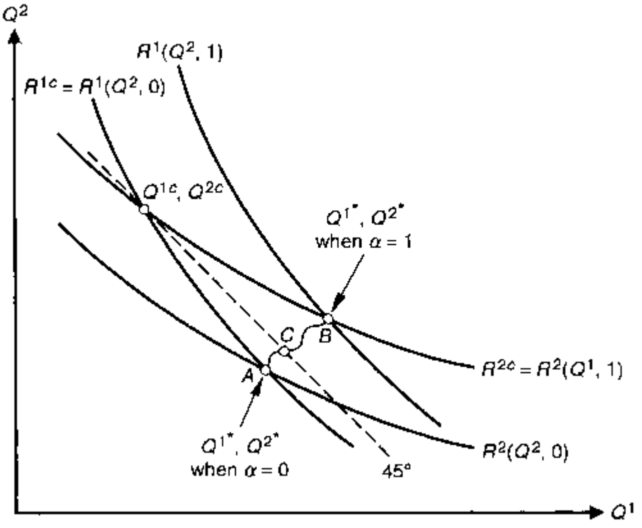
[11] To get a sense of the magnitude of $\bar{\alpha}$, suppose that $P(Q) = 1 - Q$ and $C(Q) = Q^2/n^i$. Then, a straightforward calculation shows that $Q^c = 2(2n + 1)/(7n + 8)$, and

$$Q^* = 2(1 + n)(1 - \alpha + n + \alpha n)/[8(1 - \alpha) + n(3 + 4\alpha)(1 + n) + 6n].$$

A comparison of the two quantities, reveals that $\bar{\alpha} = (n + 2)/(n + 5)$. Hence, $\bar{\alpha} = ½$ when $n = 1$ and $\bar{\alpha} \rightarrow 1$ as $n \rightarrow \infty$. Thus, as firm 1's upstream production efficiency increases, the equilibrium industry output is more likely to exceed its Cournot level.

FIGURE 2

EX POST HORIZONTAL SUBCONTRACTING: THE ASYMMETRIC DOWNSTREAM COSTS CASE



either larger or smaller than Q^c depending on α. As in the proof of Proposition 3, one can define $\bar{\bar{\alpha}} \in (0, 1)$ such that $Q^* = Q^c$ if $\alpha = \bar{\bar{\alpha}}$. However, in contrast with the asymmetric upstream costs case, now $Q^* \gtreqless Q^c$ as $\alpha \gtreqless \bar{\bar{\alpha}}$. That is, now the industry output exceeds its Cournot level when α is relatively large, but is smaller otherwise.[12] An immediate implication of this is the following:

Proposition 5. When firms have asymmetric marketing costs, *ex post* horizontal subcontracting increases welfare over its traditional Cournot level when $\alpha \geq \bar{\bar{\alpha}}$, but may either increase it or decrease it otherwise.

Proof. See the Appendix.

To assess the welfare implications of horizontal subcontracting in this case, note that now it has three effects on welfare. First, it affects the industry output and hence consumers' welfare; second, it enhances the efficiency of upstream production; and third, it induces a shift in downstream production from firm 2, the less efficient in marketing, to firm 1, thereby enhancing the overall efficiency of marketing as well. While the last two effects are unambiguously positive, the first effect may be either positive or negative, depending on α. When α is small, this effect is negative and may outweigh the two positive effects. Indeed, the example in the proof of Proposition 5 shows that when $\alpha = 0$, the reduction in industry output is the dominating effect if m^2 is not too large, and that the reduction in the industry's marketing cost is the dominating effect otherwise.

4. *Ex ante* subcontracting

■ This section analyzes the second variant of the model. In this variant, firms are assumed to sign a subcontracting agreement, $(\tilde{q}, \tilde{t})$, before they choose their downstream quantities. As argued above, this variant can describe industries in which firms sign long-term contracts with inputs' suppliers (i.e., subcontractors) but can make output decisions on a frequent

[12] The formal proof is omitted since it is similar to the one obtained for the upstream asymmetric costs case. To get a sense of the magnitude of $\bar{\bar{\alpha}}$, let $P(Q) = 1 - Q$, $C(Q) = Q^2$, and $m^1 = 0 < m^2 = m$. Then, a straightforward calculation shows that $Q^c = (2 - m)/5$, and $Q^* = [4 - m(3 - 2\alpha)]/10$. A comparison of the two quantities reveals that $\bar{\bar{\alpha}} = 1/2$, so $Q^* \gtreqless Q^c$ as $\alpha \gtreqless 1/2$.

basis. In addition, it represents situations in which firms sign an agreement that specify explicitly the amount that will be subcontracted.

Given the stage 1 subcontract $(\tilde{q}, \tilde{t})$, firms 1 and 2 choose their respective quantities in the quantity-setting stage of the game to maximize their respective profit functions

$$\pi^1(Q^1, Q^2, \tilde{q}) = (P - m^1)Q^1 + \tilde{t} - nC\left(\frac{Q^1 + \tilde{q}}{n}\right) \tag{9}$$

and

$$\pi^2(Q^1, Q^2, \tilde{q}) = (P - m^2)Q^2 - \tilde{t} - C(Q^2 - \tilde{q}), \tag{10}$$

where $\tilde{q}, \tilde{t} > 0$, $(\tilde{q}, \tilde{t} < 0)$, if firm 1 (firm 2) is the subcontractor. Differentiating each firm's profit function with respect to its own quantity yields the following first-order conditions:

$$\pi_1^1(Q^1, Q^2, \tilde{q}) \equiv P - m^1 + P'Q^1 - C'\left(\frac{Q^1 + \tilde{q}}{n}\right) \leq 0; \qquad \pi_1^1(Q^1, Q^2, \tilde{q})Q^1 = 0 \tag{11}$$

and

$$\pi_2^2(Q^1, Q^2, \tilde{q}) \equiv P - m^2 + P'Q^2 - C'(Q^2 - \tilde{q}) \leq 0; \qquad \pi_2^2(Q^1, Q^2, \tilde{q})Q^2 = 0. \tag{12}$$

Assumptions 2 and 3 guarantee that, for all $\tilde{q}$, (11) and (12) are necessary and sufficient for a maximum. The two first-order conditions define the equilibrium downstream quantities as functions of $\tilde{q}$, $Q^{1*}(\tilde{q})$, and $Q^{2*}(\tilde{q})$. It can be shown that Assumptions 1 through 3 ensure that for every $\tilde{q}$, the pair $(Q^{1*}(\tilde{q}), Q^{2*}(\tilde{q}))$ is unique. Note that when $\tilde{q} = 0$, (11) and (12) become identical to the first-order conditions for a Cournot equilibrium. Thus, $Q^{1*}(0) = Q^{1c}$ and $Q^{2*}(0) = Q^{2c}$. To examine the effects of horizontal subcontracting on firms' behavior in the downstream market, assume that m^2 is not too large, so that both (11) and (12) hold with equality, i.e., both firms are active in the downstream market. Differentiating the system of the two equations totally yields

$$\frac{dQ^{1*}(\tilde{q}^*)}{d\tilde{q}} = \frac{\psi + \frac{1}{n}P'C''\left(\frac{Q^{1*}(\tilde{q}^*) + \tilde{q}^*}{n}\right)}{\tilde{H}} \tag{13}$$

and

$$\frac{dQ^{2*}(\tilde{q}^*)}{d\tilde{q}} = -\frac{\psi + P'C''(Q^{2*}(\tilde{q}^*) - \tilde{q}^*)}{\tilde{H}}, \tag{14}$$

where

$$\psi \equiv (P' + P''Q^{1*}(\tilde{q}^*))C''(Q^{2*}(\tilde{q}^*) - \tilde{q}^*) + (P' + P''Q^{2*}(\tilde{q}^*))\frac{1}{n}C''\left(\frac{Q^{1*}(\tilde{q}^*) + \tilde{q}^*}{n}\right)$$
$$- \frac{1}{n}C''\left(\frac{Q^{1*}(\tilde{q}^*) + \tilde{q}^*}{n}\right)C''(Q^{2*}(\tilde{q}^*) - \tilde{q}^*),$$

and $\tilde{H} \equiv \pi_{11}^1\pi_{22}^2 - \pi_{12}^1\pi_{21}^2$. Assumptions 1 through 3 ensure that $\tilde{H} > 0 > \psi$, so $dQ^{1*}(\tilde{q}^*)/d\tilde{q} < 0 < dQ^{2*}(\tilde{q}^*)/d\tilde{q}$. This establishes the following result:

Proposition 6. Assume that in equilibrium the downstream market is a duopoly. Then the subcontractor supplies a smaller quantity and his rival supplies a larger quantity than they respectively supply in the traditional Cournot model.

Now, given $\tilde{q}$, the equilibrium stage-2 profits are given by

$$\pi^i(\tilde{q}) \equiv \pi^i(Q^{1*}(\tilde{q}), Q^{2*}(\tilde{q}), \tilde{q}), \qquad i = 1, 2.$$

Anticipating the outcome of the quantity competition in the downstream market, the two firms choose the *ex ante* subcontract with the objective of maximizing their joint profits, $\Pi(\tilde{q}) = \pi^1(\tilde{q}) + \pi^2(\tilde{q})$. Assume that $\Pi(\tilde{q})$ is strictly concave,[13] and denote its (unique) maximum by $\tilde{q}^*$. Using the envelope theorem, the first-order condition for $\tilde{q}^*$ is

$$\Pi'(\tilde{q}) = \begin{cases} P'\left[Q^{1*}(\tilde{q})\dfrac{dQ^{2*}(\tilde{q})}{d\tilde{q}} + Q^{2*}(\tilde{q})\dfrac{dQ^{1*}(\tilde{q})}{d\tilde{q}}\right] \\ \quad -\left[C'\left(\dfrac{Q^{1*}(\tilde{q}) + \tilde{q}}{n}\right) - C'(Q^{2*}(\tilde{q}) - \tilde{q})\right] = 0, \\ \qquad\qquad \text{if} \quad Q^{1*}(\tilde{q}) > 0 \text{ and } Q^{2*}(\tilde{q}) > 0, \\ -\left[C'\left(\dfrac{Q^{1*}(\tilde{q}) + \tilde{q}}{n}\right) - C'(Q^{2*}(\tilde{q}) - \tilde{q})\right] = 0, \qquad \text{otherwise.} \end{cases} \tag{15}$$

The equilibrium transfer payment, $\tilde{t}^*$, is chosen to split the gains from *ex ante* subcontracting, $\Pi(\tilde{q}^*) - \Pi(0)$, between the two firms according to their respective bargaining powers.

An equilibrium in the overall game is the quadruple $(\tilde{q}^*, \tilde{t}^*, Q^{1*}(\tilde{q}^*), Q^{2*}(\tilde{q}^*))$. In equilibrium, the downstream market is either a duopoly (i.e., $Q^{1*}(\tilde{q}^*), Q^{2*}(\tilde{q}^*) > 0$) or a monopoly with firm 1 as the sole provider of the final good (i.e., $Q^{1*}(\tilde{q}^*) > 0$ and $Q^{2*}(\tilde{q}^*) = 0$). A necessary and sufficient condition for the latter case is that m^2 is sufficiently larger than m^1.[14]

□ **The symmetric costs case.** The analysis begins by considering the case where firms have the same cost structures.

Proposition 7. Assume that $\Pi(\tilde{q})$ is strictly concave. Then, when firms have symmetric cost structures, they do not engage in *ex ante* horizontal subcontracting, i.e., $\tilde{q}^* = 0$. Consequently, both firms produce the same quantities that are equal to their traditional Cournot quantities.

Proof. See the Appendix.

Proposition 7 indicates that the result of Proposition 1 is robust to a reversal in the sequence in which the quantity-setting stage and the subcontracting stage are played. The conclusion is that firms with symmetric cost structures do not engage in horizontal subcontracting regardless of whether they can sign an agreement before or after they compete in the downstream market. An implication of this is that firms never sign horizontal subcontracting agreements with the sole intention of supporting collusion. These agreements are, therefore, necessarily associated with enhanced upstream production efficiency.

Before proceeding, it should be noted that the opposite of Proposition 7 does not necessarily hold. That is, in contrast with the *ex post* horizontal subcontracting variant, here, firms with asymmetric cost structures do not always engage in subcontracting: the example in the Appendix demonstrates that there is a nonempty set of parameters for which $\tilde{q}^* = 0$.

□ **The asymmetric costs case.** First, consider the case where m^2 is sufficiently larger than m^1 so that firm 2 prefers to stay out of the downstream market. Let $\tilde{Q}^*$ be the industry

[13] This assumption is satisfied, for example, when the demand for the final good is linear and the upstream cost functions are quadratic.

[14] To see why, note that when $Q^{2*}(\tilde{q}^*) = 0$, equation (15) implies that $\tilde{q}^* = Q^{1*}(\tilde{q}^*)/(n+1)$. Thus, a necessary condition for $Q^{2*}(\tilde{q}^*) = 0$ is $P - m^2 - C'(Q^{1*}(\tilde{q}^*)/(n+1)) < 0$. Given this inequality, the first-order condition for Q^{1*} is satisfied if and only if $m^2 > m^1 - P'Q^{1*}(\tilde{q}^*)$. When this condition fails, i.e., m^2 is close to m^1, the downstream market is a duopoly.

output in this case. Equation (15) implies that $\tilde{q}^* = -\tilde{Q}^*/(n+1) < 0$, where from equation (11) it follows that $\tilde{Q}^*$ is the solution to

$$P(\tilde{Q}^*) + P'(\tilde{Q}^*)\tilde{Q}^* = m^1 + C'(\tilde{Q}^*/(n+1)).$$

Thus, $\tilde{Q}^*$ is produced efficiently and is identical to the output of a monopolist with $n + 1$ plants. Now, define $\bar{m}^2(\tilde{q}^*)$ as the lowest m^2 for which firm 2 stays out of the downstream market under (optimal) *ex ante* horizontal subcontracting. Similarly, define $\bar{m}^2$ for the *ex post* horizontal subcontracting case and $\bar{m}^{2c}$ for the traditional Cournot case. Then,

Proposition 8. Assume that, in equilibrium, firm 1 is a monopolist in the downstream market. Then, $\tilde{Q}^* = Q^* > Q^c$ and $\bar{m}^2(\tilde{q}^*) = \bar{m}^2 < \bar{m}^{2c}$ if $\alpha = 1$; $\tilde{Q}^* > Q^* = Q^c$ and $\bar{m}^2(\tilde{q}^*) < \bar{m}^2 = \bar{m}^{2c}$ if $\alpha = 0$; and $\tilde{Q}^* > Q^* > Q^c$ and $\bar{m}^2(\tilde{q}^*) < \bar{m}^2 < \bar{m}^{2c}$ otherwise.

Proof. See the Appendix.

Proposition 8 shows that the range of m^2 for which the downstream market becomes a monopoly is wider under *ex ante* horizontal subcontracting than it is under *ex post* horizontal subcontracting and that the latter is wider than the corresponding range under traditional Cournot competition. On the other hand, the proposition also indicates that when the downstream market is a monopoly, the industry output is ranked in the reverse order.

Next, consider the case in which the downstream market is a duopoly. Then, $\tilde{q}^*$ is given by the first line in (15). From (15) it is clear that the contracting firms have two objectives when they choose $\tilde{q}^*$. The first objective, captured by the first term in (15), is to internalize the negative externalities that the firms inflict on one another in the downstream market. The second objective, captured by the second term in (15), is to increase the efficiency of upstream production by shifting production from the high-marginal-cost firm to the low-marginal-cost firm. As the next proposition shows, the second objective is never fully achieved.

Proposition 9. Assume that $\Pi(\tilde{q})$ is strictly concave, that firms have asymmetric cost structures, and that in equilibrium, the downstream market is a duopoly. Then, if *ex ante* subcontracting takes place, i.e., $\tilde{q}^* \neq 0$, the industry output is produced inefficiently, in the sense that upstream marginal costs are not equalized across firms.

Proof. See the Appendix.

Proposition 9 stands in contrast with the case in which the agreement is set *ex post*, since here subcontracting does not lead to full efficiency in upstream production. The reason for this difference is that while in the *ex post* horizontal case the agreement is used only to allocate production efficiently between the two firms, here it is also utilized to affect their downstream production levels.

To evaluate the social desirability of *ex ante* horizontal subcontracting, let $W(\tilde{q}) \equiv W(Q^{1*}(\tilde{q}), Q^{2*}(\tilde{q}))$, and assume that it is strictly concave.[15] Differentiating $W(\tilde{q})$ with respect to $\tilde{q}$ and using the envelope theorem yields

$$W'(\tilde{q}) = P'\left[Q^{1*}(\tilde{q})\frac{dQ^{1*}(\tilde{q})}{d\tilde{q}} + Q^{2*}(\tilde{q})\frac{dQ^{2*}(\tilde{q})}{d\tilde{q}}\right] - \left[C'\left(\frac{Q^{1*}(\tilde{q}) + \tilde{q}}{n}\right) - C'(Q^{2*}(\tilde{q}) - \tilde{q})\right]. \quad (16)$$

Recall that when $\tilde{q} = 0$, the equilibrium is identical to the traditional Cournot equilibrium.

[15] Again, this assumption is satisfied, for example, when the demand for the final good is linear and the upstream cost functions are quadratic.

Thus, by the strict concavity of $W(\tilde{q})$, *ex ante* horizontal subcontracting is welfare enhancing if and only if $W'(0)$ and $\tilde{q}^*$ have the same sign. The following proposition establishes a sufficient condition for *ex ante* horizontal subcontracting to become welfare enhancing.

Proposition 10. Assume that $W(\tilde{q})$ is strictly concave. Then, *ex ante* subcontracting is welfare enhancing if $\tilde{q}^* > 0$, i.e., if firm 1 becomes the subcontractor.

Proof. See the Appendix.

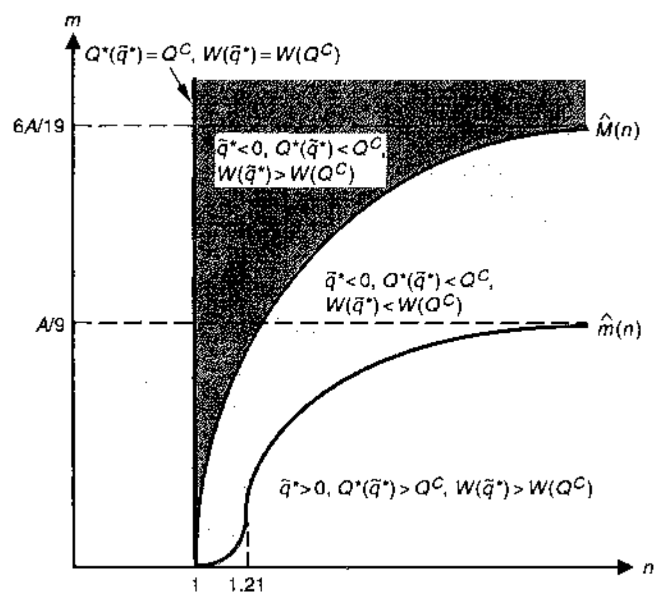
Proposition 10 gives only a partial answer to the question of whether *ex ante* horizontal subcontracting is socially desirable, since it does not say what happens when $\tilde{q}^* < 0$, and moreover, it does not specify the circumstances under which $\tilde{q}^* > 0$. To show that in general either firm can become a subcontractor and to evaluate the welfare implications of *ex ante* horizontal subcontracting when $\tilde{q}^* < 0$, let $P(Q) = A - Q$, $m^2 = m > 0 = m^1$, and $C(Q) = Q^2/n^i$, where $n^1 = n > 1 = n^2$. The full solution for this example appears in the Appendix. The main results are illustrated in Figure 3 in the (m, n) space. When $m < \hat{m}(n)$, then $\tilde{q}^* > 0$, i.e., firm 1 is the subcontractor. In this case, the industry output and consumer surplus exceed their respective Cournot levels and consequently, *ex ante* horizontal subcontracting is welfare enhancing. When $m > \hat{m}(n)$, then $\tilde{q}^* < 0$, i.e., firm 2 is the subcontractor. Now, the industry output and consumer surplus are below their respective Cournot levels, but when $m > \hat{M}(n)$, welfare still exceeds its traditional Cournot level because the industry's cost savings are sufficiently large to outweigh the loss to consumer surplus. Finally, when $\hat{m}(n) < m < \hat{M}(n)$, horizontal subcontracting is welfare decreasing.

5. The effects of horizontal subcontracting on entry

■ So far, both firms were assumed to have an access to both the upstream and the downstream processes. However, in many cases, firms sign subcontract agreements with partners who lack the technological ability to produce a final good on their own. A case in point is the recent agreement between Boeing and three of Japan's biggest industrial giants: Mitsubishi

FIGURE 3

THE WELFARE IMPLICATIONS OF *EX ANTE* HORIZONTAL SUBCONTRACTING FOR DIFFERENT COMBINATIONS OF n AND m



Heavy Industries, Kawasaki Heavy Industries LTD, and Fugi Heavy Industries. These firms expressed their interest in entering the market for commercial aircraft. According to the agreement, Boeing will subcontract to the Japanese firms the production of 15–20% of the 767-X fuselage.[16] This section suggests that by subcontracting production to a potential rival, an incumbent firm can lessen the incentive of the former to develop its own final product and enter the downstream market as a competitor.

To examine the role of horizontal subcontracting in entry deterrence, consider the following modified version of the model described in Section 2. Suppose that initially, only firm 1 has the technology to operate in the downstream market, while firm 2 is considering an entry. For simplicity, let $m^1 = 0$ and assume that by investing K, firm 2 can also develop the technology to produce a final good, in which case $m^2 = 0$. Otherwise, $m^2 = \infty$. If firm 2 invests K and enters the downstream market, the two firms engage in a Cournot quantity competition. Since at this point the firms are symmetric, each produces $Q^c/2$, where Q^c is the industry output, and each earns a profit of $\pi(Q^c)$.[17] To prevent the situation of blockaded entry, assume that $\pi(Q^c) - K > 0$. From earlier analysis, it is clear that since the firms are symmetric, they do not engage in horizontal subcontracting.

If firm 2 does not invest K and stays out of the downstream market, then firm 1 is a monopolist producing Q^m. Its revenues are $P(Q^m)Q^m$. Since C is strictly convex, the two firms can benefit from subcontracting the amount $Q^m/2$ to firm 2. Let t be the transfer payment that firm 2 receives when it produces $Q^m/2$ for firm 1. Clearly, firm 2 prefers to stay out of the downstream market and subcontract for firm 1 if and only if

$$t - C\left(\frac{Q^m}{2}\right) \geq \pi(Q^c) - K. \tag{18}$$

Now, consider firm 1, the incumbent. If it subcontracts production to firm 2, it earns $P(Q^m) - C(Q^m) - t$. If, however, it lets firm 2 enter the downstream market, its profits are $\pi(Q^c)$. Thus, firm 1 prefers to subcontract production to firm 2 if and only if

$$P(Q^m)Q^m - C\left(\frac{Q^m}{2}\right) - t \geq \pi(Q^c). \tag{19}$$

Adding conditions (18) and (19) and rearranging terms, it follows that entry can be deterred if and only if

$$P(Q^m)Q^m - 2C\left(\frac{Q^m}{2}\right) \geq 2\pi(Q^c) - K. \tag{20}$$

Since the left side of (20) represents the industry profits with perfect collusion, it exceeds the right side. This implies that horizontal subcontracting can always deter entry because it makes both the incumbent and the potential entrant better off. The analysis therefore suggests that Boeing may have used the agreement with the Japanese firms to weaken their incentives to enter the commercial aircraft market as prime contractors.

The role of horizontal subcontracting in deterring entry is similar to the role of licensing in Gallini (1984), where an incumbent firm may prefer to license its technology to an entrant rather than let the latter develop its own, potentially superior, product. The main difference, however, is that with licensing the market becomes a duopoly, whereas here the market remains a monopoly.

[16] *Chicago Tribune*, April 14, 1990, sec. 2, p. 1.

[17] Assumptions 1 through 4 ensure the existence of a unique symmetric equilibrium.

To evaluate the welfare implications of horizontal subcontracting in this case, note that if entry occurs,

$$W(Q^c) = \int_0^{Q^c} P(Q)dQ - 2C(Q^c) - K. \tag{21}$$

If, on the other hand, entry is deterred,

$$W(Q^m) = \int_0^{Q^m} P(Q)dQ - 2C\left(\frac{Q^m}{2}\right). \tag{22}$$

A comparison of (21) with (22) shows that in general, it is impossible to determine whether $W(Q^c) > W(Q^m)$ or vice versa. For example, if $P = 1 - Q$ and $C(Q) = Q^2/2$, then straightforward calculations show that $Q^m = 2/5$ and $W(Q^m) = Q^m(4 - 3Q^m)/4 = 7/25$, while $Q^c = 1/2$ and $W(Q^c) = Q^c(4 - 3Q^c)/4 - K = 5/16 - K$. Hence, horizontal subcontracting is welfare enhancing if and only if $K \geq 13/400$, i.e., entry into the downstream market is sufficiently costly.[18]

6. Conclusion

■ This article examines two types of horizontal subcontracting agreements between two actual or potential rivals: *ex ante* agreements that are signed before the firms compete in the market and *ex post* agreements that are signed after competition takes place. The driving force of either type of agreement is the convexity of upstream costs. Thus, when the marginal upstream costs are not equalized across firms, subcontracting generates a mutually beneficial surplus. Both *ex ante* and *ex post* subcontracting, however, arise and affect the market equilibrium only when firms have asymmetric cost structures. Otherwise, firms produce the same (Cournot) quantities and have no incentive to cooperate.

Horizontal subcontracting tends to increase the subcontractor's cost and lower its rival's cost. Therefore, the former contracts his output while the latter expands. The overall effect on industry output, however, is in general ambiguous. Since horizontal subcontracting enhances the efficiency of upstream production, it also enhances welfare if industry output increases. If industry output falls, however, welfare can still increase if the induced efficiency gains are sufficiently large.

Although this article relies on asymmetry in the firms' cost structures to explain horizontal subcontracting, other types of asymmetry will also give rise to this practice. For example, when the firms engage in a Stackelberg competition in the downstream market, the leader operates at a higher marginal cost level if no subcontracting occurs, so the firms will find it mutually beneficial to let the follower produce some of the leader's output. Similarly, if besides competing in a joint market each firm also sells in a separate market, horizontal subcontracting will be mutually beneficial if the firm whose output in its own separate market is low will produce some of its rival's output.

In the introduction, the analysis was motivated by several puzzling examples of horizontal subcontracting agreements. An examination of these examples in light of the formal analysis shows that they are consistent with the results of this article: Mazda is more efficient than Ford in the production of small and midsize cars.[19] The substantial network of Mazda dealerships can explain why Mazda sells its own MX-6 along with subcontracting for Ford. The agreement between Zenith and Hewlett-Packard can be explained similarly, since it

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[18] Though not too costly, since entry is blockaded whenever $K > 25/400$.

[19] For an item of evidence, see *Business Week*, March 26, 1990. The article discusses the cooperation of Ford and Mazda in designing and building a new version of the Ford Escort. It is estimated that by teaming up with Mazda, Ford was able to save at least $1 billion.

too involves two firms that have different production abilities (Zenith is more efficient in the production of laptop PCs) but similar marketing abilities. Lockheed apparently preferred to enter the market for commercial aircraft as a subcontractor because it has not produced any commercial aircraft since 1981. Its costs of assembling a complete commercial aircraft and marketing it are therefore substantially higher than those of Boeing. At the same time, it has excess production capacity that Boeing lacks. Furthermore, it is conceivable that becoming a subcontractor may have lessened Lockheed's incentives to develop its own commercial aircraft.

The analysis suggests that such agreements are an important vehicle in promoting production efficiency and, more important, are in many cases welfare enhancing.

Appendix

■ Below are proofs for Propositions 1, 3, 4, 5, and 7–10 and Lemmas 1–3, along with the solution for the linear-quadratic example of *ex ante* horizontal subcontracting with asymmetric costs.

Proof of Proposition 1. Assume by way of negation that an equilibrium exists in which $Q^{1*} > Q^{2*}$. Subtracting (7) from (6),

$$P'(Q^{1*} - Q^{2*}) + (1 - \alpha)[C'(Q^*/2) - C'(Q^{1*})] + \alpha[C'(Q^{2*}) - C'(Q^*/2)] = 0, \tag{A1}$$

where $Q^* = Q^{1*} + Q^{2*}$. Since $P' < 0$, the first term in (A1) is negative. The strict convexity of C together with $Q^{1*} > Q^{2*}$ imply that the other two terms in (A1) are also negative, a contradiction. Thus $Q^{1*} = Q^{2*}$. This implies that $q^* = Q^{1*} - Q^*/2 = 0$, so in equilibrium no subcontracting occurs. In this case, equations (6) and (7) reduce to the first-order conditions for a Cournot equilibrium. *Q.E.D.*

Proof of Lemma 1. Subtracting (7) from (6) yields

$$P'(Q^{1*} - Q^{2*}) + (1 - \alpha)[C'(Q^*/(n + 1)) - C'(Q^{1*}/n)] + \alpha[C'(Q^{2*}) - C'(Q^*/(n + 1))] = 0. \tag{A2}$$

First, assume by way of negation that $Q^{1*} \le Q^{2*}$. Since $P' < 0$, the first term in equation (A2) is nonnegative. The second and third terms are strictly positive, since $Q^{2*} > Q^*/(n + 1) > Q^{1*}/n$. Hence, the left side of (A2) is strictly positive, a contradiction.

To prove the second part of the lemma, note that since $Q^{1*} > Q^{2*}$, the first term in (A2) is strictly negative, implying that the sum of the second and third terms is strictly positive. This implies, in turn, that $Q^{1*}/n < Q^{2*}$. *Q.E.D.*

Proof of Lemma 2. Differentiating equations (6) and (7) totally yields

$$R_2^1 = \frac{P' + P''Q^{1*} - \dfrac{\alpha}{n+1} C''\left(\dfrac{Q^*}{n+1}\right)}{2P' + P''Q^{1*} - \dfrac{1-\alpha}{n} C''\left(\dfrac{Q^{1*}}{n}\right) - \dfrac{\alpha}{n+1} C''\left(\dfrac{Q^*}{n+1}\right)}$$

and

$$R_1^2 = \frac{P' + P''Q^{2*} - \dfrac{1-\alpha}{n+1} C''\left(\dfrac{Q^*}{n+1}\right)}{2P' + P''Q^{2*} - \alpha C''(Q^{2*}) - \dfrac{1-\alpha}{n+1} C''\left(\dfrac{Q^*}{n+1}\right)}.$$

From Assumptions 1 through 3 it follows that $0 > R_j^i > -1$, for $i, j = 1, 2$.

As for the second part of the lemma, fixing Q^2 and differentiating equation (6) totally yields

$$R_\alpha^1 = \frac{C'\left(\dfrac{Q^*}{n+1}\right) - C'\left(\dfrac{Q^{1*}}{n}\right)}{2P' + P''Q^{1*} - \dfrac{1-\alpha}{n} C''\left(\dfrac{Q^{1*}}{n}\right) - \dfrac{\alpha}{n+1} C''\left(\dfrac{Q^*}{n+1}\right)}.$$

Similarly, fixing Q^1 and differentiating equation (7) totally yields

$$R_\alpha^2 = \frac{C'(Q^{2*}) - C'\left(\frac{Q^*}{n+1}\right)}{2P' + P''Q^{2*} - \alpha C''(Q^{2*}) - \frac{1-\alpha}{n+1} C''\left(\frac{Q^*}{n+1}\right)}.$$

By Assumptions 1 through 3, the denominators of both expressions are strictly negative. Therefore, since Lemma 1 and the convexity of C imply that $C'(Q^{1*}/n) < C'(Q^*/(n+1)) < C'(Q^{2*})$, it follows that $R_\alpha^1 < 0$ and $R_\alpha^2 < 0$. *Q.E.D.*

Proof of Proposition 3. In the traditional Cournot model, the equilibrium quantities are determined by the following two first-order conditions:

$$\pi_1^1(Q^1, Q^2) = P - m^1 + P'Q^1 - C'(Q^1/n) = 0, \tag{A3}$$

$$\pi_2^2(Q^1, Q^2) = P - m^2 + P'Q^2 - C'(Q^2) = 0. \tag{A4}$$

Denote the (unique) solution to these equations by (Q^{1c}, Q^{2c}). Equations (A3) and (A4) define the Cournot reaction functions of firm 1, $R^{1c}(Q^2)$, and firm 2, $R^{2c}(Q^1)$, respectively. Note that equation (A3) is identical to equation (6) when $\alpha = 0$, and equation (A4) is identical to equation (7) when $\alpha = 1$. Hence, $R^{1c}(Q^2) = R^1(Q^2, 0)$ and $R^{2c}(Q^1) = R^2(Q^1, 1)$. Since (Q^{1c}, Q^{2c}) is the intersection point of $R^{1c}(Q^2)$ and $R^{2c}(Q^1)$, then $Q^{1c} \equiv R^{1c}(Q^{2c}) = R^1(R^2(Q^1, 1), 0)$, and $Q^{2c} \equiv R^{2c}(Q^{1c}) = R^2(R^1(Q^2, 0), 1)$. In the presence of *ex post* horizontal subcontracting, the equilibrium is given by $Q^{1*} \equiv R^1(R^2(Q^1, \alpha), \alpha)$ and $Q^{2*} \equiv R^2(R^1(Q^2, \alpha), \alpha)$. By Lemma 2, $R^i(Q^j, \alpha)$ decreases in both arguments. Thus, the following chain of inequalities can be established:

$$Q^{1c} \equiv R^1(R^2(Q^1, 1), 0) > R^1(R^2(Q^1, 1), \alpha) > R^1(R^2(Q^1, \alpha), \alpha) \equiv Q^{1*}.$$

Similarly,

$$Q^{2c} \equiv R^2(R^1(Q^2, 0), 1) < R^2(R^1(Q^2, 0), \alpha) < R^2(R^1(Q^2, \alpha), \alpha) \equiv Q^{2*}.$$

As for the industry output, differentiating the system of first-order equations (equations (6) and (7)) with respect to α and using the Crémer rule,

$$\frac{dQ^{1*}}{d\alpha} = \frac{\pi_{22}^2\left[C'\left(\frac{Q^*}{n+1}\right) - C'\left(\frac{Q^{1*}}{n}\right)\right] - \pi_{12}^1\left[C'(Q^{2*}) - C'\left(\frac{Q^*}{n+1}\right)\right]}{H}$$

and

$$\frac{dQ^{2*}}{d\alpha} = \frac{\pi_{11}^1\left[C'(Q^{2*}) - C'\left(\frac{Q^*}{n+1}\right)\right] - \pi_{21}^2\left[C'\left(\frac{Q^*}{n+1}\right) - C'\left(\frac{Q^{1*}}{n}\right)\right]}{H},$$

where $H \equiv \pi_{11}^1\pi_{22}^2 - \pi_{12}^1\pi_{21}^2$. Adding the two expressions and rearranging terms,

$$\frac{dQ^*}{d\alpha} = \frac{1}{H}\left\{\frac{1-\alpha}{n} C''\left(\frac{Q^{1*}}{n}\right)\left[C'\left(\frac{Q^*}{n+1}\right) - C'(Q^{2*})\right]\right.$$
$$\left. - \alpha C''(Q^{2*})\left[C'\left(\frac{Q^*}{n+1}\right) - C'\left(\frac{Q^{1*}}{n}\right)\right] - P'\left[C'(Q^{2*}) - C'\left(\frac{Q^{1*}}{n}\right)\right]\right\}.$$

Using Assumptions 1 through 3, it is straightforward to verify that $H > 0$. Since Lemma 1 and the convexity of C imply that $C'(Q^{1*}/n) < C'(Q^*/(n+1)) < C'(Q^{2*})$, it follows that $dQ^*/d\alpha < 0$.

Now, by the first part of the proposition, $Q^1 = Q^{1c}$ and $Q^2 > Q^{2c}$ if $\alpha = 0$, and $Q^1 < Q^{1c}$ and $Q^2 = Q^{2c}$ if $\alpha = 1$. Thus, $Q^* > Q^c$ if $\alpha = 0$ and the converse if $\alpha = 1$. Since Q^* is continuously decreasing in α, this implies that there exists a unique $\bar{\alpha} \in [0, 1]$ such that $Q^* \gtreqless Q^c$ as $\alpha \lesseqgtr \bar{\alpha}$. *Q.E.D.*

Proof of Proposition 4. First, note that *ex post* horizontal subcontracting enhances the efficiency of upstream production. Second, note that $W(Q)$ is strictly concave and denote its (unique) maximum by Q^{fb}. It can easily be shown that $Q^* < Q^{fb}$, i.e., *ex post* subcontracting never leads to overproduction of the final good. Hence, welfare is unambiguously increased if $Q^* \geq Q^c$. As Proposition 3 shows, this occurs for $\alpha \leq \bar{\alpha}$. Otherwise, however, $Q^* < Q^c$, so although upstream production becomes more efficient in the presence of *ex post* horizontal subcontracting, industry's output and hence consumer surplus become smaller. Hence, when $\alpha > \bar{\alpha}$, the welfare consequences of *ex post* horizontal subcontracting are in general ambiguous.

To demonstrate, consider the following example: $P(Q) = 1 - Q$, $C(Q) = \gamma Q^2/n^i$, where $n^1 = n > 1 = n^2$, and $m^1 = m^2 = 0$. A straightforward calculation shows that $Q^{1c} = (1 + 2\gamma)/H^c$, $Q^{2c} = (1 + 2\gamma/n)/H^c$, where $H^c \equiv 4(1 + \gamma/n)(1 + \gamma) - 1$. Substituting into the social welfare function,

$$W(Q^{1c}, Q^{2c}) = \frac{2(n + \gamma(n+1))(2n + 4\gamma^2 + 3\gamma(n+1)) - \gamma n(1 + 2\gamma)^2 - \gamma(n + 2\gamma)^2}{(nH^c)^2}. \tag{A5}$$

Now suppose that the firms subcontract production *ex post* and let $\alpha = 1$. Then, $Q^{1*} = (1 + 2n\gamma/(n+1))/H^*$ and $Q^{2*} = (1 + 2\gamma/(n+1))/H^*$, where $H^* \equiv 4(1 + \gamma)(1 + \gamma/(n+1)) - (1 + 2\gamma/(n+1))$. Substituting into the social welfare function,

$$W(Q^{1*}, Q^{2*}) = \frac{2(1+\gamma)\left(2 + 3\gamma + \dfrac{2\gamma^2}{n+1}\right)}{(H^*)^2}. \tag{A6}$$

A comparison of (A5) with (A6) reveals that

(i) When $\gamma = 1$ and $n = 2$ (i.e., firm 1 is twice as large as firm 2),

$$W(Q^{1c}, Q^{2c}) = 68/242 > W(Q^{1*}, Q^{2*}) = 68/243.$$

(ii) When $\gamma \to 0$, upstream production becomes costless, so the firms cannot gain from *ex post* subcontracting. In this case, $W(Q^{1c}, Q^{2c}) = W(Q^{1*}, Q^{2*}) = 4/9$.

(iii) When $\gamma = 1$ and $n \to \infty$, firm 1's upstream production becomes costless, while firm 2's does not. Hence, *ex post* subcontracting yields a substantial upstream cost saving. In this case,

$$W(Q^{1c}, Q^{2c}) = 19/49 < W(Q^{1*}, Q^{2*}) = 20/49. \qquad \textit{Q.E.D.}$$

Proof of Lemma 3. The proof of the first part of the lemma is similar to the proof of Lemma 2. To prove the second part, fixing Q^2 and differentiating equation (6) totally yields

$$R^1_\alpha = \frac{C'\left(\dfrac{Q^*}{2}\right) - C'(Q^{1*})}{2P' + P''Q^{1*} - (1-\alpha)C''(Q^{1*}) - \dfrac{\alpha}{2}C''\left(\dfrac{Q^*}{2}\right)}.$$

Similarly, fixing Q^1 and differentiating equation (7) totally yields

$$R^2_\alpha = \frac{C'(Q^{2*}) - C'\left(\dfrac{Q^*}{2}\right)}{2P' + P''Q^{2*} - \alpha C''(Q^{2*}) - \dfrac{1-\alpha}{2}C''\left(\dfrac{Q^*}{2}\right)}.$$

The proof is completed by observing that Assumptions 1 through 3 ensure that the denominators of both expressions are strictly negative, and that the convexity of C and the fact that $Q^{1*} > Q^* > Q^{2*}$ imply that $C'(Q^{1*}) > C'(Q^*/2) > C'(Q^{2*})$. *Q.E.D.*

Proof of Proposition 5. Again, notice that *ex post* subcontracting enhances the upstream production efficiency and never leads to overproduction. Hence, welfare increases unambiguously when $\alpha \geq \bar{\bar{\alpha}}$, because then $Q^* \geq Q^c$. Otherwise, however, $Q^* < Q^c$, so the overall effect of *ex post* subcontracting is ambiguous.

To demonstrate, consider the following example: $P(Q) = 1 - Q$, $C(Q) = Q^2$, $m^1 = 0$, and $m^2 = m > 0$. A straightforward calculation shows that $Q^{1c} = (3 + m)/15$ and $Q^{2c} = (3 - 4m)/15$. Substituting into the social welfare function,

$$W(Q^{1c}, Q^{2c}) = (108 - 108m + 77m^2)/450. \tag{A7}$$

Now suppose that the firms subcontract production *ex post* and let $\alpha = 0$. Then, $Q^{1*} = (2 + m)/10$ and $Q^{2*} = (1 - 2m)/10$. Substituting into the social welfare function yields

$$W(Q^{1*}, Q^{2*}) = (24 - 26m + 31m^2)/100. \tag{A8}$$

A comparison of (A7) with (A8) reveals that $W(Q^{1*}, Q^{2*}) \gtreqless W(Q^{1c}, Q^{2c})$ as $m \gtreqless 18/125$, so *ex post* horizontal subcontracting is welfare improving when firm 2's marketing costs are sufficiently high. *Q.E.D.*

Proof of Proposition 7. Since $m^1 = m^2$, equations (11) and (12) evaluated at $\tilde{q}^*$ both hold in equality (see footnote 14). Thus, one can substitute from (13) and (14) into the first line in equation (15),

$$\Pi'(\tilde{q}^*) = -\frac{P'}{\tilde{H}}\left[\psi(Q^{1*}(\tilde{q}^*) - Q^{2*}(\tilde{q}^*)) + P'\left(\frac{Q^{2*}(\tilde{q}^*)}{n}C''\left(\frac{Q^{1*}(\tilde{q}^*)+\tilde{q}^*}{n}\right) - Q^{1*}(\tilde{q}^*)C''(Q^{2*}(\tilde{q}^*) - \tilde{q}^*)\right)\right]$$
$$-\left[C'\left(\frac{Q^{1*}(\tilde{q}^*)+\tilde{q}^*}{n}\right) - C'(Q^{2*}(\tilde{q}^*) - \tilde{q}^*)\right] = 0. \quad \text{(A9)}$$

Now, assume that $\tilde{q}^* = 0$. In this case, it follows from equations (11) and (12) that the (unique) stage-2 equilibrium is symmetric, i.e., $Q^{1*}(0) = Q^{2*}(0)$. Since $n = 1$, $\Pi'(0) = 0$, so the strict concavity of $\Pi(\tilde{q})$ implies that $\tilde{q}^* = 0$ is the unique optimum. The proof is completed by observing that when $\tilde{q}^* = 0$, equations (11) and (12) become identical to the first-order conditions for a Cournot equilibrium. *Q.E.D.*

Proof of Proposition 8. Assume that firm 1 is a monopolist in the downstream market, (i.e., $Q^{2*} = 0$), and let Q^* and Q^c be the industry outputs under *ex post* subcontracting and in the Cournot case, respectively. Then, Q^* solves equation (6) and Q^c solves equation (6) when $\alpha = 0$. The first part of the proposition follows by observing that the convexity of C implies $C'(Q/n) \geq (1-\alpha)C'(Q/n) + \alpha C'(Q/(n+1)) \geq C'(Q/(n+1))$ (with strict inequalities for $\alpha \in (0, 1)$), and also that Assumption 2 ensures $P + P'Q$ is decreasing in Q.

As for the second part of the proof, let $Q^2 = 0$ and let (12) and (7) hold with equality. Then, $\bar{m}^2(\tilde{q}^*) = P(\hat{Q}^*) - C'(\hat{Q}^*/(n+1))$, $\bar{m}^2 = P(Q^*)) - \alpha C'(0) - (1-\alpha)C'(Q^*/(n+1))$ and $\bar{m}^c = P(Q^c) - C'(0)$ (the latter definition is obtained by setting $\alpha = 1$ in (7)). Using the fact that $\hat{Q}^* \geq Q^* \geq Q^c$, and the convexity of C, the result follows. *Q.E.D.*

Proof of Proposition 9. Assume by way of negation that $\tilde{q}^* \neq 0$ and that industry output is produced efficiently, so that $C'((Q^{1*}(\tilde{q}^*) + \tilde{q}^*)/n) = C'(Q^{2*}(\tilde{q}^*) - \tilde{q}^*)$. Using this equality, equations (11) and (12) imply that for all $n \geq 1$, $Q^{1*}(\tilde{q}^*) = Q^{2*}(\tilde{q}^*)$ if $m^1 = m^2$, and $Q^{1*}(\tilde{q}^*) > Q^{2*}(\tilde{q}^*)$ if $m^1 < m^2$. Thus, when $n > 1$ and $m^1 = m^2$, (A9) implies

$$\Pi'(\tilde{q}^*) = \frac{(P')^2 Q^*(\tilde{q}^*)}{2\tilde{H}}\left[\frac{1}{n}C''\left(\frac{Q^{1*}(\tilde{q}^*)+\tilde{q}^*}{n}\right) - C''(Q^{2*}(\tilde{q}^*) - \tilde{q}^*)\right] < 0,$$

a contradiction to the optimality of $\tilde{q}^*$.

Similarly, when $m^1 < m^2$ and $n = 1$, it follows from (A9) that

$$\Pi'(\tilde{q}^*) = -\frac{P'\psi}{\tilde{H}}(Q^{1*}(\tilde{q}^*) - Q^{2*}(\tilde{q}^*)) < 0,$$

again, a contradiction to the optimality of $\tilde{q}^*$. *Q.E.D.*

Proof of Proposition 10. Assume that $\tilde{q}^* > 0$. Then, since $W(\tilde{q})$ is strictly concave, it is sufficient to show that

$$W'(0) = P'\left[Q^{1*}(0)\frac{dQ^{1*}(0)}{d\tilde{q}} + Q^{2*}(0)\frac{dQ^{2*}(0)}{d\tilde{q}}\right] - \left[C'\left(\frac{Q^{1*}(0)}{n}\right) - C'(Q^{2*}(0))\right] > 0.$$

But since $\Pi(\tilde{q})$ is strictly concave, $\tilde{q}^* > 0$ implies that

$$\Pi'(0) = P'\left[Q^{1*}(0)\frac{dQ^{2*}(0)}{d\tilde{q}} + Q^{2*}(0)\frac{dQ^{1*}(0)}{d\tilde{q}}\right] - \left[C'\left(\frac{Q^{1*}(0)}{n}\right) - C'(Q^{2*}(0))\right] > 0.$$

Using this expression,

$$W'(0) > P'(Q^{1*}(0) - Q^{2*}(0))\left[\frac{dQ^{1*}(0)}{d\tilde{q}} - \frac{dQ^{2*}(0)}{d\tilde{q}}\right].$$

From equations (11) and (12) it is easy to verify that $Q^{1*}(0) > Q^{2*}(0)$ when either $n > 1$ or $m^2 > m^1$. The proof is thus completed by observing from (13) and (14) that $dQ^{1*}(0)/d\tilde{q} < 0 < dQ^{2*}(0)/d\tilde{q}$, so that $W'(0) > 0$. *Q.E.D.*

The solution for the linear-quadratic example of ex ante *horizontal subcontracting with asymmetric costs.* Assume that $P(Q) = A - Q$, $m^2 = m > 0 = m^1$, and $C(Q) = Q^2/n^i$, where $n^1 = n > 1 = n^2$. Then, from equations (11) and (12) it follows that

$$Q^{1*}(\tilde{q}) = \frac{3An + mn - 2\tilde{q}(4+n)}{8+7n}; \qquad Q^{2*}(\tilde{q}) = \frac{A(2+n) - 2m(n+1) + 2\tilde{q}(3+2n)}{8+7n}. \quad \text{(A10)}$$

The industry output is therefore given by

$$Q^*(\tilde{q}) = Q^{1*}(\tilde{q}) + Q^{2*}(\tilde{q}) = \frac{2A(1+2n) - m(2+n) + 2\tilde{q}(n-1)}{8+7n}. \quad \text{(A11)}$$

Clearly, when firms have the same upstream cost functions, i.e., $n = 1$, $Q^*(\tilde{q})$ is independent of $\tilde{q}$, in which case, $Q^*(\tilde{q}) = Q^c$. When $n > 1$, $dQ^*(\tilde{q})/d\tilde{q} > 0$ for all $\tilde{q}$, so $Q^*(\tilde{q}) > Q^c$ if and only if $\tilde{q}^* > 0$. To determine the sign of $\tilde{q}^*$, substitute from (A10) into the first line in equation (15) and rearrange terms to obtain

$$\tilde{q}^* = \frac{2An(n-1) - 2m(9n^2 + 25n + 16)}{13n^2 + 29n + 8}.$$

Using this expression, define

$$\hat{m}(n) \equiv \frac{An(n-1)}{9n^2 + 25n + 16}$$

as the critical value of m below which $\tilde{q}^* > 0$, i.e., firm 1 is the subcontractor. Note that firm 1 is the subcontractor when n is relatively large (firm 1's advantage in upstream production is large) and m is relatively small (firm 2's advantage in marketing is small). When $m = \hat{m}(n)$, $\tilde{q}^* = 0$, so no *ex ante* subcontracting takes place.

In this example, the welfare function is given by

$$W(\tilde{q}) = AQ^*(\tilde{q}) - \frac{Q^*(\tilde{q})^2}{2} - mQ^{2*}(\tilde{q}) - \frac{(Q^{1*}(\tilde{q}) + \tilde{q})^2}{n} - (Q^{2*}(\tilde{q}) - \tilde{q})^2.$$

One can verify that $W(\tilde{q})$ is strictly concave. Thus, Proposition 10 applies, so $W(\tilde{q}^*) > W(Q^c)$ if $\tilde{q}^* > 0$, i.e., if $m < \hat{m}(n)$. Otherwise, $\tilde{q}^* < 0$, so as (A11) shows, the industry output and hence consumer surplus are below their respective Cournot levels. To compare $W(\tilde{q}^*)$ with $W(Q^c)$ in this case, recall that $W(0) = W(Q^c)$. Since $W(\tilde{q})$ is strictly concave and $\tilde{q}^* < 0$, this implies that $W(\tilde{q}^*) > W(Q^c)$ if and only if $W'(0) < 0$. Differentiating $W(\tilde{q})$ and evaluating at $\tilde{q} = 0$ yields

$$W'(0) = \frac{2[2A(3n^2 - 2n - 1) - m(19n^2 + 51n + 30)]}{8 + 7n}.$$

Using this expression, define

$$\hat{M}(n) \equiv \frac{2A(3n^2 - 2n - 1)}{19n^2 + 51n + 30}$$

as the critical value of m above which $W'(0) < 0$. Note that for all $n > 1$, $\hat{M}(n) > \hat{m}(n)$. Hence, whenever $\tilde{q}^* < 0$, *ex ante* horizontal subcontracting is welfare enhancing if $m > \hat{M}(n)$, but it is welfare decreasing if $\hat{M}(n) > m > \hat{m}(n)$. *Q.E.D.*

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