Zipf’s Law: The Case of Institutional Real Estate

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Abstract

This study uses data on the spatial distribution of institutional real estate investments both inside the U.S. and internationally over the period 2005 to 2012 to examine the applicability of Zipf’s Law. A key finding is that there is a distinct log-log rank-size relationship between size of institutional property investments and rank inside the US. This finding holds for both private institutions and publicly-traded REITs. It is also shown that there is a specific relationship between size of institutional property investments and rank globally, and that the stratum of observations globally is more concentrated at the upper tail than predicted by Zipf’s Law.

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1 Introduction

This paper is related to a large literature that has sought to test Zipf’s Law. Most of the existing papers have used growth patterns of cities to test Zipf’s Law (see Zipf (1949), Krugman (1996), and Gabaix (1999)). Zipf’s Law has also been tested using the relative size of business firms (see Ijiri and Simon (1977)) and the relative frequency of English word usage (see Irmay (1997)). Interestingly enough, Zipf’s Law has been successfully applied to each case, usually producing a good estimation fit. It is the purpose of this paper to test the proposition that Zipf’s Law applies to the relative size of institutional real estate investments (with a focus inside the U.S. and internationally). We are the first to conduct an empirical analysis of Zipf’s Law applied to institutional real estate investments.

Zipf’s Law is an empirical regularity that manifests itself in the form of an inverse relation between size and rank of discrete phenomena. It is a variant of Pareto’s Law, which tells us that elements in a distribution can be broken down into the trivial many, and the vital few. As such, Zipf’s Law is of interest in itself. Specifically, it is of direct interest to investigate when and where Zipf’s law holds, and whether the functional relationship between size and rank is a power-law function or a linear function. However, it is also of interest to connect Zipf’s Law to the relative size of institutional real estate investments both inside the U.S. and internationally and to use it to provide a useful way of thinking about the flow of funds into real estate.

We follow the approach of Rosen and Resnick (1980) and base our analysis on two regressions: a linear and nonlinear specification. The explanatory variables are identical in both models. Such tests are particularly appropriate when there are sample selection issues. Our U.S. data come from the following two websites: SNL Real Estate, http://www.snl.com, and the National Council of Real Estate Investment Fiduciaries, http://www.ncreif.org. The former has data on individual real estate property holdings for a large set of public and private Real Estate Investment Trusts (REITs). The latter has data on individual real estate property
holdings for investment managers and plan sponsors. The U.S. data are aggregated to the MSA or micropolitan statistical area as the case may be. The sample period is from 2005 to 2012. Our international data set covers twenty-two countries with at least seven years of annual data on the size of the institutional real estate market (denominated in U.S. dollars) within each country from 2005 to 2011.¹ These data come from International Property Database, http://www1.ipd.com. We take advantage of the inter-metropolitan (both MSA and micropolitan statistical area as the case may be) and inter-country variation in institutional real estate investments to generate a distribution of rankings of markets. We obtain an average slope of -0.999 and an $R^2$ of 0.86 for private institutional property investments in the U.S. For investments made by publicly-traded REITs in the US, we obtain an average slope of -1.192 and an $R^2$ of 0.89. We cannot statistically distinguish these slope coefficients from that of minus one at the 0.05 level.² In contrast, for global property investments we obtain an average slope of -0.610 and an $R^2$ of 0.85. This negative slope coefficient is statistically different from that of minus one at the 0.05 level. Hence, we reject Zipf’s Law in favor of a Pareto-like view of the world.

We explain these results with a random growth model in which productivity-enhancing and/or quality-of-life-enhancing attributes induce firms, at least initially, to set up in a city, employing workers and hiring physical capital. We follow Rossi-Hansberg and Wright (2007) and posit that agglomeration effects may result (treated here as positive productivity shocks), as a high inflow of both labor and capital induces high capital returns and high wages and mutually reinforcing inflows of both capital and labor. The model helps to explain not only why the spatial distribution of institutional real estate investments inside the U.S. follows a Zipf’s Law-like distribution but also why the spatial distribution of institutional real estate investments.

¹The lack of data makes it impossible to construct institutional real estate holdings at the MSA-level outside of the U.S. Instead, we use country-level data to analyze the behavior of institutional investors outside the U.S. Fortunately, this level of aggregation is not necessarily a problem. Existing findings suggest that the well-known characteristics of city-size distributions work about as well when one considers countries instead of cities (see, e.g., Rose (2006)).

²These findings are related to Coval and Moskowitz’s (1999) finding that institutions prefer to concentrate their stock market holdings more locally, which is evidence that location matters and that, for example, America and Japanese’s dominance of the institutional real estate investment market comes from size as well. Additionally, with a home bias in investment demand and a preference for thick markets if, and when, home markets become satiated, we are not surprised to find that Zipf’s Law applies to the spatial distribution of institutional real estate investment at the national level at the same time.
investment internationally is more Pareto-like.

Several implications follow from the present research. The first implication concerns how to define what institutional real estate investments are. Here, it is important to keep in mind the functional relationship between size and rank, and why this relationship exists. For instance, instead of defining institutional real estate investments as assets that exhibit “institutional qualities,” as in assets that are “well located within their local and regional markets and of high quality design and construction,” one should speak of investing in “desirable” markets where there are self-reinforcing agglomeration effects. The second implication centers in on diversification by geography, and whether institutional investors should diversify their real estate portfolios internationally. Here, instead of speaking of domestic versus international investments and of many different types of economies (e.g., developed versus developing, developing versus emerging, etc.), one should again speak of highly desirable markets where both domestic and international firms wish to locate. For example, if we look beneath the surface, investments in domestic and international markets are generally made in markets with similar growth features. To see just how highly concentrated institutional investment is in certain markets, consider Figure 1, which displays data on cross-border real estate investments during the last five years. These data come from Real Capital Analytics, https://www.rcanalytics.com. Markets are ordered by the size of their foreign direct investment, with larger circles indicating higher inflows. Highly desirable markets in Advanced Economies (AEs), such as London, Paris, New York City, Tokyo, Berlin, Sydney, and Frankfort, tend to have high inflows. Similarly, highly desirable markets in Emerging Market and Developing Economies (EMDEs), such as Shanghai and Singapore, also have high inflows. Third, the very specific and consistent relationship between size and rank tells us something about core, value-add, and opportunistic investment real estate strategies. Ten to fifteen years ago, institutional real estate investment was largely a matter of investing in core assets; that is, investing in assets having low leverage and generally low risk/return profiles. Since then, institutional investors have moved up the risk spectrum into value-add and
opportunistic investments in search of high returns. However, there is no question about it, the Zipf distribution has not been altered by this shift toward value-add and opportunistic investments. In fact, throughout our sample period the Zipf coefficient estimates are quite stable over time, something we would not expect if investors targeting value-add and opportunistic projects were investing exclusively in secondary locations in search of high returns. Rather, the results here are consistent with a hierarchical structure, in which investors first choose to invest mostly in desirable markets, and then, for effective risk management, divide their investments into core, value-add, and opportunistic.

The remainder of the paper is structured as follows. Section 2 describes Zipf’s Law and discusses what happens when the rank-size distribution is more or less dispersed than that predicted by Zipf’s Law. Section 3 performs a test of Zipf’s Law using data on real estate property holdings for private institutions and publicly-traded REITs inside the US. Section 4 performs a test of Zipf’s Law using data for institutional property investments globally. Section 5 provides a discussion of the results obtained and of possible impacts. Section 6 concludes.

2 Zipf’s Law

Zipf’s Law is a law about the size of an occurrence of an event relative to its rank. The log-log depiction of Zipf’s Law is

$$\ln(R_j) = C - \alpha \ln(S_j) + \epsilon_j \quad j = 1, 2, \ldots, N$$

(1)

where $R_j$ refers to the rank of an observation, $S_j$ is its size (where $S_1$, the largest value, is given rank 1, $S_2$, the second largest value, is given rank 2, and so on), $C > 0$ is a normalization constant, $\epsilon_j$ is an error term, $N$ denotes the total number of observations, and $\alpha > 0$. The special case of Zipf’s Law arises when the value of $\alpha = 1$, that is, when the size of the largest observation is precisely $k$ times as large as the $k$th largest observation. In this case, the graph of equation (1) is a straight line with slope -1 on a log-log plot.
The more general rank-size distribution arises when the value of \( \alpha \) is either smaller or greater than 1. A value of \( \alpha < 1 \) indicates that the stratum of observations is more concentrated at the upper tail and the slope becomes flatter as a reflection of the huge dichotomy that exists between the most and least common observation. In contrast, a value of \( \alpha > 1 \) indicates that the size distribution is more dispersed and there is a small decline in the size of observations with their ranks.

The rank-size distribution in equation (1) has been explained in terms of the Pareto distribution. Adamic (2003), Gabaix (1999), among others show that the rank-size distribution in equation (1) is related to the probability density function

\[ \Pr(S > S_j) - S_j^{-\alpha} \]  

(2)

where \( \Pr(S > S_j) \) is the cumulative distribution function (CDF) of \( S \). Equation (2) implies that the number of observations larger than \( S_j \) is an inverse power of \( S_j \). The latter means

\[ \Pr(S = S_j) - S_j^{-(1+\alpha)} = S_j^{-\alpha} \]  

(3)

where \( \Pr(S = S_j) \) is the probability distribution (PDF) function associated with the cumulative distribution function given in equation (2) and \( \alpha = 1 + \alpha \). In equation (2), the frequency of objects and their sizes obey a Pareto distribution, while in equation (3) there is a power-law PDF for \( S_j \), which implies that the process generating a Zipf size-rank distribution must have a strictly power-law probability density function with \( \alpha = 2 \).

Most of the work on Zipf’s Law and rank-size distributions in economics has dealt with city size distributions. The classical examples here are Zipf (1949), Krugman (1996), and Gabaix (1999). What Zipf argues is that the distribution of city size in any country is determined by two kinds of forces: forces of diversification and forces of unification. The former makes for a larger number of smaller cities, while the latter makes for a smaller number of large cities. Zipf computes the value of \( \alpha \) using the one hundred largest metropolitan districts in the U.S. in 1940 as reported by the Sixteenth Census. Zipf’s estimate of equation (1) yields a slope of -0.98. Others like Krugman (1996), and Gabaix (1999) have
repeated these calculations in subsequent research and have found similar results both in level and in pattern.³ This similarity between Zipf’s results and those of later investigators led Krugman (1996) to conclude that the rank-size rule for cities is a very fundamental and robust outcome.

Rosen and Resnick (1980) use the same methods to estimate a rank-size relationship for OECD countries. Rosen and Resnick find that Zipf’s Law reasonably holds for a variety of these countries. Their slope estimates range from -0.81 in Morocco to -1.96 in Australia, with a sample mean of -1.14. Rozman (1990) report a slope coefficient around –1.00 for China. Brakman et al. (1999) report a slope coefficient of -0.55 for the Netherlands in 1600, –1.03 for the Netherlands in 1900, and –0.72 for the Netherlands in 1990. These estimates indicate that Dutch cities for the years 1600 and 1990 were more even-sized than for the year 1900.

In a more recent article, Soo (2005) estimates Zipf’s Law for over 100 countries for the year 1972 through 2001. Soo (2004) finds that 14 of his 73 countries had a slope coefficient significantly greater than -1, of which thirteen are in Africa, North or South America, or Asia and one is in Europe. Thirty-nine countries had a slope coefficient significantly less than -1, of which twenty-one are in Europe and the rest are primarily in North or South America, or Asia. Eeckhout (2004) finds a slope coefficient of -0.999 for the U.S. for the year 2000 (using a truncated sample of observations that excludes the bottom half of the MSAs). Rossi-Hansberg and Wright (2007) find that some countries have a rank-size distribution that is more or less dispersed than that predicted by Zipf’s Law, which is reflected in flatter or steeper plots of log-rank against log-size. Rossi-Hansberg and Wright (2007) also find that there is a tendency for an approximately concave relationship, reflecting the absence of very large cities relative to Zipf’s Law. Rose (2006) finds that Zipf’s Law works well for countries as well. Rose (2006) investigates the size distribution of countries’ population from 1900 through 2004. His results suggest that the relationship between country rank and size is close to -1 and linear, with the exception of the year 1900, which has a slope coefficient of -0.78.


3 Investment-Size Distributions Inside the U.S.

The standard data sources on institutional real estate investments inside the U.S. are SNL Real Estate (SNL) for public institutions and the National Council of Real Estate Investment Fiduciaries (NCREIF) for private institutions. Both are relatively recent. SNL launched its real estate coverage in 1994, but did not start providing real estate investments information until 1999 (when it acquired ASSETrac Information Services, a provider of financial information on the REIT industry). NCREIF launched its real estate coverage in 1978 for a sample group of private and public pension sponsors (holding retirement assets of private and public employees), life insurance companies (investing separate and commingled employee benefit plan trusts individual agency accounts), and independent real estate investment managers (not affiliated with life insurance companies) who provide customized investments for pension plans.

We use these two databases to construct a total size distribution of real estate investments for the U.S. from 2005 to 2012. We measure investment size in two ways. These choices were dictated by data availability. For publicly-traded REITs, investment size is measured in terms of square feet. For private institutions, investment size is measured in terms of market value. One cannot overlook the fact that publicly-traded REITs in general are larger than private institutions. The amount of square feet of real estate space held by publicly-traded...
REITs is over 2.9 million square feet. In contrast, the amount of real estate assets held by private institutions totals just about $375 billion, or around 1.5 million square feet, which is about one-half the size of publicly-traded REITs.4

There are 226 MSAs represented in the NCREIF data set and 362 MSAs in the SNL data set. The SNL data set also includes a detailed accounting of investments disaggregated according to 527 micropolitan statistical areas (urban clusters with a population of at least 10,000 but no more than 49,999), allowing us to study not only the presence of publicly-traded REIT investment in MSAs, but in micro areas as well. The individual investments made by both private and public institutions are aggregated by MSA and micropolitan statistical area (metro and micro areas) as the case may be for each year from 2005. Table 1 summarizes the data for the year 2012. The data show that private institutions and publicly-traded REITs generally invest in the same markets. For private institutions, the largest market is the Washington-Arlington-Alexandria, DC-VA-MD-WV area. The second largest market is the Los Angeles-Long Beach-Santa Ana, CA area. The third largest market is the New York-Northern New Jersey-Long Island, NY-NJ-PA area. The fourth largest market is Chicago-Joliet-Naperville, IL-IN-WI area.

For publicly-traded REITs, the top four largest markets are (the rank among institutional real estate markets is in parentheses): New York-Northern New Jersey-Long Island, NY-NJ-PA (3), Washington-Arlington-Alexandria, DC-VA-MD-WV (1), Los Angeles-Long Beach-Santa Ana, CA (2), and Chicago-Joliet-Naperville, IL-IN-WI (4). MSAs in the top 15 largest publicly-traded REIT markets by rank that are not in the top 15 largest institutional real estate markets include Phoenix-Mesa-Glendale, AZ, Baltimore-Towson, MD, and Minneapolis-St. Paul-Bloomington, MN-WI. In turn, MSAs in the top 15 largest institutional real estate markets that are not in the top 15 publicly-traded REIT markets include Seattle-Tacoma-Bellevue, WA, Riverside-San Bernardino, CA, and Denver-Aurora-Broomfield, CO.

4 This evidence is not, however, equivalent to saying that the public real estate market is larger than the private real estate market. We do not consider in these calculations the real estate holdings of private equity investors.
Several technical points must be made before considering the estimates of equation (1). Gabaix and Ioannides (2002) show that ordinary least-squares (OLS) estimation of equation (1) produces parameters estimates that are biased downward and standard errors underestimated for sample sizes in the range that is usually considered for city size distributions. Gabaix and Ioannides (2011) propose a simple way to improve the OLS estimation of equation (1). Gabaix and Ioannides (2011) find that using $R_j - 1/2$ and estimating the regression

$$\ln\left(R_j - 1/2\right) = C - \alpha \ln(S_j) + \epsilon_j \quad j = 1, 2, ..., N$$  \hspace{1cm} (4)

can be used to correct for the OLS bias.

The potential investment-size distribution for both public and private real estate investments seems to display an extreme power-law distribution. For example, looking at the raw data on direct institutional real estate investments inside the U.S. from 2012, a few MSAs have investments in excess of $10$ billion. However, most MSAs have investments less than $55$ million. The full range of observations is summarized in Figure 2. The curve is a near-perfect L shape. This pattern has been shown to be a characteristic signature of a power-law PDF. Nonetheless, there is a problem of fitting a power-law PDF to data of this type. Because there are so few data points with investments in excess of $10$ billion, simply fitting a log-linear line to the data in Figure 2 gives a slope that is too shallow. There is an interesting example in Adamic (2003) which deals with this problem for the distribution of AOL users’ visits to various sites on a December day in 1997. Adamic’s (2003) solution to this problem is first to bin the data to smooth out the data points in the right tail of the distribution, and then to fit a power-law PDF to the resulting distribution. We take exactly the same approach to smooth out the data points in the right tail of our investment-size sample. We place the data into bins, where the width of each bin $j$ (e.g., 1, 2, 3, ...) is a constant ($w = S_{j+1} - S_j$). We then count the number of observations in each bin (i.e., with values of $S$ between $S_j$ and $S_j + w$). We also estimate the value of $S$ at the center of the bin ($S_j/2 + S_{j+1}/2$). We then
fit equation (4) to these data, with the slope coefficient giving an estimate of $\alpha$. Figure 3 shows the log-transformed values for 2012.

In Figure 4, we reproduce a similar plot to Figure 2 for investments made by publicly-traded REITs by MSAs and micro areas for 2012. The overall pattern to the plot in Figure 4 is very similar to the plot in Figure 2, with a near perfect L shape relation between investment size and rank for publicly-traded REITs inside the US. Next, we apply the same process as above to bin the data points, to form a log-log investment size-rank profile for publicly-traded REITs. These data are plotted in Figure 5. Here the plot is remarkably similar to that in Figure 3 above.

Table 2 reports the estimates of equation (4) for the two samples. We present two sets of estimates. First, we present the (unbiased) linear estimates of equation (4). Next, we present the (unbiased) nonlinear estimates of equation (4), where the regression estimated is

$$ \ln \left( R_j - 1/2 \right) = C - \alpha' \ln \left( S_j \right) + \beta' \left( \ln \left( S_j \right) \right)^2 + \varepsilon_j, \quad j = 1, 2, \ldots, N \quad (5) $$

Equation (5) seeks to test for nonlinearities in the relationship between $\ln \left( R_j - 1/2 \right)$ and $\ln \left( S_j \right)$ by including a quadratic investment size term on the RHS.

The results in Table 2 are quite interesting. Estimates of equation (4) show that the coefficient on size (Table 2, columns (1) and (3)) is negative and statistically significant for both samples. This negative coefficient is -0.999 for private institutions and -1.192 for publicly-traded REITs. Both coefficients are close to and generally not statistically different from -1. These results confirm the existence of an investment rank-size distribution for both private institutions and publicly-traded REITs. For values of $C$, we find estimates of 10.943 for private institutions and 10.391 for publicly-traded REITs. These values can be converted to anti-log values to give $87.7$ billion and $24.6$ billion square feet, respectively. These intercept values are large, appearing to be significantly larger than the size of the largest market across the two samples (compare these values to the values given in Table 1).
The remaining results in Table 2 seek to uncover any nonlinearities in the relationship between \( \ln(R_j - 1/2) \) and \( \ln(S_j) \). Three general conclusions stand out. First, the value of the quadratic term is negative and statistically significant for both samples, thus indicating concavity of the log-rank-log-size plot for most of the relevant values of \( \ln(S_j) \). Second, including a quadratic term dramatically changes the value of and the sign of the coefficient of the linear term. In columns (2) and (4) in Table 2, the linear term is positive and statistically significant for both samples, while the intercept term is now negative and statistically significant. The intuitive explanation of this result can be made with reference to equation (5) (see Soo (2004)). If the relationship between \( \ln(R_j - 1/2) \) and \( \ln(S_j) \) is
\[
\ln\left(R_j - \frac{1}{2}\right) = C + \alpha' \ln(S_j) + \beta' \left(\ln(S_j)\right)^2,
\]
then \( \ln\left(R_j - \frac{1}{2}\right) \) is maximized, with \( \beta' < 0 \). When \( \ln(S_j) = -\left(\alpha'/2\beta'\right) \), thus implying a positive value of \( \alpha' \). Third, and lastly, we judge the effect of quadratic term in Table 2 to be slight. The \( R^2 \) values shown in Table 2 suggest that the percentage change in \( R^2 \) caused by the removal of the quadratic term is around 10 percent or less in both samples, indicating the curve for the log-rank-log-investment-size is in actual fact quite shallow.

4 Investment-Size Distributions Internationally

International data on institutional real estate investments are difficult to obtain. Our data source for this paper is the International Property Databank (IPD). On an annual basis, IPD estimates the professionally managed market size of the institutional real estate market for 24 countries. For 22 countries in our sample, we have complete data for 7 years over the period 2005 through 2011. We delete the two countries in which we lack complete data about size of market and proceed here with a sample of 22 countries. We employ this data to determine whether the characteristics of institutional real estate investment-size distributions at the MSA level work about as well when one considers countries instead of MSAs. The harbinger for
this analysis is Rose (2006), who examines if cities and countries have similar population-size distributions.

Table 3 gives the investment statistics for twenty-two countries over the period 2005-2011. Size is measured in terms of market capitalization (converted to U.S. dollars). Table 3 reveals two important facts. First, the US, Japan, the UK, Germany, and France have consistently been the largest institutional real estate markets across all time periods, although the ordering varies slightly in 2011 in relation to 2005. In 2005, the US, Japan, the UK, Germany, and France had institutional real estate holdings of $2.6 trillion, which represented about three-quarters of the world total. In 2011, the US, Japan, the UK, Germany, and France held over $3.7 trillion of institutional real estate, which, again, represents about three-quarters of the world total. Second, the U.S. single-handedly has $1.98 trillion of institutional real estate, which is about 40 percent of the world total. From 2005 to 2011, total institutional real estate market capitalization in the US rose 46 percent.

Table 4 presents the detailed results of regressing (4) and (5) for countries. When we do not control for nonlinearities, we find a negative and statistically significant relationship between log rank and log size (Table 4, column (1)). This negative coefficient is -0.610, suggesting that the stratum of observations globally is more concentrated at the upper tail than predicted by Zipf’s Law. The coefficient of the quadratic term in Table 4 is negative and statistically significant (Table 4, column (2)). However, there is no evidence to suggest that the departure from the power-law distribution is significant. The percentage change in $R^2$ caused by the removal of the quadratic term is just around 13 percent. This very small quadratic term increases our confidence that the relationship between size and rank at the international level is a power-law function.

5 Discussion and Implications

The two questions that beg answering here are, Why does Zipf’s Law apply to the spatial distribution of institutional real estate investment inside the US? In addition: Why does a power-law function appear to apply to the spatial distribution of institutional real estate
globally? In this section, we offer one such explanation. The explanation is along the lines mentioned in the introduction. Before giving this explanation, though, we start by reviewing why Zipf’s Law holds in the case of city size. It is instructive to compare the city size-rank relationship, as one will see below, to the institutional real estate investment-rank relationship.

Various studies have provided an explanation for why city size is related to its rank. Geographic theories feature underlying natural advantages. In Ellison and Glaeser (1999), for example, more plants will locate in countries, regions, or cities with observed cost advantages; more plants will locate in countries, regions, or cities with unobserved cost advantages; and plants will cluster if spillovers are geographically localized. Static but non-geographic theories feature agglomeration economies (e.g., scale economies in intermediate inputs, labor-market pooling, knowledge spillovers, etc.) and dispersion forces. In Fujita (1988) and Krugman (1991), for example, firms that locate in densely populated countries, regions, or cities economize on fixed costs and transport costs. These location decisions give rise to agglomerations, which promote the further spatial concentration of economic activity and sustain the conditions for further growth. Offsetting this spatial concentration are dispersion forces, which favor an equal distribution of economic activity. These dispersion forces arise from diseconomies or congestion costs for each firm (e.g., firms must compensate workers by paying them high wages relative to outlying areas). In this case, Zipf’s Law emerges as an equilibrium outcome given a nearly Pareto distribution of talent.

Other explanations for the existence of Zipf’s Law focus on random growth theory. In Gabaix (1999, 2008), for example, Zipf’s Law derives from Gibrat’s Law, where each city grows at some common mean rate (independent of size) and the same standard deviation, but with some idiosyncratic shocks to the growth rate. The idiosyncratic shocks are introduced to account for cities that experience a history of above- or below-average productivity shocks over time, and thus either shrink over time, like Detroit throughout the 1990s and 2000s, or have higher-than-average population growth, like the San Francisco Bay area over the same

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5 Among others using this random growth modeling approach are Davis and Weinstein (2002), Eeckhout (2004), and Duranton (2006).
time period. With each city growing at an arbitrary mean rate, on average, the distribution of cities automatically, in the steady state, under very general conditions, converges to Zipf’s Law.\(^6\)

Still other explanations center on locational fundamentals including physical geography, market access and comparative advantage, and under-emphasized agglomeration economies. Krugman (1996), for example, posits that these location-specific characteristics are distributed according to the same random growth process as in Gabaix (1999). Thus, instead of city growth itself being random, locational fundamentals theory assumes that it is the fundamental economic characteristics of locations that are random. Cordoba (2004) develops a model of city growth in which either tastes or technologies follow a Brownian motion. Rossi-Hansberg and Wright (2007) develop a model of city growth in which there are increasing returns at the local level and constant returns in the aggregate. In both cases, cities specialize and then trade with other cities. As cities reach their efficient size given their specialization, Zipf’s Law emerges.

We now turn to lessons on what happens to the stock of real estate in these models. Under conditions of “normal” growth, where production depends entirely on the amount of physical capital available to workers, Rossi-Hansberg and Wright (2007) show that the physical capital stock will essentially grow at the rate of deterministic growth in the aggregate labor supply, \(g_N\), with

\[
\ln n_{t+1, j} - \ln n_j = g_N \left[ \ln K_{t+1, j} - \ln K_{j} \right]
\]  

(6)

where \(n_j\) is establishment size measured in terms of number of employees per establishment in city \(j\) and \(K_j\) is the stock of real estate capital in city \(j\) (which is equation (10) in Rossi-Hansberg and Wright (2007), with \(\alpha_j = 1\) and \(\beta_j = 0\)). With no change in establishment size, the stock of real estate capital in each city should grow at some common mean rate, \(g_N\),

\(^6\) Solution requires only an arbitrarily small reflecting barrier that prevents cities from getting too small.
that is independent of the initial size $K_j$ of city $j$.\(^7\) However, by changing $n_j$, due perhaps to information technology, which leads to spinoffs and smaller establishments (i.e., declines in \(\ln n_{t+1,j} - \ln n_j\)), the stock of physical capital ought to grow faster than $g_N$ (i.e., \(\ln K_{t+1,j} - \ln K_j\) must exceed $g_N$ for the equality in (6) to hold). This last result seems obvious. The entry of smaller, more productive firms into market $j$ effectively triggers an increased demand for real estate, which, when coupled with the normal growth in $K_j$, causes \(\ln K_{t+1,j} - \ln K_j\) to exceed $g_N$, so that $K_{t+1,j}$ is chosen that sets $g_N - \left[\ln K_{t+1,j} - \ln K_j\right]$ at the level where $\ln n_{t+1,j} - \ln n_j = g_N - \left[\ln K_{t+1,j} - \ln K_j\right]$.

With $\left[\ln K_{t+1,j} - \ln K_j\right] = g_N + \ln n_{t+1,j} - \ln n_j = g_N + \varepsilon_j = g_j$, where $\varepsilon_j$ and $g_j$ are random variables, and where $\varepsilon_j$ is used to measure $\ln n_{t+1,j} - \ln n_j$, the workings of the model suggests that $K_{t+1,j}$ should grow or shrink by a random amount,

$$K_{t+1,j} = e^{\varepsilon_j} K_j \quad (7)$$

In this approach, we shall only deal with $K_{t+1,j}$ exceeding some lower bound, $K_{\min}$. This approach can be interpreted as stating that when the physical capital stock reaches some lower bound, the city goes bankrupt (like Detroit) and does not continue to invest in physical capital. This assumption simplifies the model.

The process (7) implies that the physical capital stock of city $j$ changes from $K_j$ at time $t$ to $\lambda_j K_j$ at time $t+1$, where $\lambda_j = e^{\varepsilon_j}$. Denote the normalized physical capital stock

\(^7\) Note that in Rossi-Hansberg and Wright’s (2007) model the initial size $K_j$ in city $j$ is simply a constant proportion of output.

\(^8\) An excellent common-sense explanation of the above is the San Francisco Bay Area and Seattle. Small technology firms continue to drive demand for commercial real estate in both of these markets. High prices for commercial real estate in downtown San Francisco (due to excess demand) have spurred some firms to migrate to more affordable areas of the region.
distribution at time \( t \) and at time \( t + 1 \) by \( F(K,t) \) and \( F(K,t+1) \), respectively. The value of \( F(K,t+1) \) is given by

\[
F(K,t+1) = \int_0^{\infty} F(\frac{K}{\lambda}, t)h(\lambda) d\lambda
\]  

(8)

The right-hand side of (8) is integrated over all values of \( \lambda \) in the array defined by \( e^{\delta t} \), where \( h(\lambda) \) is a common distribution of \( \lambda \) for all cities.

In the presence of a lower bound on the physical capital stock (\( K_{g} \geq K_{min} \)), equation (8) describes a process which is analogous to diffusion towards a barrier. This process is known to lead to the convergence of \( F(K,t) \) to a stationary distribution, as indeed suggested by, for example, Boltzmann (1964). For the limiting stationary physical capital distribution, we have \( F(K,t+1) = F(K,t) = F(K) \), giving

\[
F(K) = \int_0^{\infty} F(\frac{K}{\lambda})h(\lambda) d\lambda
\]  

(9)

Differentiating (9) with respect to \( K \) yields

\[
f(K) = \int_0^{\infty} f(\frac{K}{\lambda}) \frac{1}{\lambda} h(\lambda) d\lambda
\]  

(10)

The Pareto distribution is a solution to (10). The Pareto distribution is given by

\[
f(K) = \frac{cK^{c} K_{min}}{K^{c+1}} = BK^{-(c+1)}
\]  

(11)

where \( B = cK_{min}^{-c} \), and \( B \), \( c \), and \( K_{min} \) are constants. Substituting (11) into (10) yields

\[
BK^{-(c+1)} = \int_0^{\infty} BK^{-(c+1)} \lambda^c h(\lambda) d\lambda = BK^{-(c+1)} \int_0^{\infty} \lambda^c h(\lambda) d\lambda
\]  

(12)

With \( c \) satisfying \( \int_0^{\infty} \lambda^c h(\lambda) d\lambda = 1 \), it follows that we are dealing with a process that has a Pareto distribution.

As Adamic (2003) and others point out, the shape parameter of the Pareto distribution, \( c \), equals \( a - 1 \), where \( a \) is the power law slope as expressed in (3). Thus, if \( \varepsilon_0 \), which we
used as a measure $\ln n_{t+1} - \ln n_t$, is a random variable, a plot of log-rank against log-investment-size should give a straight line of slope $-\left[\frac{1}{(a-1)}\right]$. This result also implies that if Zipf’s Law applies to the spatial distribution of institutional real estate investment inside the US, then in this case a plot of log-rank against log-investment-size will have a slope of -1, implying a power-law exponent of $a = 2$ and a Pareto exponent of $c = 1$. Further, another distinguishing characteristic is the tendency for $\varepsilon_t$, which can be caused by various causal factors, acting independently or in a linear factor, to reverse itself over time. The more $\varepsilon_t$ tends to reverse itself over time, the more the spatial distribution of physical capital will be characterized by Zipf’s distribution.

However, it is worth noting that $\varepsilon_t$ need not mean-revert over time. Rather, $\varepsilon_t$ could show a significant trend in the same direction over time. For example, Rappaport and Sachs (2003) provide a model of how firms choose where to locate that leads to increased concentration of both capital and labor. In their model, growth is function of a set of productivity-enhancing and/or quality-of-life-enhancing attributes. Productivity-enhancing attributes in this case include factors like access to navigable water, abundant natural resources, temperate weather, and rule of law. In contrast, quality-of-life-enhancing attributes include factors like ocean vistas, pleasant weather, and low crime. A high productivity-enhancing attribute induces an inflow of both labor and capital, in turn inducing high capital returns and high wages and mutually reinforcing inflows of both capital and labor. Similarly, a quality of life induces an inflow of labor, which in turn induces an inflow of capital. As a result of self-reinforcing agglomeration effects, both capital and labor in “desirable” markets may consistently grow at an above-average rate, (i.e., for some markets or counties, we could observe $g_{t+1} > g_t$ over time).

These ideas easily reconcile the results documented above. According to (6), for example, shocks enter the investment demand function only through the term $\ln n_{t+1} - \ln n_t$. This
implies that, given a mean of zero for $\ln n_{t+1j} - \ln n_y$, the physical capital stock will in equilibrium grow at a constant rate independent of market size (i.e., in equilibrium, $K_y$ must be chosen such that (6) holds at all times). However, it is possible for some markets to experience a history of above- or below-average shocks to $\ln n_{t+1j} - \ln n_y$ over time, and thus for $K_y$ to grow must faster than $g_N$ over time, especially if there are self-reinforcing agglomeration effects influencing $\ln n_{t+1j} - \ln n_y$ over time. With respect to this reasoning, unless a large number of markets experience a history of above- or below-average shocks to $\ln n_{t+1j} - \ln n_y$ over time, we would expect the spatial distribution of $K_y$ to be Zipf-like. At the same time, however, it does not require any stretch of the imagination to see what will happen to the distribution of $K_y$ if there are stark locational fundamentals and self-reinforcing agglomeration effects consistently influencing $\ln n_{t+1j} - \ln n_y$, especially across countries. In this case, the spatial distribution of $K_y$ will become Pareto-like, with capital becoming more concentrated in certain countries than predicted by Zipf’s Law.\footnote{Differences in locational fundamentals can be quite stark across countries, and they are of a magnitude that can lead to a Pareto-like spatial distribution of $K_y$. A considerable number of surveys of quality-of-life are relevant to this problem. Polls by the United Nations, for example, gauge a country’s level of human development, while polls by Newsweek and the Economist Intelligence Unit rank standard of living and quality of life, measuring education, health, quality of life, economic dynamism, and political environment in over 100 countries. Two noteworthy observations can be made about these surveys. First, most use a diverse group of rating criteria. Second, results indicate that there are large and persistent differences in quality of life across countries. Countries that rank very high, for example, in terms of quality of life include Finland, followed by Switzerland and Sweden. In contrast, countries that rank very low in terms of quality of life include the African nations of Burkina Faso, followed by Nigeria and Cameroon. See Newsweek’s the world’s best countries Index of 2010 (http://www.newsweek.com/interactive-infographic-worlds-best-countries-71323).} Of course, there may be other reasons why the international data may deviate from Zipf’s Law. These reasons may be the extra transactions costs, taxes and regulation, ownership restrictions, currency fluctuations, and social and economic instability (Stulz (1981); Errunza and Losq (1985)).

The insights gained in this exercise seem to be the following. The results appear quite at odds with admonitions to institutional investors to tilt their portfolios towards “prudent assets that are large with high information flow” or to limit their investments to “assets that are more liquid” or to open the door to international diversification, in order to buy extra amounts of
assets that “carry a credible high average return.” Advocates of prudent investor rules argue that institutional investors should invest and manage property held in a trust as a prudent investor would, by tilting their portfolios towards high-quality assets that are easy to defend in court. This position can be understood as advocating an investment strategy based primarily on the characteristics of assets in isolation (see Del Guercio (1996)). However, this theory cannot explain the near-perfect inverse relation between size and rank for private institutions and publicly-traded REITs inside the U.S., nor the considerably less-than-perfect inverse relation between size and rank for institutional investment globally.

The liquidity issue goes beyond the laws and regulations that govern fiduciaries who manage assets on behalf of others. The liquidity issue states that institutional investors should tailor their assets holdings by rebalancing their portfolios toward assets that are more liquid (presumably with large market capitalizations and thick markets), since they generally turn over their portfolios and trade more often than individuals do. This view is attributable to Longstaff (2009) and others. However, tilting one’s portfolio toward assets that are more liquid would seem to imply that institutional investors should have portfolios overweighted in publicly-traded REIT stocks rather than in private real estate equity investments. A problem with this argument is that institutional ownership of publicly-traded REIT stocks (as a percentage of total institutional real estate holdings) is quite small. It typically is around 25 percent of total institutional real estate holdings.

The introduction of international diversification into the analysis creates a number of quandaries. Standard economic theory posits that capital should freely flow from rich countries (large markets) where capital is abundant and rates of return are low to poorer countries (small markets) where capital is scarce and rates of return are high. Such a concept is intuitively reasonable. However, capital does not flow from rich (large markets) to poor (small markets) countries in the quantities suggested by theory. Rather, most institutional investors generally stick to large markets (rich countries), an observational point shown in
Figures 1 above. Further, as seen in Figure 1, cross-border real estate capital flows are highly concentrated in a few metropolitan areas.

In the current work, as we have touched on earlier, we view institutions, along with other primary suppliers of capital to the real estate market, as profit-seeking entities. These entities simply adjust the supply of physical capital in order to respond to demand changes. They also concentrate their investments in markets where demand for space is strong and growing – precisely where $\ln n_{t+1j} - \ln n_{ij}$ is large. We might add some realism to the model by assuming that institutions differ from other suppliers of capital principally in terms of wealth. These differences would naturally lead institutions to tilt their asset holdings toward large assets to take advantage of the lack of competition that prevails when high value is a market-limiting factor (thereby explaining the tilt toward assets with large market capitalizations and thick markets). Although most institutions now have both core and high-yield investment components within their portfolios, this has not always been the case. During most of the 1980s and the first half of the 1990s, institutions were generally only in core real estate. Now institutions invest across the risk/reward spectrum in core, value-add and opportunistic real estate investments. Interestingly enough, this shift toward value-add and opportunistic investments has not caused a structural shift in the coefficients in (4). Stability, in this sense, is consistent with a hierarchical structure, in which we need not assume that all value-add and opportunistic investments are made in secondary markets with high yields. We can imagine, instead, investors first choosing to invest in desirable markets, and then, for effective risk management, dividing their investments into core, value-add, and opportunistic.

The model also reinforces the present consensus that investment capital can be quite immobile internationally. Investment capital in the model does not freely flow in large quantities to markets or countries unless there are large values of $\ln n_{t+1j} - \ln n_{ij}$ over time. Of course, to a certain sense large values of $\ln n_{t+1j} - \ln n_{ij}$ are not possible unless there is something – some productivity-enhancing or quality-of-life-enhancing attribute, as Rappaport
and Sachs (2003) would argue – which sparks an interest in the location in the first place and gets things kicked off. This condition makes the spatial distribution of $K_t$ quite resilient over time (another one of our findings) and can help explain the lack of capital flows to poor countries (and also to what extent poorer nations might wish to subsidize the underlying productivity of certain locations to induce an inflow of institutional capital).

6 Conclusions

Our analysis of the spatial pattern of institutional investment inside the US and internationally in 2005-2012 makes clear several important generalizations. The first of these is that there is a distinct log-log rank-size relationship between size of private institutional real estate and rank inside the US. That is, we find that the second and subsequently smaller private institutional real estate markets inside the US represent a proportion of the largest market. The second generalization is that we find similar results as for real estate investments made by publicly-traded REITs in the US. The findings confirm the existence of substantive spatial regularities, both for private institutions and publicly-traded REITs in the US. Two additional findings that emerge are as follows. First, we find a negative and statistically significant relationship between log rank and log size globally. Second, a power law distribution appears to fit the global data better than Zipf’s Law. That is, we find the stratum of observations globally is more concentrated at the upper tail than predicted by Zipf’s Law.

To explain these phenomena, we turned to random models of establishment size and enterprise growth emerging from the economic geography literature (Rossi-Hansberg and Wright (2007)), in which the physical capital stock essentially grows, in the absence of random productivity-enhancing or quality-of-life-enhancing shocks, at the rate of deterministic growth in the aggregate labor supply. Of particular interest is the pattern of enterprise development that occur in presence (or absence) of changes in, for example, information technology, which leads to spinoffs and smaller establishments, and a faster (or slower) average rate of growth in the stock of physical capital over time. When these shocks are truly random (with mean-reverting errors), the physical capital stock will in equilibrium grow at a
constant rate independent of market size and the resulting spatial distribution of the physical capital stock will be Zipf-like. However, when enterprise growth is propelled by some productivity-enhancing or quality-of-life-enhancing attribute, and if there self-reinforcing agglomeration effects, the physical capital stock in these markets will grow at a rate in excess of the rate of deterministic growth in the aggregate labor supply and the resulting spatial distribution of the physical capital stock will be more Pareto-like. We use the former to explain the spatial pattern of institutional investment inside the US, and the latter (along with stark differences in locational fundamentals) to explain the spatial pattern of institutional investment internationally.

The reported results do suggest some implications. There is a great deal of literature which addresses the question of what is institutional real estate. A related body of literature examines how and where institutional investors should invest their wealth – e.g., direct ownership of property through pooled investment funds versus indirect ownership of property through publicly-traded REITs, large versus small investment properties, core versus core-plus, value-added, or opportunistic strategies, international versus domestic real estate investments, etc. Del Guercio (1996) has attempted to approach institutional investments in theoretical terms by arguing that prudence considerations ought to lead institutions to tilt their portfolios towards prudent assets that are large with high information flow. Liquidity considerations also matter for institutional investors. Longstaff (2009) argues that institutional investors should tailor their assets holdings by rebalancing their portfolios toward assets that are more liquid (presumably with large market capitalizations and thick markets), since they generally turn over their portfolios and trade more often than individuals do. It has also been argued repeatedly in recent years that institutions should mix in broad international diversification, thereby reducing the variance of their real estate portfolio.

Our approach assumes institutions are profit maximizers who do not necessarily behave on the basis of prudence or liquidity considerations, but instead simply respond to random productivity-enhancing or quality-of-life-enhancing shocks by supplying physical capital to
the private sector. Institutions therefore do not start out thinking that they are going to tilt their portfolios toward “desirable” (large) markets, but end up doing so anyway because “desirable” (large) markets are where firms want to locate. We do recognize (while not part of the model) vast wealth differences among institutions and other suppliers of physical capital. By adding this assumption to the model, we have that institutions will want to tilt their asset holdings toward large assets to take advantage of the lack of competition that prevails when high value is a market-limiting factor. As a result, institutions end up focusing somewhat exclusively on assets with large market capitalizations and thick markets.

Finally, the results support the consensus view that investment capital can be quite immobile internationally. Productivity-enhancing and quality-of-life-enhancing attributes as well as self-reinforcing agglomeration effects determine the direction and magnitude of investment capital flows. These effects are self-reinforcing because a high productivity-enhancing or quality-of-life-enhancing attribute induces an inflow of labor or capital, or both, which, in turn, induces high capital returns and high wages and mutually reinforcing inflows of capital and labor. Once these effects become self-reinforcing, they tend to persist, which causes the spatial distribution of physical capital to become quite resilient to change. An implication for small (poor) nations is that subsidizing the underlying productivity of certain locations raises the attractiveness of these locations to firms as well as the inflow of capital and therefore can lead to increased capital flows into the economy.
References


Figure 1. International real estate capital flows over the period 2007-2013.
Figure 2. Linear plot of the investment-size-rank distribution for US private institutions among MSAs. This figure gives a graphical representation of investments by private institutions in real estate in 226 MSAs inside the U.S. as of 2012. Investment size is measured in market value.
Figure 4. Linear plot of the investment-size-rank distribution for US publicly-traded REITs among MSAs and micro areas. This figure gives a graphical representation of investments by publicly-traded REITs in real estate in 887 MSAs and micro areas inside the U.S. as of 2012. Investment size is measured in square feet.
Table 1. Top 15 Largest Private Institution and Public Real Estate Markets Inside the U.S., 2012

<table>
<thead>
<tr>
<th>Rank</th>
<th>Private Institutions</th>
<th>Market value</th>
<th>Publicly-traded REITs</th>
<th>Rank</th>
<th>Square feet</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Washington-Arlington-Alexandria, DC-VA-MD-WV</td>
<td>$38,502,932,701</td>
<td>2</td>
<td>202,028,534</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Los Angeles-Long Beach-Santa Ana, CA</td>
<td>$34,948,673,569</td>
<td>3</td>
<td>187,709,083</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>New York-Northern New Jersey-Long Island, NY-NJ-PA</td>
<td>$30,748,474,555</td>
<td>1</td>
<td>327,955,937</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Chicago-Joliet-Naperville, IL-IN-WI</td>
<td>$24,099,110,545</td>
<td>4</td>
<td>175,478,928</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Boston-Cambridge-Quincy, MA-NH</td>
<td>$20,223,590,333</td>
<td>10</td>
<td>99,365,741</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Miami-Fort Lauderdale-Pompano Beach, FL</td>
<td>$19,252,655,813</td>
<td>9</td>
<td>99,998,316</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Houston-Sugar Land-Baytown, TX</td>
<td>$14,820,677,778</td>
<td>6</td>
<td>148,428,371</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Dallas-Fort Worth-Arlington, TX</td>
<td>$14,298,687,278</td>
<td>5</td>
<td>170,592,566</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>Seattle-Tacoma-Bellevue, WA</td>
<td>$12,874,325,900</td>
<td>--</td>
<td>--</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>Atlanta-Sandy Springs-Marietta, GA</td>
<td>$12,193,706,773</td>
<td>7</td>
<td>142,235,726</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>Riverside-San Bernardino PMSA, CA</td>
<td>$9,967,884,196</td>
<td>--</td>
<td>--</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>San Diego-Carlsbad-San Marcos, CA</td>
<td>$9,387,229,035</td>
<td>15</td>
<td>61,080,623</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>Denver-Aurora-Broomfield, CO</td>
<td>$9,194,234,781</td>
<td>--</td>
<td>--</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>San Francisco-Oakland-Fremont, CA</td>
<td>$7,854,115,990</td>
<td>11</td>
<td>92,895,637</td>
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<tr>
<td>15</td>
<td>Philadelphia-Camden-Wilmington, PA-NJ-DE-MD</td>
<td>$6,460,290,448</td>
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<td>117,240,589</td>
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<tr>
<td>--</td>
<td>Phoenix-Mesa-Glendale, AZ</td>
<td>--</td>
<td>12</td>
<td>86,239,689</td>
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<tr>
<td>--</td>
<td>Baltimore-Towson, MD</td>
<td>--</td>
<td>13</td>
<td>74,608,073</td>
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<tr>
<td>--</td>
<td>Minneapolis-St. Paul-Bloomington, MN-WI</td>
<td>--</td>
<td>14</td>
<td>61,687,891</td>
<td></td>
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</tbody>
</table>


Table 2. Average Zipf Coefficients for Private Institutions and Publicly-traded REITs Inside the U.S., 2005-2012

<table>
<thead>
<tr>
<th></th>
<th>Private institutions</th>
<th>Publicly-traded REITs</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Model (1)</td>
<td>Model (2)</td>
</tr>
<tr>
<td>( C )</td>
<td>10.78</td>
<td>-51.11</td>
</tr>
<tr>
<td></td>
<td>(16.5)</td>
<td>(-5.6)</td>
</tr>
<tr>
<td>( \ln\left(S_j\right) )</td>
<td>-0.989</td>
<td>11.804</td>
</tr>
<tr>
<td></td>
<td>(-14.8)</td>
<td>(12.7)</td>
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<tr>
<td>( \left(\ln\left(S_j\right)\right)^2 )</td>
<td>-0.660</td>
<td>(-0.489)</td>
</tr>
<tr>
<td></td>
<td>(-13.7)</td>
<td></td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.87</td>
<td>0.98</td>
</tr>
<tr>
<td>( SE )</td>
<td>0.16</td>
<td>0.06</td>
</tr>
</tbody>
</table>

Note: The samples include real estate investments by private institutions and publicly-traded REITs over the time period 2005 through 2012. The data come from SNL Real Estate, http://www.snl.com, and the National Council of Real Estate Investment Fiduciaries, http://www.ncreif.org. Dependent variable is \( \ln\left(R_j - 1/2\right) \), where \( R_j \) refers to the rank of an observation. Estimates show the average effect of size of investment, \( S_j \), on rank. Estimates use binned data. t-statistics are shown in parentheses.
<table>
<thead>
<tr>
<th>Rank</th>
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<th>Market Value, USD, in thousands</th>
<th>Rank</th>
<th>2005</th>
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<td>Japan</td>
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<td>Portugal</td>
<td>$19,566</td>
<td>19</td>
<td>Portugal</td>
<td>$14,578</td>
</tr>
<tr>
<td>21</td>
<td>Czech Republic</td>
<td>$14,477</td>
<td>20</td>
<td>Czech Republic</td>
<td>$13,572</td>
</tr>
<tr>
<td>22</td>
<td>Ireland</td>
<td>$3,274</td>
<td>22</td>
<td>Ireland</td>
<td>$6,943</td>
</tr>
<tr>
<td></td>
<td>Model (1)</td>
<td>Model (2)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>---------------</td>
<td>-----------</td>
<td>-----------</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>$C$</td>
<td>4.192</td>
<td>-2.796</td>
<td></td>
<td></td>
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</tr>
<tr>
<td></td>
<td>(17.0)</td>
<td>(-4.6)</td>
<td></td>
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</tr>
<tr>
<td>$\ln(S_j)$</td>
<td>-0.667</td>
<td>2.177</td>
<td></td>
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<tr>
<td></td>
<td>(-13.4)</td>
<td>(8.9)</td>
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</tr>
<tr>
<td>$(\ln(S_j))^2$</td>
<td></td>
<td>-0.285</td>
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<tr>
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<td></td>
<td>(-11.6)</td>
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<td></td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.89</td>
<td>.99</td>
<td></td>
<td></td>
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<tr>
<td>SE</td>
<td>0.14</td>
<td>0.05</td>
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</table>

Note: The sample covers 22 countries with at least seven years of annual data on the size of the institutional real estate market (denominated in U.S. dollars) within each country, 2005-2011. These data come from International Property Database, http://www1.ipd.com. Dependent variable is $\ln(R_j - 1/2)$, where $R_j$ refers to the rank of an observation. Estimates show the effect of size of investment, $S_j$, on rank. $t$-statistics are shown in parentheses.