Abstract

We build a risk aversion sensitive RBC model through endogenous state-contingent technology choices. With plausible parameter values, the risk averse agent optimally chooses productivity which is amplified and moves counter to the exogenous technology shock to smooth consumption across states. Such an amplification mechanism creates more volatile output, investment, and equity returns. In equilibrium, we find a high price of risk, a low and smooth risk-free rate, and a sizable unlevered equity premium. Various preference specifications, including CRRA, recursive preferences, and external habit, reasonably match moments of asset prices and business cycle statistics once we allow for state-contingent technology choices.

Keywords: State-contingent technology; Risk aversion; Business cycles; Asset pricing

JEL Classification: E23; E32; E37; G12

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In a standard production economy, macroeconomic quantities are sensitive to the elasticity of intertemporal substitution (EIS) but not to risk aversion. Tallarini (2000) demonstrates this feature by forcefully choosing a high risk aversion to match asset price moments without negatively affecting the fit of macroeconomic quantities. Cochrane (2008b) calls this feature of RBC models the divorce between asset pricing and macroeconomics. In this paper, we attempt to reunite the peculiar couple by allowing for flexible technology choice, introduced in Cochrane (1993). Technology choice allows to substitute output across states of nature, which reflects the desire of risk averse agents to smooth consumption across states. Thus risk aversion drives production substitution and through that channel it affects macroeconomic quantities such as consumption, output, and investment. Hence, our model generates risk aversion sensitive real business cycles.

In an attempt to build a pure production-based pricing kernel, Cochrane (1993) observes that the standard production function only allows firms to transform outputs in fixed proportions across states. Therefore, the marginal rate of transformation across states is not well defined. To address this shortcoming, Cochrane proposes to allow firms to choose state contingent productivity endogenously subject to a constraint set. In closely related work Cochrane (1988) and Jermann (2010) back out the stochastic discount factor from producers’ first-order conditions assuming that producers have access to as many technologies as the number of states of nature in the economy. For tractability, they explore an economy with two states of nature. Belo (2010) successfully applies the idea in Cochrane (1993) to derive a pure production-based pricing kernel in a partial equilibrium setting. He estimates producers’ ability to transform output across states to be high. In turn, we study the general equilibrium implications of risk aversion motivated production substitution across states of nature. Our calibration confirms Belo’s estimate of producers’ ability to transform output across states.

Consumption smoothing across states is limited in the standard RBC models, since investment largely smooths consumption over time only. In our model, however, the risk averse
agent can smooth consumption across states by modifying the underlying technology shocks. With plausible parameters, a risk averse agent optimally chooses productivity which is amplified and moves counter to the underlying technology process. Intuitively, the agent tends to choose low productivity realizations in states in which it is difficult to produce, namely in states when the underlying exogenous shocks are low. This force entails technology chosen to move in the same direction as the underlying process. Meanwhile, risk aversion motivates the agent to smooth consumption across states. That is, the agent prefers a high productivity technology in states in which consumption is more valuable at the expense of states in which consumption is less valuable. This implies a technology choice that moves counter to the underlying technology process. When the agent is sufficiently risk averse, the second force dominates the first one. As technology modifications increase in risk aversion, amplifications occur. Therefore, in the model, we see volatile investment and output even given small exogenous shocks. This amplification mechanism helps address concerns of unrealistically large technology shocks needed in the standard RBC models (Burnside and Eichenbaum (1996)).

From an asset pricing perspective, state contingent technology choice can quantitatively justify observed features of asset prices, namely, a large equity premium and market price of risk and a low and smooth risk-free rate. In this economy, real investment becomes volatile as risk averse agent smooths her consumption, especially when amplification occurs. This implies volatile investment returns and equity returns, which can be seen from the equivalence between stock returns and investment returns established in Cochrane (1991), Restoy and Rockinger (1994), and Liu, Whited and Zhang (2009). In addition, amplified shocks drive up the price of risk in the economy because the marginal rate of transformation is volatile. Therefore, the amplification mechanism drives up the equity returns. Moreover, since technology choice mainly increases variations of marginal rate of substitution across states instead of over time, the risk-free rate can be smooth in this economy. In fact, the risk-free rate is significantly lower and smoother than in a benchmark RBC economy, which is consistent with a consumption smoothing motive. Therefore, state contingent technology
choice generates a sizeable equity premium through equity returns that are elevated by its volatility and a low risk-free rate.

Preference specifications usually play crucial roles in asset pricing models. We, therefore, consider a general equilibrium model under various preference specifications, including the constant relative risk aversion utility (CRRA), recursive preferences, and external habit, to understand how they might interplay with state contingent technology choice. Surprisingly, we find that once we introduce technology choice, all preference specifications generate similar results and match both macroeconomic quantities and asset prices reasonably well. For example, a CRRA risk aversion of 7.5 generates significant amplification in the economy. Small shocks (with a volatility of 1.2% per quarter) generate output volatility comparable to the data (with a volatility of 1.57% per quarter). In addition, the amplification mechanism generates a large price of risk (Sharpe ratio is around 0.3) and a sizable unlevered equity premium of 2.82% per year. We note that recursive and external habit preferences help to further elevate the unlevered equity premium through a somewhat higher mean equity return. However, relative to the importance of risk aversion and technology choice channel for the fit of the model to the data, additional improvements through preferences appear small in our calibrations. This suggests that the simple preference specifications often used in the macroeconomics literature, e.g., CRRA, instead of more sophisticated preferences, are sufficient for many applications once we allow for endogenous technology choice.

This paper is closely related to the pure production-based asset pricing literature, e.g., Cochrane (1988, 1993), Belo (2010), and Jermann (2010). Pure production-based asset pricing models attempt to infer asset returns from production technology without specifying unobservable preferences. For example, Belo (2010) examines the cross-sectional pricing ability of a pure-production based pricing kernel and Jermann (2010) considers the equity premium in a production economy with a complete set of technology. Our results confirm and complement the findings in this strand of the literature in a general equilibrium setting in which macroeconomic quantities are determined endogenously. Moreover, this paper
highlights the importance of recognizing the impact of risk attitude jointly on macroeconomic quantities and asset prices. In addition, we propose a perturbation method to solve the optimal technology choice problem when shocks are continuously distributed instead of discretely distributed as considered in the literature.

This paper also belongs to the large and growing literature of production-based general equilibrium asset pricing models (Jermann (1998), Tallarini (2000), Boldrin, Christiano and Fisher (2001), Gomes, Kogan and Zhang (2003), Kogan (2004), Kuehn (2009), Guvenen (2009), Campanale, Castro and Clementi (2010), Croce (2010), Kaltenbrunner and Lochstoer (2010), and many others). These models generate a sizable equity premium through investment frictions (e.g., convex capital adjustment costs, investment irreversibility, investment commitment, and capital immobility) or stochastic productivity shocks together with habit formation or recursive preferences. Many of these models, however, show difficulties in fitting volatilities of asset prices or business cycle statistics. Often, for example, either the risk-free rate is too volatile or macroeconomic volatility is too low in the economy. We instead propose an alternative explanation for volatile investment and equity returns stemming exclusively from state-contingent technology choices that complements other production-based general equilibrium asset pricing models.

1 A Model with State-Contingent Technology Choices

Consider a representative agent who owns an all-equity representative firm, which uses productive capital to generate one real good and operates in a discrete and infinite time horizon.
1.1 Firms

Let $\theta_t$ be an exogenous technology shock at time $t$. To focus on business cycle fluctuations, we assume $\log \theta_t$ follows an AR(1) process without a trend,

$$\log \theta_{t+1} = \rho \log \theta_t + \sigma \varepsilon_{t+1}, \tag{1}$$

where $0 < \rho < 1$, $\varepsilon_{t+1}$ denotes a standard normal distributed shock, and $\sigma$ scales the shock.

Departing from the standard production technology, we assume that it is feasible for the representative firm to modify the underlying technology shocks. Following Cochrane (1993) and Belo (2010), at time $t$ a state contingent technology $A_{t+1}$ is chosen through a CES aggregator

$$\mathbb{E}_t \left[ \frac{A_{t+1}^{(1-\alpha)} \nu}{\theta_{t+1}^{(1-\alpha) \nu}} \right] \leq 1 \tag{2}$$

where $\mathbb{E}_t$ is the conditional expectation operator at time $t$, $\alpha \in (0, 1)$ stands for the capital share in output and the curvature $\nu$ captures the representative firm’s technical ability to modify technology. The probabilities in this constraint reflect the uncertainty regarding the future state. Such potential technology modifications reflect the fact that producers either at the firm level or at the aggregate level diversify among different technologies to optimally choose future productivity. The constraint reflects the fact that distorting the exogenous shocks is costly, which represents the tradeoff between static efficiency and flexibility (see Mills and Schumann (1985)). No modification is always feasible, and as $\nu$ increases, it becomes harder and harder to distort the underlying shocks. When $\nu \to +\infty$, no modifications occurs, i.e., $A_{t+1} = \theta_{t+1}$. Constraint (2) implies that firms can choose higher realizations of shocks in some states at time $t + 1$ at the expense of lower realizations in other states. Such production substitution across states of nature can smooth consumption across states as well.

Output, $Y_t$, is given by

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where $K_t$ denotes the capital stock at the beginning of period $t$.\footnote{We abstract from labor decisions; hence, labor input is normalized to 1.}

Capital accumulates according to

$$K_{t+1} = (1 - \delta)K_t + g_t \tag{4}$$

where $\delta$ is the depreciation rate and $g_t$ stands for the capital formation function. We specify it as in Jermann (1998), i.e.,

$$g_t = \left[ \frac{a_1}{1 - 1/\chi} \left( \frac{I_t}{K_t} \right)^{1-1/\chi} + a_2 \right] K_t, \tag{5}$$

where $I_t$ denotes investment at time $t$, the curvature $\chi$ governs capital adjustment costs ($\chi > 0$), and $a_1$ and $a_2$ are constants.\footnote{Alternatively, we can specify a quadratic function for the capital adjustment costs, which generates similar numerical results.} These specifications imply that capital adjustment costs are high when $\chi$ is low and that capital adjustments are costless when $\chi \to \infty$. Following Boldrin, Christiano and Fisher (2001), we set $a_1$ and $a_2$ such that there is no capital adjustment costs in the deterministic steady state

$$a_1 = \delta^{1/\chi}, \quad a_2 = \frac{1}{1 - \delta}.$$ 

1.2 Households

To separate EIS from risk aversion, our main analysis uses recursive preferences (Kreps and Porteus (1978), Epstein and Zin (1989, 1991), and Weil (1989)). The representative agent’s utility at time $t$ is given by

$$U_t = \left\{ (1 - \beta) C_{t}^{1-1/\psi} + \beta \mathbb{E}_t \left[ U_{t+1}^{1-1/\gamma} \right]^{1-1/\gamma} \right\}^{\frac{1}{1-1/\psi}}, \tag{6}$$
where $\beta$ denotes the subjective time discount factor ($0 < \beta < 1$), $C_t$ stands for aggregate consumption at time $t$, $\psi$ represents the elasticity of intertemporal substitution (EIS, $\psi > 0$), and the relative risk aversion is given by $\gamma > 0$.

The pricing kernel for recursive preferences is given by

$$M_{t,t+1} = \beta \left[ \frac{C_{t+1}}{C_t} \right]^{-\frac{\psi}{\psi + 1}} \left[ \frac{U_{t+1}^{1-\gamma}}{E_t U_{t+1}^{1-\gamma}} \right]^{\frac{1}{\psi + 1}}. \tag{7}$$

Since the representative agent owns and runs the firm, she consumes all dividends paid out by her firm

$$C_t = D_t = Y_t - I_t. \tag{8}$$

Therefore, the return to equity over one period is defined as

$$R_{t+1} = \frac{P_{t+1} + C_{t+1}}{P_t}, \tag{9}$$

where $P_t$ denotes the stock price at time $t$.

Given the transitory underlying shock process described by (1), the representative agent optimally chooses $A_{t+1}$, $C_t$, and $I_t$ to maximize her utility (6), subject to constraints (8), (4) and (2), where the state variables are $K_t$, $A_t$, and $\theta_t$.

### 1.3 The Equilibrium Conditions

With recursive preferences, the current value Lagrangian function of the maximization problem can be written as

$$L_t = \left\{ (1 - \beta) C_t^{1-\psi} + \beta E_t \left[ U_{t+1}^{1-\gamma} \right]^{\frac{1}{\psi + 1}} \right\}^{\frac{\psi}{\psi + 1}} - \lambda_t (C_t - K_t A_t^{1-\alpha} + I_t)$$

$$- \mu_t [K_{t+1} - (1 - \delta) K_t - g_t] - \xi_t \left\{ E_t \left[ \frac{A_{t+1}^{(1-\alpha)\nu}}{\theta_{t+1}^{(1-\alpha)\nu}} \right] - 1 \right\}, \tag{10}$$
where \( \{\lambda_t, \mu_t, \xi_t\} \) are the current value Lagrangian multipliers associated with constraints (8), (4) and (2), respectively.

From the first order condition with respect to \( C_t \), we have the following condition:

\[
\lambda_t = (1 - \beta)U_t^{\frac{1}{\psi}}C_t^{-\frac{1}{\psi}}. \tag{11}
\]

This equation defines the marginal utility from consuming one additional unit of good at time \( t \).

The optimal investment policy satisfies the following first order condition,

\[
\lambda_t = \mu_t g_t', \tag{12}
\]

where \( g_t' \) stands for the partial derivative of the capital formation function with respect to \( I_t \).

The optimal capital stock at time \( t + 1 \) satisfies

\[
\frac{1}{g_{t+1}} = \frac{\mu_t}{\lambda_t} = E_t \left\{ M_{t,t+1} \left[ \alpha A_{t+1}^{1-\alpha} K_{t+1}^{\alpha-1} + \frac{\mu_{t+1}}{\lambda_{t+1}} \left( 1 - \delta + g_{K_{t+1}}' \right) \right] \right\}, \tag{13}
\]

where \( g_{K_{t+1}}' \) is the partial derivative of capital formation function with respect to \( K_{t+1} \). The left hand side of (13) shows the marginal costs of investment while the right hand side of (13) describes its marginal benefits (i.e., marginal \( q \)). Thus, in equilibrium the firm equates the marginal costs with the marginal benefits from adding one additional unit of productive capital.

At time \( t \), firms optimally choose technology state-by-state for time \( t + 1 \). The technology shock profile \( A_{t+1} \) satisfies

\[
M_{t,t+1} K_{t+1}^\alpha - \frac{\xi_t}{\lambda_t} \nu A_{t+1}^{(1-\alpha)(\nu-1)} \theta_{t+1}^{\nu(\alpha-1)} = 0. \tag{14}
\]
The above equation describes a tradeoff that relates to every possible state at time $t+1$: (1) It can be optimal to choose a high $A_{t+1}$ in states in which it is easier to produce (i.e., when $\theta_{t+1}$ is high). Thus, technology chosen moves in the same direction as the underlying process. (2) It can be optimal to choose a high $A_{t+1}$ in states with high contingent claim prices (states of high marginal utility, i.e., when $M_{t,t+1}$ is high). Thus, technology chosen moves reversely to the underlying process. Which effect dominates depends partly on risk aversion as it considerably drives consumption smoothing across states. In our calibrations, the second effect dominates the first effect. That is, the risk averse agent choose a state contingent productivity that moves counter to the exogenous technology process as an “insurance” against exogenous fluctuations.

2 Inspecting the Mechanism through Log-linearization

This section summarizes the main results from a log-linearization exercise that allows to examine the mechanism behind technology choice. Appendix A contains the proofs and additional details of the log-linearization. By convention, the percentage deviation of variable $x_t$ from its steady state value ($\bar{x}$) is defined as $\hat{x}_t = \log x_t - \log \bar{x}$. For example, the underlying technology shock process can be rewritten as $\hat{\theta}_t = \rho \hat{\theta}_{t-1} + \epsilon_t$ where $\epsilon \sim N(0, \sigma^2)$.

2.1 Risk Aversion Sensitive Real Business Cycles

Before studying the economy with technology choice, we first recall the case of standard production without technology choice in the following corollary.

Corollary 1. For the standard production function under recursive preferences, the percentage deviations of utility, consumption, and investment from the steady state can be written as

$$\hat{U}_t = \bar{U}_K \hat{K}_t + \bar{U}_\theta \hat{\theta}_t.$$
\[ \hat{C}_t = \tilde{C}_K \hat{K}_t + \tilde{C}_\theta \hat{\theta}_t, \]
\[ \hat{I}_t = \tilde{I}_K \hat{K}_t + \tilde{I}_\theta \hat{\theta}_t, \]

where coefficients \( \tilde{U}_K, \tilde{U}_\theta, \tilde{C}_K, \tilde{C}_\theta, \tilde{I}_K, \) and \( \tilde{I}_\theta \) represent constants that depend on EIS (\( \psi \)) but are independent of risk aversion (\( \gamma \)).

Substituting the underlying technology shock process into the above equations gives
\[ \hat{U}_t = \tilde{U}_K \hat{K}_t + \tilde{U}_\theta \hat{\theta}_{t-1} + \tilde{U}_\theta \epsilon_t, \]
\[ \hat{C}_t = \tilde{C}_K \hat{K}_t + \tilde{C}_\theta \hat{\theta}_{t-1} + \tilde{C}_\theta \epsilon_t, \]
\[ \hat{I}_t = \tilde{I}_K \hat{K}_t + \tilde{I}_\theta \hat{\theta}_{t-1} + \tilde{I}_\theta \epsilon_t. \]

Since as shown in Corollary 1 all coefficients depend on EIS (\( \psi \)) but are independent of risk aversion (\( \gamma \)), it is evident that risk aversion has no first-order effects on macroeconomic quantities.\(^3\) Consequently, macroeconomic quantities are only risk (\( \epsilon_t \)) sensitive but not risk aversion (\( \gamma \)) sensitive. This feature of the standard RBC model represents the root of the separation between asset prices and macroeconomic quantities highlighted in Tallarini (2000). The separation appears to suggest that in a model economy we can fit asset prices by adjusting risk aversion without negatively affecting the fit to macroeconomic quantities. For example, Tallarini (2000) chooses \( \gamma = 100 \) to match asset prices. However, as shown below, this separation does not hold in our model with technology choice.

**Corollary 2.** In the production economy with technology choice under recursive preferences, let the percentage deviations of utility, consumption, and investment from the steady state be
\[ \hat{U}_t = U_K \hat{K}_t + U_A \hat{A}_t + U_\theta \hat{\theta}_t, \]
\[ \hat{C}_t = C_K \hat{K}_t + C_A \hat{A}_t + C_\theta \hat{\theta}_t, \]
\[ \hat{I}_t = I_K \hat{K}_t + I_A \hat{A}_t + I_\theta \hat{\theta}_t, \]

\(^3\)One exception is with CRRA utility in which EIS mechanically equals to \( 1/\gamma \).
where coefficients $U_K, U_A, U_\theta, C_K, C_A, C_\theta, I_K, I_A, I_\theta$ are constants. These coefficients depend on EIS ($\psi$) but are independent of risk aversion ($\gamma$) and technology choice curvature ($\nu$).

Does this imply that macroeconomic quantities are insensitive to risk aversion in a production economy with technology choice? No. In our model, macroeconomic quantities are sensitive to risk aversion through technology choice, as illustrated below.

**Corollary 3.** The percentage deviation of technology choice from the steady state is given by

$$\hat{A}_{t+1} = A_{\theta_1} \hat{\theta}_t + A_{\theta_2} \hat{\theta}_{t+1},$$

(16)

where $A_{\theta_1}$ and $A_{\theta_2}$ are constants satisfying $A_{\theta_1} + \rho A_{\theta_2} = \rho$, $A_{\theta_1} \geq 0$, and $A_{\theta_2} \leq 1$. Moreover, coefficients $A_{\theta_1}$ and $A_{\theta_2}$ directly depend on risk aversion ($\gamma$) and technology choice curvature ($\nu$), which can be seen from

$$A_{\theta_1} = \frac{-\rho(1 - \alpha) + \frac{\psi}{\psi} (C_A + C_\theta) + \rho(\gamma - \frac{1}{\psi})(U_A + U_\theta)}{(1 - \alpha)(\nu - 1) + \frac{1}{\psi} C_A + (\gamma - \frac{1}{\psi})U_A};$$

$$A_{\theta_2} = \frac{(1 - \alpha)\nu - \frac{1}{\psi} C_\theta + (\frac{1}{\psi} - \gamma)U_\theta}{(1 - \alpha)(\nu - 1) + \frac{1}{\psi} C_A + (\gamma - \frac{1}{\psi})U_A}.$$ 

(17)

Corollary (3) states that the technology chosen for time $t + 1$, $A_{t+1}$, can be viewed as a weighted average of the current underlying shock and a future underlying shock. For example, when $\rho$ is close to 1 then $A_{\theta_1}$ and $A_{\theta_2}$ are simple weights on the current and next period’s underlying shocks. Note that $A_{\theta_1}$ is nonnegative, reflecting the fact that choosing a high productivity technology is easier when the underlying technology shocks ($\hat{\theta}_t$) is also high, since the exogenous process is persistent. Also, when $\rho$ is zero so is $A_{\theta_1}$. This is because when $\rho$ is zero there is no information in $\hat{\theta}_t$ about $\hat{\theta}_{t+1}$. However, $A_{\theta_2}$ can be negative due to risk aversion, which could lead the agent to choose a shock opposite to the underlying one.
Substituting (16) into (15) yields

\[
\hat{U}_t = U_K \hat{K}_t + \rho(U_A + U)\hat{\theta}_{t-1} + (U_A A_\theta + U)\epsilon_t, \tag{18}
\]

\[
\hat{C}_t = C_K \hat{K}_t + \rho(C_A + C)\hat{\theta}_{t-1} + (C_A A_\theta + C)\epsilon_t, \tag{19}
\]

\[
\hat{I}_t = I_K \hat{K}_t + \rho(I_A + I)\hat{\theta}_{t-1} + (I_A A_\theta + I)\epsilon_t. \tag{20}
\]

Therefore, macroeconomic quantities depend on both risk ($\epsilon_t$) and risk aversion ($\gamma$) simply because $A_{\theta_2}$ depends on risk aversion. Specifically, risk aversion drives technology choice, which then drives macroeconomic quantities. Hence, the separation observed in Tallarini (2000) does not hold in our model.

### 2.2 Amplification Mechanism

Next, we explore why and when technology choice amplifies underlying shocks. From (17), we obtain the following results:

**Corollary 4.** When $\psi < 1$, $A_{\theta_1}$ ($A_{\theta_2}$) monotonically decreases (increases) with curvature of technology choice ($\nu$) and increases (decreases) with risk aversion ($\gamma$), i.e.,

\[
\frac{\partial A_{\theta_1}}{\partial \nu} < 0 \text{ and } \lim_{\nu \to +\infty} A_{\theta_1} = 0, \tag{21}
\]

\[
\frac{\partial A_{\theta_2}}{\partial \nu} > 0 \text{ and } \lim_{\nu \to +\infty} A_{\theta_2} = 1, \tag{22}
\]

\[
\frac{\partial A_{\theta_1}}{\partial \gamma} > 0 \text{ and } \lim_{\gamma \to +\infty} A_{\theta_1} = \rho \left(1 + \frac{U_\theta}{U_A}\right), \tag{23}
\]

\[
\frac{\partial A_{\theta_2}}{\partial \gamma} < 0 \text{ and } \lim_{\gamma \to +\infty} A_{\theta_2} = -\frac{U_\theta}{U_A}. \tag{24}
\]

These results are intuitive. When $\nu \to +\infty$, it is impossible to alter the technology shock, hence we obtain $A_{t+1} = \theta_{t+1}$ as $A_{\theta_1} = 0$ and $A_{\theta_2} = 1$. Since the technology choice rule is set one period ahead, the higher the risk aversion, the more weight the agent puts on the current underlying shock ($\theta_t$), which is an observable, instead of putting weight on the
future underlying shock \((\theta_{t+1})\), which is uncertain. When the agent is infinitely risk averse \((\gamma \to +\infty)\), her utility is predictable, as can be seen from (18).\(^4\)

From (19) and (20), conditional volatilities of consumption and investment are

\[
\begin{align*}
\text{Var}_{t-1}(\hat{C}_t) &= (C_A A_{\theta_2} + C_{\theta})^2 \sigma^2, \quad (25) \\
\text{Var}_{t-1}(\hat{I}_t) &= (I_A A_{\theta_2} + I_{\theta})^2 \sigma^2, \quad (26)
\end{align*}
\]

respectively. As shown in the Appendix A, \(C_A A_{\theta_2} + C_{\theta} > 0, C_A > 0, I_A > 0,\) and \(I_{\theta} < 0\). Therefore, equation (25) implies that \(\text{Var}_{t-1}(\hat{C}_t)\) increases with \(A_{\theta_2}\). We learn that the larger the technology choice (when \(A_{\theta_2}\) is small), the smoother is the consumption stream. In fact, for plausible parameter values we can get \(A_{\theta_2} < -1\). This shows that the agent chooses amplified shocks that move counter to the underlying shocks to smooth her consumption. In this case, consumption is particularly smooth while investment is more volatile, as seen from equation (26). Hence, technology choice is simultaneously a smoothing device and an amplification mechanism. To see this, compare the conditional volatilities of underlying technology shocks to the shocks chosen:

\[
\text{Var}_{t}(\hat{A}_{t+1}) - \text{Var}_{t}(\hat{\theta}_{t+1}) = (A_{\theta_2}^2 - 1) \sigma^2. \quad (27)
\]

Based on Corollary 4, since \(A_{\theta_2}\) cannot be larger than 1, amplification occurs only when \(A_{\theta_2} < -1\). Together with (17), we reach the following condition for amplification to occur.

**Corollary 5.** When \(\psi < 1\), optimal technology choice amplifies the underlying shocks, i.e., \(A_{\theta_2} < -1\), if risk aversion and technology choice curvature satisfy

\[
(1 - \gamma \psi)(U_\theta - U_A) + \psi(1 - \alpha)(2\nu - 1) < C_\theta - C_A. \quad (28)
\]

Note that the right hand side of (28) is independent of risk aversion \((\gamma)\) and technology.

\(^4\)This is the case of utility smoothing discussed in Backus, Routledge and Zin (2013).
choice curvature \((\nu)\). Since \(U_0 > U_A\), condition (28) holds when \(\nu\) is small and \(\gamma\) is large. Intuitively, when it is relatively easy to modify underlying technology shocks and the agent is risk averse enough, then amplification occurs. In this case, since the shocks chosen are larger, output is more volatile, comparing with an economy without technology choice. Investment is more volatile as well since the volatility of output is carried over. Thus the agent chooses larger shocks that move counter to the underlying technology shocks, which smooth her consumption at the expense of investment and output volatilities.

### 2.3 Amplification and Investment Returns

Macroeconomic amplification is useful in generating a sizable equity premium, because amplification increases both price of risk and return volatility. First, when shocks chosen are amplified, the marginal rate of transformation becomes more volatile. From the pure production perspective, this implies a more volatile pricing kernel. Therefore, the price of risk increases in our model. Second, the impact of amplification on return volatility can be understood from the investment returns. Define the investment returns, \(R^i_{t+1}\), as follows

\[
R^i_{t+1} = \alpha A^{1-\alpha} K^{\alpha-1} g_t' + g_t' \left[ \frac{1 - \delta + a_2}{a_1} \left( \frac{I_{t+1}}{K_{t+1}} \right)^\chi + \frac{1}{\chi - 1} I_{t+1} \right].
\]

(29)

The first term of the right hand side can be viewed as dividend yields while the second term is capital gains. So, the conditional volatility of investment returns depends on the volatilities and covariance of these two terms, which are governed by the technology chosen. Clearly, the volatility of the first term increases with more volatile shocks chosen. Additionally, since \(I_A > 0\), investment volatility increases with the shock volatility. Therefore, the second term is more volatile when amplification occurs. Lastly, since \(I_A > 0\), investment increases with shocks chosen. Thus these two terms are positively correlated and the covariance also increases with the shock volatility.
3 The Numerical Solution

It is numerically challenging to solve our model since the number of state-by-state equilibrium conditions is infinite when shocks are continuously distributed, as can be seen from (14).\(^5\) Value function iteration is not applicable to our economy unless one is willing to highly discretize the model. Instead, we solve our model with perturbation methods (Judd and Guu (1997) and Schmitt-Grohé and Uribe (2004)). Importantly, the perturbation method has proven to be highly accurate for recursive preferences (Caldara et al. (2012)). Moreover, the perturbation method provides a perfect solution to our model when uncertainty is continuously distributed. In addition, the perturbation method also allows to include additional variables of interest in the optimization problem easily. For example, we add basic asset pricing equations to identify the risk-free rate and stock price,\(^6\)

\[ \mathbb{E}_t [M_{t,t+1}(P_{t+1} + C_{t+1})] = P_t, \quad (30) \]
\[ \mathbb{E}_t [M_{t,t+1} R_{f,t+1}] = 1. \quad (31) \]

Further, we can introduce an auxiliary equation that accommodates recursive preferences

\[ eu_t = \mathbb{E}_t \left[ U_{t+1}^{1-\gamma} \right]. \quad (32) \]

The complete equilibrium conditions are summarized in Appendix A.

The main idea of the perturbation method is to approximate a function around its non-stochastic steady state (when \( \sigma = 0 \)) via a Taylor expansion. Therefore, the main task is to compute the steady state and partial derivatives around the steady state. Let us rewrite

\(^5\)We note that this is similar to the identification problem in Belo (2010). Cochrane (1993) and Jermann (2010) consider a setting with finite states to handle the identification problem.

\(^6\)In the interest of numerical accuracy, we do not compute stock returns from (29) directly.
equilibrium conditions other than the technology choice equation (14) as

$$\mathbb{E}_t F (X_t, X_{t+1}, Z_t, Z_{t+1}) = 0, \quad (33)$$

where $X_t = \{K_t, A_t, \theta_t, \sigma\}$ and $Z_t = \{U_t, C_t, I_t, \mu_t, \lambda_t, K_{t+1}, P_t, R_{f,t+1}, \varepsilon u_t\}$. We apply the standard perturbation method to solve the partial derivatives to approximate $Z_t$. However, note that the equilibrium condition in (14) holds state-by-state, instead of by expectation as in (33). Again, standard perturbation procedures are not applicable for this equilibrium condition.

Let us rewrite the equilibrium condition (14) as

$$G(K_t, A_t, \theta_t, \theta_{t+1}, \sigma) \equiv M_{t,t+1} K_t^\alpha - \frac{\xi_t}{\lambda_t} A_t^{(1-\alpha)(\nu-1)} \theta_{t+1}^{\nu(\alpha-1)} = 0. \quad (34)$$

For illustration, consider the first-order partial derivatives evaluated at the steady state of the above equation. For example, the first order partial derivative with respect to $\sigma$ gives

$$G^{(0,0,0,0,1)} + G^{(0,0,0,1,0)} \varepsilon_{t+1} = 0, \quad (35)$$

where $G^{(0,0,0,0,1)}$ and $G^{(0,0,0,1,0)}$ are direct partial derivatives. Since this holds state-by-state, the constant term and the coefficient of $\varepsilon_{t+1}$ must be zeros. That is,

$$G^{(0,0,0,0,1)} = 0, \quad (36)$$

$$G^{(0,0,0,1,0)} = 0. \quad (37)$$

As we obtain the first-order partial derivatives with respect to $\theta_{t+1}$ and $\sigma$ simultaneously, it is, therefore, redundant to compute the partial derivative with respect to $\theta_{t+1}$ directly. We can similarly compute the first-order partial derivatives with respect to $K_t, A_t, \theta_t$. The second-order partial derivatives can be also computed following the same procedure.
We implement a second-order perturbation of the model and find that it performs well and introduces only small Euler equation errors (See Appendix C). As in Aruoba, Fernández-Villaverde and Rubio-Ramírez (2006), the model is simulated for 1000 paths. Starting from the non-stochastic steady state, each path has 300 periods, and the first 100 periods are a burn-in to eliminate the transition from the deterministic steady state to the ergodic distribution. One unit of time equals a quarter of a calendar year. Hence, each simulated path is 50 years long, which is roughly identical to the length of the empirical sample.

4 Calibration Approach

4.1 Empirical Data

The model is calibrated to US data for the period 1964 to 2011. The annual market return and the risk-free rate data are from the annual Fama-French factor data file from Kenneth French’s website. Since we abstract from capital structure choice, we calculate the unlevered market return from the data by assuming a debt-to-equity ratio of 0.5 (see, e.g., Barro (2006)). Macroeconomic data are collected from the NIPA tables. Details about the data are at Appendix B. The key moments of macroeconomic quantities and asset prices are reported in Table 2. The volatilities of output, consumption, and investment are computed employing the Hodrick-Prescott filter.

4.2 Parameters

We use standard parameters from the literature and summarize these in Table 1.\footnote{We acknowledge that it would be ideal to calibrate all parameters. However, it is infeasible. So, we directly build on the success of those commonly used parameter values in the literature.} The capital share ($\alpha$) is set at 0.36, which is similar to the capital share in Boldrin, Christiano and Fisher (2001) and Kaltenbrunner and Lochstoer (2010). The quarterly depreciation rate ($\delta$) is computed from the average investment/capital ratio over 1964-2011 from the NIPA
tables as 0.026. We follow Belo (2010) and employ a technology choice curvature ($\nu$) of 1.04. Together with $\nu$, the persistence of the technology shock ($\rho$) is set at 0.99 to ensure that the technology chosen in the main model is similar to the productivity process observed in the data. The volatility of the technology shock ($\sigma$) is 0.012, which is set to fit the volatility of output.

The subjective time discount factor ($\beta$), the curvature of capital adjustment costs ($\chi$), the EIS ($\psi$), and the relative risk aversion ($\gamma$) are calibrated to match asset prices. Based on our calibrations, we set $\beta = 0.988$. The curvature of capital adjustment costs ($\chi$) is set to 1.5, which is within the range employed in the literature. The EIS ($\psi$) of 0.03 is similar to Yogo (2004, 2006) and Gomes, Kogan and Yogo (2009). In the main model, the relative risk aversion ($\gamma$) is set to 7.5, which is similar to Bansal and Yaron (2004).

5 Results

5.1 CRRA Utility

We first examine the CRRA utility case. Panels A and B of Table 2 report calibration results from models with and without technology choice. To see the impact of risk aversion, in addition to the main case ($\gamma=7.5$), we also examine a case with low risk aversion ($\gamma=2$). First, the CRRA model with technology choice and high risk aversion (Model (1)) matches both asset prices and macroeconomic quantities reasonably well. This implies that it might be indeed sufficient for many macroeconomic models to adopt a simple preference specification, such as CRRA, instead of more sophisticated preferences. For example, the model generates an equity premium of 2.82% per year, which is close to the unlevered equity premium (3.79%) observed in the data. The volatility of stock returns, 9.43% per year, is also close to the data. Model (1) also yields a Sharpe ratio of 0.3, which is very close to

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8For example, Jermann (1998), and Boldrin, Christiano and Fisher (2001) choose 0.23; Kaltenbrunner and Lochstoer (2010) set $\chi$ as 0.7 and 18.
that (0.31) in the data. In contrast, the CRRA model without technology choice (Model (3)) generates a tiny equity premium of 0.28% per year and the volatility of stock returns is also too small. One might wonder why Model (1) generates an equity premium larger than Model (3), given a slightly smaller consumption volatility in Model (1). This is rooted in the fact that price of risks in Model (1) is much higher, because its marginal rate of transformation is more volatile due to the amplification mechanism. Second, comparing Models (1) and (2), we see that volatilities of output and investment increase with risk aversion while the volatility of consumption decreases with risk aversion, as predicted in (25). However, there is no significant change between Models (3) and (4). Hence, from a macroeconomic perspective, technology choice makes macroeconomic quantities sensitive to risk aversion. This differs from standard production models in which risk aversion has no first order impact on macroeconomic quantities (as shown in Models (3) and (4)).

Third, technology choice amplifies the volatilities of output and investment when risk aversion is large, comparing Models (1) and (2). Under our parameter values, the agent chooses amplified and counter to the underlying shocks \( A_{\theta 1} = 2.37 \) and \( A_{\theta 2} = -1.39 \). In fact, in Model (1), volatility of output is 1.62%, which is larger than the volatility of underlying shock (1.2%). But, there is no amplification when risk aversion is small in Model (2).

Comparing the volatility of the underlying shock from our calibration, for example, to Boldrin, Christiano and Fisher (2001) we note that they employ a permanent shock with a standard deviation of 1.8% per quarter whereas ours is 1.2% per quarter. Kaltenbrunner and Lochstoer (2010) even use a shock with a standard deviation as high as 4.1%. From our calibrations we see that with amplification through technology choice, the volatility of exogenous technology shocks required to reproduce realistic business cycle fluctuations is lower than that in a standard RBC model. We, therefore, provide one potential solution to concerns over unrealistically large shocks needed in standard RBC models (Burnside and Eichenbaum (1996)). Because of the amplification mechanism, output becomes more volatile

\[ \text{In fact, since } EIS = 1/\gamma \text{ for CRRA utility, macroeconomic quantities are not completely independent of risk aversion. Yet, the difference between Models (3) and (4) appears to be trivial.} \]
when the risk averse agent chooses larger shocks that move counter to the underlying shocks to smooth her consumption. Hence, investment becomes much riskier as the volatility of output carries over to that of investment. From an asset pricing perspective, Model (1) also confirms the intuition from Section 2.3 that amplified investment risks together with higher price of risks generate a sizable equity premium.

5.2 Recursive Preferences

Panels A and B of Table 3 report simulation results for the models with recursive preferences. We learn that Model (1) (recursive preferences with technology choice and a high risk aversion) generates asset prices and macroeconomic quantities in line with the data. Similar to the case of CRRA utility, we observe that technology choice makes macroeconomic quantities sensitive to the risk aversion. When risk aversion is large ($\gamma = 7.5$), a significant amplification effect occurs in Model (1). Actually, under our parameter values, we have $A_{\theta 1} = 2.41$ and $A_{\theta 2} = -1.43$. That is, an amplified shock that moves counter to the underlying technology shock. One might wonder whether the technology chosen in the model shares similar properties with the productivity process observed in the data. Table 4 compares the statistical properties of technology chosen in the main model with TFP observed in the data.\(^{10}\) Specifically, we are interested in its autocorrelation and the cross-correlations with output, investment, and consumption. Overall, the technology chosen in the main model matches the data fairly well for the first-order autocorrelation and cross-correlations. For example, technology chosen in the main model shows an AR(1) of 0.78, close to the autocorrelation observed in the data (0.83). Also, the contemporaneous correlation of technology chosen and output, investment, and consumption are 0.89, 0.78, and 0.82, respectively, while they are 0.80, 0.78, and 0.68 in the data. We note that the higher order correlations are more stable in the main model than those in the data.\(^{11}\)

\(^{10}\)The total factor productivity data are obtained from the website of the Federal Reserve Bank of San Francisco.

\(^{11}\)Two reasons could contribute to this discrepancy. First, the technology choice is assumed to be one-period ahead in the model, so the higher order correlations stabilize after one-period. Second, the HP filter
Since in the model shocks chosen are amplified and macroeconomic variables are risk aversion sensitive, one might wonder whether this implies large welfare costs associated with business fluctuations. Appendix E reports the welfare costs of various preferences with and without technology choice. Disciplined by the match of the model to asset price data, we see welfare costs of 0.97% consumption in the main model. This value is an order of magnitude larger than Lucas’ estimate but smaller than Tallarini’s estimate; it is close to the estimates in Alvarez and Jermann (2004) and Barro (2009).

Compared to the CRRA case, the model with recursive preferences generates a larger and more volatile stock return. For example, the equity premium is 3.35% per year and the volatility of stock returns is 10.84%, while these quantities are 2.82% and 9.43%, respectively, in the case of CRRA utility. The risk-free rate is very smooth, with a volatility of 1.38%. This is because state-dependent technology choice mainly increases variation of marginal rate of substitution across states instead of over time. But why do recursive preferences generate a larger equity premium than CRRA utility? First, note that recursive preferences are state-dependent and technology choice is also state-dependent, hence, there is an interplay. Therefore, recursive preferences enhance the sensitivity of investment to risk and risk aversion. For example, volatility of investment under recursive preferences is 6%, which is larger than that under CRRA utility (5.04%). Second, recursive preferences allow a separation between risk aversion and the elasticity of intertemporal substitution. Since technology choice is one-period ahead and technology shocks are transitory, a smaller EIS helps to generate a larger equity premium. As shown in Appendix D.1, consumption volatility increases with EIS as the agent is less averse to intertemporal consumption variations when EIS is large. For example, when we set EIS=1.5, consumption volatility is as large as 3.87% per quarter, which is much larger than that observed in the data, though such highly

\[ \text{Lucas (1987, 2003) finds that welfare costs associated with business fluctuations are trivially small, about 0.05-0.1\% of consumption. Tallarini (2000) finds instead a very high welfare cost, about 26\% of consumption loss, when risk aversion is as high as 100. Barro (2009) finds that it costs only 1.6\% of GDP to remove consumption volatilities when risk aversion is 4. Alvarez and Jermann (2004) estimate that eliminating business cycle fluctuations costs only 0.08-0.49\% of consumption.} \]
volatile consumption also leads to a sizeable equity premium (2.25% per year).

Similar results from exercises in Tables 2 and 3 appear to imply that in a risk aversion sensitive economy, the utility specification is less important than in the standard RBC models. As further explored in Appendix D.2, an external habit formation specification also generates results similar to our previous calibrations in Table 3. Therefore, our results show low sensitivity to the preferences specifications. This might revalidate the simple preference specifications often used in the macroeconomics literature, e.g., CRRA, instead of more sophisticated preferences once we allow for endogenous technology choice.

5.3 Impulse Responses

To complement the loglinearization results, we now inspect the impulse responses of key variables. Figure 1 compares the impulse responses of output, investment, consumption, and capital stock from models of recursive preferences with and without technology choice. When there is no technology choice, after a positive shock at time 1, output, investment, and consumption increase at time 1 and then stabilize as the shock decays. However, when technology choice is allowed for, the risk averse agent chooses amplified shocks that move counter to the underlying shocks. That is, since the exogenous shock at time 0 is zero, for a positive exogenous shock at time 1 (1.2%), the risk averse agent chooses a negative and larger shock at time 1 (−1.72%), which leads to a temporary drop in output. Consumption rises because future shocks chosen will be high, given the persistence in shocks. Investment at time 1 decreases since output decreases and consumption increases. The decrease in investment also can be understood as follows. Expected consumption is high (relative to the steady state) because of the expected rise in the technology choice. This implies that the future marginal utility is expected to be low relative to today’s marginal utility, leading to a fall in marginal $q$ and in investment. Note that consumption increases first at time 1 and then again at time 2 in Figure 1(f). Comparing to the case without technology choice, consumption with technology choice is smoother. Thus, technology choice allows consumption smoothing

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relative to the standard RBC model. The shocks chosen become an AR(1) process after time 1, so output, investment, and consumption from the models with and without technology choice converge. Figure 1(b) demonstrates the amplification of shocks. This can also be seen from the more volatile investment in Figure 1(d), while Figure 1(f) shows the consumption smoothing role of technology choice. Therefore, our agent pays for her smooth consumption with volatile investment. This mechanism of our model with technology choice is the key to understand why our model matches the data well.

5.4 Different Technology Choice Abilities

Technology choice curvature $\nu$ governs the extent of technology choice. Specially, it is easier to modify the underlying shocks when $\nu$ is small. Table 5 compares models with different abilities to alter technology. As $\nu$ increases from 0.5 in Model (1) to 5 in Model (3), it is more difficult to smooth consumption via technology modifications. Hence, volatility of consumption increases with $\nu$. Meanwhile, the volatility of output decreases from Model (1) to Model (3). Actually, strong amplifications occur when $\nu$ is small. For example, quarterly output volatility is 1.93% and 1.58% in Models (1) and (2), respectively, comparing with 0.94% in Model (3). Putting output and consumption dynamics together, we see that the volatility of investment declines significantly with $\nu$.

Turning to the asset prices, Panel B shows that technology choice curvature alone mainly affects asset volatilities. For example, the Sharpe ratio only increases from 0.28 in Model (3) to 0.30 in Model (1), but the annual volatility of equity returns jumps from 4.07% in Model (3) to 13.87% in Model (1). This reflects the large increase in investment volatility from Model (3) to Model (1). As a result, Model (1) presents an annualized equity premium of 4.09%, comparing with 1.13% in Model (3).
5.5 Capital Adjustment Costs and Technology Choice

In this economy, the agent has two different yet related consumption smoothing devices. Technology choice mainly smooths consumption across states while consumption smoothing over time is achieved by investment. These two devices are complementary as technology choice makes investment more state sensitive. Therefore, when the capital adjustment costs are low, the agent can intensify the use of both devices. Figure 2 confirms this intuition. $A_{\theta_2}$ decreases with the capital adjustment costs. That is, the agent modifies underlying shocks more when the capital adjustment costs are lower, which leads to significant amplifications. Consequently, consumption is smoother while investment is more volatile. Table 6 further compares the macro quantities and asset prices of models with different capital adjustment costs. This table examines cases with capital adjustment costs as high as $\chi=0.23$ (Jermann (1998) and Boldrin, Christiano and Fisher (2001)) or as low as $\chi=18$ (Kaltenbrunner and Lochstoer (2010)). When there is no technology choice, consumption volatility increases while investment is less volatile when capital adjustment costs increase from Model (6) to Model (4). However, even with very large capital adjustment costs ($\chi=0.23$), the equity premium is still too small (1.37%) together with too smooth investment and a small Sharpe ratio (0.14). In contrast, when technology choice is allowed, both output and investment are more volatile while consumption is less volatile when the capital adjustment costs decrease from Model (1) to (3). Significant amplification occurs when capital adjustment costs are not too large. For example, when the capital adjustment costs are very small ($\chi=18$), the volatilities of output and investment are as high as 7.67% and 33.62%, respectively. However, the equity premium is small in this case (1.30%). Therefore, disciplined by moments of asset prices, capital adjustment costs can not be too small, though low capital adjustment costs are useful in generating amplifications. When capital adjustment costs are large, investment is too smooth in Model (1) although large capital adjustment costs are useful in generating a large equity premium (5.23% per year). Overall, with moderate capital adjustment costs, the main model, Model (2), fits both asset prices and macroeconomic quantities quite well.
5.6 Return Predictability

Despite of debates over measurement errors and econometric methodology problems, many studies show that future returns can be predicted by the dividend-price ratio (see Cochrane (2008a) for a recent summary). We, therefore, assess the model for its ability to replicate this feature of the data. Table 7 reports predictability regressions of the dividend-price ratio. The empirical data are the NYSE/AMEX/NASDAQ annual market returns over 1925-2011 obtained from CRSP, deflated by the CPI. The replication regression results are close to Cochrane (2008a). In particular, we see the predictability of dividend-price ratio over the excess returns in Panel B. This implies that predictability of future returns in Panel A is driven by the predictability of equity premium, instead of the risk-free rate. This feature challenges many production-based asset pricing models. For example, Campanale, Castro and Clementi (2010) show the lack of predictability over the excess returns when agents are endowed with disappointment aversion, though they find predictability over future returns. The right panels of Table 7 demonstrate that the model with recursive preferences and technology choice replicates the return predictability very well, especially the predictability over the excess returns. The reason is that in our model, the equity premium is generated through volatile investment across states (and over time) while the risk-free rate is relatively smooth.

6 Conclusions

We model a risk aversion sensitive RBC economy driven by the interplay between endogenous state-contingent technology choices and risk aversion. We build on the insight in Cochrane (1993) that the standard production function only allows firms to transform output in fixed proportions across states. Specifically, Cochrane (1993) proposes to allow firms to choose technology shocks endogenously to solve this singularity problem. In a general equilibrium setting, this implies that the agent smooths consumption across states through technology
choices. Therefore, risk aversion drives production substitution and through that channel it affects macroeconomic variables such as consumption, investment, and output. The separation between asset prices and macroeconomic quantities observed in standard RBC models, e.g., Tallarini (2000), does not hold in this economy, as both asset prices and macroeconomic quantities depend on the risk aversion. In fact, under plausible parameter values, the representative agent even chooses amplified shocks that move counter to the underlying technology shocks to smooth her consumption. This provides amplification mechanism to the macroeconomic quantities. Such amplification mechanism increases the investment risks and elevates the price of risks, which generates a sizable equity premium. We compare various preferences in this economy, for example, CRRA, recursive preferences, and external habit formation. All these preferences reasonably match moments of asset prices and business cycle statistics. Therefore, in the economy with technology choice, risk aversion has first order impact on macroeconomic quantities, but the utility specification appears less important. The welfare costs analysis shows that business fluctuations cost about 1% of consumption, which is important though small.
References


Figure 1: Impulse Response Functions

This figure depicts the impulse response functions after a positive, one-standard-deviation underlying technology shock at time 1 from two versions of the model with recursive preferences: with (Choice) and without technology choice (No Choice). Each variable is plotted as a percentage deviation from its steady state value.
Figure 2: Capital Adjustment Costs and Technology Choice

Figure (a) depicts the sensitivity of technology chosen to the underlying shock as a function of capital adjustment costs. Figure (b) and (c) plot volatilities of consumption and investment as functions of capital adjustment costs. All other parameters are set as those in the main model.
This table summarizes parameters used in the calibration. The time unit is a quarter.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Fixed parameters</strong></td>
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<td>( \alpha )</td>
<td>Elasticity of capital</td>
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<td>( \delta )</td>
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<td>( \gamma )</td>
<td>Relative risk aversion</td>
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<td>( \psi )</td>
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<td>Curvature of the capital adjustment costs</td>
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<td>( S )</td>
<td>Steady state surplus consumption ratio</td>
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<tr>
<td>( \phi )</td>
<td>Habit persistence</td>
<td>0.966</td>
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Table 2: Calibrations: Constant Relative Risk Aversion

This table summarizes key moments of macroeconomic quantities (Panel A) and asset prices (Panel B) from calibrations of CRRA utility with and without technology choice, under different risk aversion ($\gamma$). The empirical data are from the NIPA tables and the annual Fama-French factor data file over 1964-2011. Unlevered returns are computed as assuming debt/equity=0.5. The macroeconomic quantities are reported as quarterly, while the asset prices are annualized. The volatilities of output, consumption, and investment are computed from the Hodrick-Prescott filter. All moments are reported in percentages, except the Sharpe ratio.

<table>
<thead>
<tr>
<th></th>
<th>Model (1)</th>
<th>Model (2)</th>
<th>Model (3)</th>
<th>Model (4)</th>
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<tr>
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<td></td>
<td>Technology Choice</td>
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<tr>
<td>Returns</td>
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<tr>
<td>$\gamma = 7.5$</td>
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<td>0.89</td>
<td>0.97</td>
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<tr>
<td>$\gamma = 2$</td>
<td>0.77</td>
<td>0.82</td>
<td>0.85</td>
<td>0.89</td>
</tr>
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</table>

Panel A Macroeconomic quantities

Volatility of output:
$\sigma(Y)$ 1.57 1.62 0.89 0.97 0.97
Volatility of consumption:
$\sigma(C)$ 0.84 0.77 0.82 0.85 0.89
Volatility of investment:
$\sigma(I)$ 7.56 5.04 1.18 1.33 1.21

Panel B Asset prices

Mean and volatility of the equity returns:
$E[R]$ 7.27 5.37 5.09 4.86 4.68 4.86
$\sigma(R)$ 18.21 12.16 9.43 1.33 2.86 1.73
Mean and volatility of the risk-free rate:
$E[R_f]$ 1.58 2.27 4.79 4.40 4.81
$\sigma(R_f)$ 2.14 1.40 1.00 0.15 0.07
Equity premium:
$E[R - R_f]$ 5.69 3.79 2.82 0.07 0.28 0.05
Sharpe ratio:
$E[R - R_f]/\sigma(R)$ 0.31 0.31 0.30 0.05 0.09 0.03
Table 3: Calibrations: Recursive Preferences with a Small EIS

This table summarizes key moments of macroeconomic quantities (Panel A) and asset prices (Panel B) from calibrations of recursive preferences models with and without technology choice, under different risk aversion (γ) and a small EIS (ψ=0.03). The empirical data are from the NIPA tables and the annual Fama-French factor data file over 1964-2011. Unlevered returns are computed as assuming debt/equity=0.5. The macroeconomic quantities are reported as quarterly, while the asset prices are annualized. The volatilities of output, consumption, and investment are computed from the Hodrick-Prescott filter. All moments are reported in percentages, except the Sharpe ratio.

<table>
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<td><strong>Panel A Macroeconomic quantities</strong></td>
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<td>γ = 7.5 γ = 2</td>
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<tr>
<td>Volatility of output</td>
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<tr>
<td>σ(Y)</td>
<td>1.57</td>
<td>1.58 0.99</td>
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<tr>
<td>Volatility of consumption</td>
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<tr>
<td>σ(C)</td>
<td>0.84</td>
<td>0.72 0.75</td>
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<tr>
<td>Volatility of investment</td>
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<tr>
<td>σ(I)</td>
<td>7.56</td>
<td>6.00 2.65</td>
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<tr>
<td><strong>Panel B Asset prices</strong></td>
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<tr>
<td>Mean and volatility of the equity returns</td>
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<td></td>
</tr>
<tr>
<td>E[R]</td>
<td>7.27</td>
<td>5.37 4.89</td>
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<td>12.16 10.84</td>
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<td>E[R_f]</td>
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<td>σ(R_f)</td>
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<td>E[R – R_f]</td>
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<td></td>
</tr>
<tr>
<td>E[R – R_f]/σ(R)</td>
<td>0.31</td>
<td>0.31 0.26</td>
</tr>
</tbody>
</table>
Table 4: Properties of Technology Chosen

This table compares the statistical properties of technology chosen from the main model and TFP observed in the data. The empirical TFP data are from the Federal Reserve Bank of San Francisco and other empirical data are from the NIPA tables, computed from the Hodrick-Prescott filter. Panel A compares the $k^{th}$-order autocorrelations. Panel B-D compare the cross-correlations of technology and output, investment, and consumption, respectively.

<table>
<thead>
<tr>
<th>Panel A</th>
<th>$AR(k)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k$</td>
<td>1</td>
</tr>
<tr>
<td>U.S. Data</td>
<td>0.83</td>
</tr>
<tr>
<td>Main Model</td>
<td>0.78</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B</th>
<th>$Corr(TFP_t, Y_{t+k})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k$</td>
<td>0</td>
</tr>
<tr>
<td>U.S. Data</td>
<td>0.80</td>
</tr>
<tr>
<td>Main Model</td>
<td>0.89</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel C</th>
<th>$Corr(TFP_t, I_{t+k})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k$</td>
<td>0</td>
</tr>
<tr>
<td>U.S. Data</td>
<td>0.78</td>
</tr>
<tr>
<td>Main Model</td>
<td>0.78</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel D</th>
<th>$Corr(TFP_t, C_{t+k})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k$</td>
<td>0</td>
</tr>
<tr>
<td>U.S. Data</td>
<td>0.68</td>
</tr>
<tr>
<td>Main Model</td>
<td>0.82</td>
</tr>
</tbody>
</table>
Table 5: **Calibrations: Recursive Preferences with Different Technology Choice Abilities**

This table summarizes key moments of macroeconomic quantities (Panel A) and asset prices (Panel B) from calibrations of recursive preferences models with different technology choice curvatures ($\nu$). The empirical data are from the NIPA tables and the annual Fama-French factor data file over 1964-2011. Unlevered returns are computed as assuming debt/equity=0.5. The macroeconomic quantities are reported as quarterly, while the asset prices are annualized. The volatilities of output, consumption, and investment are computed from the Hodrick-Prescott filter. All moments are reported in percentages, except the Sharpe ratio.

<table>
<thead>
<tr>
<th></th>
<th>U.S. Data (1964-2011)</th>
<th>Unlevered Returns</th>
<th>Model (1) $\nu = 0.5$</th>
<th>Model (2) $\nu = 1.04$</th>
<th>Model (3) $\nu = 5$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A Macroeconomic quantities</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Volatility of output</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma(Y)$</td>
<td>1.57</td>
<td>1.93</td>
<td>1.58</td>
<td>0.94</td>
<td></td>
</tr>
<tr>
<td>Volatility of consumption</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma(C)$</td>
<td>0.84</td>
<td>0.70</td>
<td>0.72</td>
<td>0.77</td>
<td></td>
</tr>
<tr>
<td>Volatility of investment</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma(I)$</td>
<td>7.56</td>
<td>7.46</td>
<td>6.00</td>
<td>2.52</td>
<td></td>
</tr>
<tr>
<td><strong>Panel B Asset prices</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean and volatility of the equity returns</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E[R]$</td>
<td>7.27</td>
<td>5.37</td>
<td>5.75</td>
<td>5.40</td>
<td>4.97</td>
</tr>
<tr>
<td>$\sigma(R)$</td>
<td>18.21</td>
<td>12.16</td>
<td>13.87</td>
<td>10.84</td>
<td>4.07</td>
</tr>
<tr>
<td>Mean and volatility of the risk-free rate</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E[R_f]$</td>
<td>1.58</td>
<td>1.66</td>
<td>2.05</td>
<td>3.84</td>
<td></td>
</tr>
<tr>
<td>$\sigma(R_f)$</td>
<td>2.14</td>
<td>1.27</td>
<td>1.38</td>
<td>1.49</td>
<td></td>
</tr>
<tr>
<td>Equity premium</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E[R - R_f]$</td>
<td>5.69</td>
<td>3.79</td>
<td>4.09</td>
<td>3.35</td>
<td>1.13</td>
</tr>
<tr>
<td>Sharpe ratio</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E[R - R_f]/\sigma(R)$</td>
<td>0.31</td>
<td>0.31</td>
<td>0.30</td>
<td>0.31</td>
<td>0.28</td>
</tr>
</tbody>
</table>
Table 6: Calibrations: Recursive Preferences with Different Capital Adjustment Costs

This table summarizes key moments of macroeconomic quantities (Panel A) and asset prices (Panel B) from calibrations of recursive preferences models with and without technology choice, under different capital adjustment costs ($\chi$). The empirical data are from the NIPA tables and the annual Fama-French factor data file over 1964-2011. Unlevered returns are computed as assuming debt/equity=0.5. The macroeconomic quantities are reported as quarterly, while the asset prices are annualized. The volatilities of output, consumption, and investment are computed from the Hodrick-Prescott filter. All moments are reported in percentages, except the Sharpe ratio.

<table>
<thead>
<tr>
<th></th>
<th>U.S. Data (1964-2011)</th>
<th>Unlevered Returns</th>
<th>Technology Choice</th>
<th>No Technology Choice</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Model (1) $\chi = 0.23$</td>
<td>Model (2) $\chi = 1.5$</td>
</tr>
<tr>
<td><strong>Panel A Macroeconomic quantities</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Volatility of output</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma(Y)$</td>
<td>1.57</td>
<td>0.91</td>
<td>1.58</td>
<td>7.67</td>
</tr>
<tr>
<td>Volatility of consumption</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma(C)$</td>
<td>0.84</td>
<td>0.85</td>
<td>0.72</td>
<td>0.52</td>
</tr>
<tr>
<td>Volatility of investment</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma(I)$</td>
<td>7.56</td>
<td>1.61</td>
<td>6.00</td>
<td>33.62</td>
</tr>
<tr>
<td><strong>Panel B Asset prices</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean and volatility of the equity returns</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E[R]$</td>
<td>7.27</td>
<td>5.37</td>
<td>6.44</td>
<td>5.40</td>
</tr>
<tr>
<td>$\sigma(R)$</td>
<td>18.21</td>
<td>12.16</td>
<td>17.06</td>
<td>10.84</td>
</tr>
<tr>
<td>Mean and volatility of the risk-free rate</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E[R_f]$</td>
<td>1.58</td>
<td>1.21</td>
<td>2.05</td>
<td>3.47</td>
</tr>
<tr>
<td>$\sigma(R_f)$</td>
<td>2.14</td>
<td>1.14</td>
<td>1.38</td>
<td>1.33</td>
</tr>
<tr>
<td>Equity premium</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E[R - R_f]$</td>
<td>5.69</td>
<td>3.79</td>
<td>5.23</td>
<td>3.35</td>
</tr>
<tr>
<td>Sharpe ratio</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E[R - R_f]/\sigma(R)$</td>
<td>0.31</td>
<td>0.31</td>
<td>0.31</td>
<td>0.31</td>
</tr>
</tbody>
</table>
Table 7: Return Predictability

This table presents predictability regressions of the dividend-price ratio. The annual U.S. market returns (including NYSE/AMEX/NASDAQ) from CRSP over 1925-2011, deflated by the CPI, are used. The simulated data from the main model are firstly aggregated into annual, and the median values of regressions over the 1000 sample paths are reported. The $t$-statistics are corrected for the heteroscedasticity and autocorrelation, using Newey-West standard errors.

Panel A

Regression: $R_{t,t+k} = a + b\frac{D_t}{P_t} + \varepsilon_{t+k}$

<table>
<thead>
<tr>
<th>Horizon (years)</th>
<th>U.S. Data (1925-2011)</th>
<th>Main Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k$</td>
<td>$b$</td>
<td>$t(b)$</td>
</tr>
<tr>
<td>1</td>
<td>3.46</td>
<td>2.43</td>
</tr>
<tr>
<td></td>
<td>5.49</td>
<td>3.04</td>
</tr>
<tr>
<td>2</td>
<td>6.47</td>
<td>2.80</td>
</tr>
<tr>
<td></td>
<td>8.11</td>
<td>3.51</td>
</tr>
</tbody>
</table>

Panel B

Regression: $R_{t,t+k} - R_{t,t+k}^f = a + b\frac{D_t}{P_t} + \varepsilon_{t+k}$

<table>
<thead>
<tr>
<th>Horizon (years)</th>
<th>U.S. Data (1925-2011)</th>
<th>Main Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k$</td>
<td>$b$</td>
<td>$t(b)$</td>
</tr>
<tr>
<td>1</td>
<td>3.83</td>
<td>2.72</td>
</tr>
<tr>
<td></td>
<td>5.00</td>
<td>2.75</td>
</tr>
<tr>
<td>2</td>
<td>7.46</td>
<td>3.27</td>
</tr>
<tr>
<td></td>
<td>7.29</td>
<td>3.10</td>
</tr>
</tbody>
</table>
Appendices

For Online Publication

A Loglinearization

The equilibrium conditions of recursive preferences with technology choice are summarized as follows:

\[ \lambda_t = (1 - \beta) U_t^\frac{1}{\psi} C_t^{-\frac{1}{\psi}}, \]  
(A.1)

\[ \lambda_t = \mu_t g_t', \]  
(A.2)

\[ \mu_t = \mathbb{E}_t \left[ \frac{\partial U_t}{\partial U_{t+1}} \frac{\partial U_{t+1}}{\partial K_{t+1}} \right], \]  
(A.3)

\[ 0 = \frac{\partial U_t}{\partial U_{t+1}} \frac{\partial U_{t+1}}{\partial A_{t+1}} - \xi_t \nu \left( \frac{A_{t+1}^{1-\alpha}}{\theta_{t+1}^{1-\alpha}} \right)^{\nu-1} \frac{1 - \alpha}{\theta_{t+1}^{1-\alpha}} A_{t+1}^\alpha, \]  
(A.4)

\[ A_t^{1-\alpha} K_t^\alpha = C_t + I_t, \]  
(A.5)

\[ K_{t+1} = (1 - \delta) K_t + g_t, \]  
(A.6)

\[ 1 = \mathbb{E}_t \left[ \frac{A_{t+1}^{1-\alpha}}{\theta_{t+1}^{1-\alpha}} \right]^\nu, \]  
(A.7)

\[ U_t = \left\{ (1 - \beta) C_t^{1-1/\psi} + \beta \mathbb{E}_t \left[ U_{t+1}^{1-1/\gamma} \right]^{1-1/\gamma} \right\}^{1-1/\psi}. \]  
(A.8)

The key variables in the deterministic steady state of the economy are described by

\[ K = \left( \frac{1 - \beta (1 - \delta)}{\alpha \beta} \right)^{\frac{1}{\alpha - 1}} A, \]

\[ C = A^{1-\alpha} K^\alpha - \delta K, \]

\[ I = \delta K, \]

\[ U = C, \]

\[ \mu = 1 - \beta, \]

\[ \lambda = 1 - \beta, \]
\[ \xi = \frac{(1 - \beta)\beta K^\alpha}{\nu A^{\alpha - 1}}, \]

where variables without subscripts indicate steady state values. Clearly, the deterministic state is independent of risk aversion \( \gamma \) and elasticity of intertemporal substitution \( \psi \), and only \( \xi \) depends on the technology choice curvature \( \nu \).

**A.1 Loglinearization: Recursive Preferences without Technology Choice**

We first recall the case of standard production under recursive preferences without technology choice. We only need to replace \( A \) with \( \theta \) and remove the equations (A.4) and (A.7). The steady state is exactly same as that when technology choice is allowed, once we replace \( A \) with \( \theta \).

By convention, the percentage deviation of variable \( x_t \) from its steady state value \( (x) \) is defined as \( \tilde{x}_t = \log x_t - \log x \). For example, the underlying technology shock process can be rewritten as \( \tilde{\theta}_t = \rho \tilde{\theta}_{t-1} + \epsilon_t \) where \( \epsilon \sim \mathcal{N}(0, \sigma^2) \). The percentage deviations of consumption and investment from the steady state can be solved from loglinearizations of the equilibrium conditions, as following

\[ \tilde{C}_t = \tilde{C}_K \tilde{K}_t + \tilde{C}_\theta \tilde{\theta}_t, \quad (A.9) \]
\[ \tilde{I}_t = \tilde{I}_K \tilde{K}_t + \tilde{I}_\theta \tilde{\theta}_t, \quad (A.10) \]

where \( \tilde{C}_K \) is the positive root from the following quadratic equation

\[ 0 = -\left[ \frac{1 - \beta(1 - \delta)}{\alpha} - \beta \delta \right] \left[ \frac{1}{\psi} + \frac{1 - \beta(1 - \delta) - \alpha \beta \delta}{\alpha \delta \chi} \right] \tilde{C}_K^2 \]
\[ + \left\{ \left[ \frac{1 - \beta(1 - \delta)}{\alpha} - \beta \delta \right] \left[ (\alpha - 1)(1 - \beta(1 - \delta)) + \frac{1 - \beta}{\chi \delta} \right] + \frac{1 - \beta}{\psi} \right\} \tilde{C}_K \]
\[ + (1 - \alpha)[1 - \beta(1 - \delta)], \quad (A.11) \]
\[
\tilde{C}_\theta = \frac{(1 - \alpha)(1 - \beta)(1 - \delta)}{(\rho + \frac{\rho^2 - 1}{\alpha \beta \delta} + B)}\left[1 \frac{\beta(1 - \delta)}{\alpha \beta \delta} - 1\right],
\]
(A.12)

\[
\tilde{I}_K = \frac{1 - \beta(1 - \delta)}{\beta \delta} - \left[\frac{1 - \beta(1 - \delta)}{\alpha \beta \delta} - 1\right] \tilde{C}_K,
\]
(A.13)

\[
\tilde{I}_\theta = (1 - \alpha)\frac{1 - \beta(1 - \delta)}{\alpha \beta \delta} + \left[1 - \frac{1 - \beta(1 - \delta)}{\alpha \beta \delta}\right] \tilde{C}_\theta,
\]
(A.14)

\[
B = (\alpha - 1)(1 - \beta)(1 - \delta) - \frac{\rho}{\psi} \tilde{C}_K
\]
\[
+ \frac{\beta}{\chi} \left[\frac{1 - \beta(1 - \delta)}{\beta \delta} - \frac{1 - \beta(1 - \delta)}{\alpha \beta \delta} - \alpha \beta \delta \tilde{C}_K - 1\right].
\]
(A.15)

Similarly, the percentage deviation of utility from the steady state can be expressed as

\[
\tilde{U}_t = \tilde{U}_K \tilde{K}_t + \tilde{U}_\theta \tilde{\theta}_t,
\]

where

\[
\tilde{U}_K = \frac{(1 - \beta)\alpha}{1 - \beta(1 - \delta) - \alpha \beta \delta},
\]
(A.17)

\[
\tilde{U}_\theta = \frac{(1 - \beta)(1 - \alpha)(1 - \beta(1 - \delta))}{(1 - \rho \beta)(1 - \beta(1 - \delta) - \alpha \beta \delta)}.
\]
(A.18)

From equations (A.11)-(A.18), we see that coefficients \(\tilde{U}_K, \tilde{U}_\theta, \tilde{C}_K, \tilde{C}_\theta, \tilde{I}_K, \tilde{I}_\theta\) are dependent on EIS \((\psi)\) but independent of risk aversion \((\gamma)\). So, risk aversion has no first-order effects on macro quantities, a separation observed in Tallarini (2000). This is summarized in Corollary 1.

### A.2 Loglinearization: Recursive Preferences with Technology Choice

Next, we consider the case when technology choice is introduced. Log-linearizing the above equilibrium conditions (A.4) and (A.7) gives

\[
\tilde{A}_{t+1} = A_{\theta 1} \tilde{\theta}_t + A_{\theta 2} \tilde{\theta}_{t+1},
\]
(A.19)
where $A_{\theta_1} + \rho A_{\theta_2} = \rho$ and

$$
A_{\theta_1} = \frac{-\rho(1 - \alpha) + \frac{\rho}{\psi}(C_A + C_{\theta}) + \rho(\gamma - \frac{1}{\psi})(U_A + U_{\theta})}{(1 - \alpha)(\nu - 1) + \frac{1}{\psi}C_A + (\gamma - \frac{1}{\psi})U_A},
$$

(A.20)

$$
A_{\theta_2} = \frac{(1 - \alpha)\nu - \frac{1}{\psi}C_{\theta} + (\frac{\psi}{\psi} - \gamma)U_{\theta}}{(1 - \alpha)(\nu - 1) + \frac{1}{\psi}C_A + (\gamma - \frac{1}{\psi})U_A}.
$$

(A.21)

The percentage deviations of consumption and investment from the steady state can be solved from loglinearizations of the equilibrium conditions, as following

$$
\hat{C}_t = C_K \hat{K}_t + C_A \hat{A}_t + C_{\theta} \hat{\theta}_t,
$$

(A.22)

$$
\hat{I}_t = I_K \hat{K}_t + I_A \hat{A}_t + I_{\theta} \hat{\theta}_t,
$$

(A.23)

where $C_K$ is the positive root from the following quadratic equation

$$
0 = -\frac{1}{\alpha} \left[ 1 - \beta(1 - \delta) - \beta \delta \right] \left[ \frac{1}{\psi} + \frac{1 - \beta(1 - \delta) - \alpha \beta \delta}{\alpha \delta \chi} \right] C_K^2
+ \left\{ \frac{1 - \beta(1 - \delta)}{\alpha} - \beta \delta \right\} \left[ (\alpha - 1)(1 - \beta(1 - \delta)) + \frac{1 - \beta}{\chi \delta} \right] + \frac{1 - \beta}{\psi} \right\} C_K
+ (1 - \alpha)(1 - \beta(1 - \delta)),
$$

(A.24)

$$
C_A = \frac{(B\delta - \frac{1}{\chi})(1 - \beta(1 - \delta))(1 - \alpha)}{\left( B\delta - \frac{1}{\chi}\right)(1 - \beta(1 - \delta)) - \alpha \beta \delta - \frac{\alpha \beta \delta}{\psi}},
$$

(A.25)

$$
C_{\theta} = \frac{\rho \left\{ (1 - \alpha)(1 - \beta(1 - \delta))(1 + \frac{1}{\alpha \delta \chi}) - \left[ \frac{1}{\psi} + \frac{1 - \beta(1 - \delta)}{\alpha \delta \chi} - \frac{\beta}{\chi} \right] C_A \right\}}{\rho \frac{1}{\psi} + \frac{1}{\chi} \left[ \frac{1 - \beta(1 - \delta)}{\alpha \beta \delta} - 1 \right] (\rho \beta - 1) + B \left[ \frac{1 - \beta(1 - \delta)}{\alpha \beta \delta} - \delta \right]},
$$

(A.26)

$$
I_K = \frac{1 - \beta(1 - \delta)}{\beta \delta} \left[ 1 - \beta(1 - \delta) \right] C_K,
$$

(A.27)

$$
I_A = (1 - \alpha) \frac{1 - \beta(1 - \delta)}{\alpha \beta \delta} + \left[ 1 - \frac{1 - \beta(1 - \delta)}{\alpha \beta \delta} \right] C_A,
$$

(A.28)

$$
I_{\theta} = \left[ 1 - \frac{1 - \beta(1 - \delta)}{\alpha \beta \delta} \right] C_{\theta},
$$

(A.29)

$$
B = (\alpha - 1)[1 - \beta(1 - \delta)] - \frac{C_K}{\psi},
$$

(A.30)
The percentage deviation of utility from the steady state can be written as
\[ \hat{U}_t = U_K \hat{K}_t + U_A \hat{A}_t + U_\theta \hat{\theta}_t, \tag{A.32} \]
where coefficients
\[
U_K = \frac{(1 - \beta)\alpha}{1 - \beta(1 - \delta) - \alpha\beta\delta}, \tag{A.33}
\]
\[
U_A = \frac{(1 - \beta)(1 - \alpha)[1 - \beta(1 - \delta)]}{1 - \beta(1 - \delta) - \alpha\beta\delta}, \tag{A.34}
\]
\[
U_\theta = \frac{\rho\beta U_A}{1 - \rho\beta}. \tag{A.35}
\]
Clearly, \( U_K > 0, U_A > 0, \) and \( U_\theta > 0. \)

From equations (A.24)-(A.35), we see that coefficients \( U_K, U_A, U_\theta, C_K, C_A, C_\theta, I_K, I_A, I_\theta \) are dependent on EIS \( (\psi) \) but independent of the risk aversion \( (\gamma) \) and technology choice curvature \( (\nu) \). This concludes in Corollary 2.

It is easy to show that when \( \psi < 1 \), we have \( 0 < \frac{\alpha(1 - \beta)}{1 - \beta + \beta\delta - \alpha\beta\delta} < C_K < 1, 0 < C_A < 1 - \alpha, 0 < I_K < 1, 0 < 1 - \alpha < I_A, C_K > U_K, \) and \( C_A > U_A. \) Also, when \( \psi < 1, \) together with a very mild condition, i.e., the capital adjustment costs are not extremely large \( (\chi > \frac{\psi[1 - \psi + \beta(-(1 + \psi - \delta\psi + \alpha\delta\psi)]}{\delta(1 - \psi + \beta\psi - \alpha\beta\psi - \beta\delta\psi + \alpha\beta\delta\psi)}), \tag{A.36} \)
then we have \( C_\theta > 0, I_\theta < 0, C_A U_\theta - C_\theta U_A > 0, \) and \( C_A A_{\theta 1} + C_\theta > 0. \)

Moreover, from equations (A.20)-(A.21), coefficients \( A_{\theta 1} \) and \( A_{\theta 2} \) depend on the risk aversion \( (\gamma) \) and technology choice curvature \( (\nu) \). Also, it is straightforward to show that , \( A_{\theta 1} \geq 0 \) and \( A_{\theta 2} \leq 1. \) This yields Corollary 3. When \( \psi < 1, \)
\[
\frac{\partial A_{\theta 1}}{\partial \nu} = \frac{\alpha - 1}{(1 - \alpha)(\nu - 1) + \frac{1}{\psi} C_A + (\gamma - \frac{1}{\psi}) U_A} A_{\theta 1} < 0, \tag{A.36}
\]
\footnote{This condition is very mild, since it is bounded only when \( \psi \) is close to 1.}
\[
\frac{\partial A_{\theta_2}}{\partial \nu} = -\frac{1}{\rho} \frac{\partial A_{\theta_1}}{\partial \nu} > 0, \quad (A.37)
\]
\[
\frac{\partial A_{\theta_1}}{\partial \gamma} = \frac{\rho (1 - \alpha)[(\nu - 1)U_\theta + \nu U_A] + \frac{\nu}{\psi} (C_A U_\theta - C_{\theta} U_A)}{[(1 - \alpha)(\nu - 1) + \frac{1}{\psi} C_A + (\gamma - \frac{1}{\psi}) U_A]^2} > 0, \quad (A.38)
\]
\[
\frac{\partial A_{\theta_2}}{\partial \gamma} = -\frac{1}{\rho} \frac{\partial A_{\theta_1}}{\partial \gamma} < 0, \quad (A.39)
\]

\[
\text{lim}_{\nu \to +\infty} A_{\theta_1} = 0, \text{lim}_{\nu \to +\infty} A_{\theta_2} = 1, \text{lim}_{\gamma \to +\infty} A_{\theta_1} = \rho \left(1 + \frac{U_\theta}{U_A}\right), \text{and lim}_{\gamma \to +\infty} A_{\theta_2} = -\frac{U_\theta}{U_A}.
\]

This is summarized in Corollary 4.

Since \(U_A > 0\), \(C_A > 0\), and \(\psi < 1\), from (A.21), it is straightforward to show Corollary 5.

**B Empirical Data**

Annual market return and the risk-free rate data are from the annual Fama-French factor data file from Kenneth French’s website. The real returns are adjusted by the CPI from the NIPA Table 2.3.4. Other macroeconomic data are mainly collected from the NIPA tables. The real output is measured as the real GDP from the NIPA Table 1.1.6. The real consumption is defined as real nondurable goods and services, computed from the NIPA Table 2.3.4 and 2.3.5. The investment is computed as the sum of the real gross private domestic investment, the government gross investment adjusted by the government gross investment price index, and the personal consumption expenditures on durable goods adjusted by the durable goods price index, computed from the NIPA Table 1.1.6, 3.9.4, 3.9.5, 2.3.4, and 2.3.5. All these quantities are quarterly and normalized by the civilian noninstitutional population with age over 16, from the Current Population Survey (Serial ID LNU00000000Q).

**C Numerical Accuracy: Euler Errors**

To evaluate the numerical accuracy of the second-order perturbation, we compute the Euler equation errors, as suggested by Judd and Guu (1997). The basic asset pricing equation
implies that

\[ P_t = \mathbb{E}_t [M_{t,t+1}(P_{t+1} + C_{t+1})] \]

\[ = \mathbb{E}_t \left[ \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\psi}} \left( \frac{U_{t+1}^{1-\gamma}}{e u_t} \right)^{-\frac{1-\gamma}{\gamma}} (P_{t+1} + C_{t+1}) \right]. \]

The Euler equation error is defined as fraction of consumption:

\[ EulerError_t = 1 - \left\{ \mathbb{E}_t \left[ \beta C_{t+1}^{-\frac{1}{\psi}} \left( \frac{U_{t+1}^{1-\gamma}}{e u_t} \right)^{-\frac{1-\gamma}{\gamma}} (P_{t+1} + C_{t+1}) \right] \right\}^{-\psi} \]

\[ \frac{1}{P_t} \cdot \frac{1}{C_t}. \]

The absolute Euler equation errors are reported in base 10 logarithm in Figure C.1. Therefore, the errors can be interpreted as the percentage of consumption. For example, a value of \(-2\) indicates the error is measured as 1% of consumption. Figure C.1(a) compares the Euler equation errors of first- and second-order perturbations for capital \(K_t\) over \([20, 40]\), assuming all other variables are in the steady state and the shock equals 0. Clearly, they show almost same Euler equation errors, about 1% of consumption. Figure C.1(b) displays the Euler equation errors for a second-order perturbation for capital \(K_t\) over \([20, 40]\) and the shock \(\theta_t\) over \([-3\sigma, 3\sigma]\), which covers the simulated distribution, assuming all other variables are in the steady state. Overall, the Euler equation error is less than 1% for wide ranges of capital and the shock. Hence, a second-order perturbation only introduces small approximation errors.
D Alternative Preferences

D.1 Recursive Preferences with Large EIS

The main model, by employing recursive preferences with a small EIS, matches both asset prices and macroeconomic quantities reasonably well. In this section, we examine the case when EIS is large, i.e., $\psi = 1.5$, which is often used in the consumption based long-run risk literature (Bansal and Yaron (2004)). For comparison, Table D.1 reports simulation results of the models with and without technology choice. To facilitate comparison, we report simulation results with a small ($\psi = 0.03$) and a large EIS ($\psi = 1.5$). Clearly, the volatility of consumption increases with EIS as the agent is less averse to intertemporal consumption variations when EIS is large. When there is no technology choice, volatility of consumption increases from Model (3) to (4), but not significantly. The equity premium is too small in Models (3) and (4). However, when technology choice is introduced, volatility of consumption increases significantly from Model (1) to (2), as large as 3.87% per quarter, which is much larger than that observed in the data (0.84%). This highly volatile consumption even leads to a sizeable equity premium (2.25% per year in Model (2)). Meanwhile, output is too volatile and investment is too smooth in Model (2).

D.2 External Habit

Next, we investigate technology choice under another popular preference specification, namely external habit formation. Following Campbell and Cochrane (1999), define utility as

$$U_0(C, H) = E_0 \left[ \sum_{t=0}^{\infty} \beta^t \frac{(C_t - H_t)^{1-\gamma} - 1}{1-\gamma} \right],$$

where $H_t$ is the level of habit. The surplus consumption ratio, $S_t = (C_t - H_t) / C_t$, conveniently summarizes the relation between consumption and habit. Assume that the log
surplus consumption ratio evolves according to

$$\ln S_{t+1} = (1 - \phi) \ln S + \phi \ln S_t + \varphi(S_t) [\ln C_{t+1} - \mathbb{E}_t [\ln C_{t+1}]]$$

(D.41)

where $\phi$ controls the speed of mean reversion and $S$ denotes the surplus consumption ratio at the steady state. The sensitivity function, $\varphi$, is specified as

$$\varphi(S_t) = \begin{cases} \frac{1}{2} \sqrt{1 - 2(\ln S_t - \ln S)} - 1 & \text{if } \ln S_t \leq S_{\text{max}} \\ 0 & \text{if } \ln S_t > S_{\text{max}} \end{cases}$$

(D.42)

where $S_{\text{max}} = \ln S + \frac{1}{2} (1 - S^2)$. Similar to Campbell and Cochrane (1999), we set $\phi = 0.966$. Since we set $\gamma = 7.5$ as the main case, we set $S = 0.125$ to reproduce a smooth risk-free rate.\footnote{Note that Campbell and Cochrane (1999) set $\gamma = 2$ and $S = 0.057$.} Table D.2 summarizes the simulation results from this exercise. Overall, the results match the empirical data reasonably well, which are quite similar to those under CRRA utility or the recursive preferences. Again, we see the amplification in Model (1). Output volatility is 1.77% per quarter, comparing with 0.98% per quarter in other models. Model (1) generates a Sharpe ratio of 0.29 and an equity premium of 2.97% per year. Therefore, exercises in Tables 2, 3, and D.2 show that in this risk aversion sensitive economy, the utility specification is less important than in the standard RBC models. This might validate the simple preference specifications often used in the macroeconomics literature, e.g., CRRA, instead of more sophisticated preferences.

E Welfare Costs

Lucas (1987, 2003) finds that welfare costs associated with business fluctuations are trivially small, about 0.05-0.1% of consumption.\footnote{See Lucas (2003) and Barlevy (2004) for a survey of this literature.} One weakness in Lucas’ calculation might be that it fails to capture the high equity premium, which implies the model might miss some
important consumption risks. Disciplined by asset price features, Tallarini (2000) finds instead a very high welfare cost, about 26% of consumption loss, when risk aversion is as high as 100. In contrast, Barro (2009) finds that it costs only 1.6% of GDP to remove consumption volatilities when risk aversion is 4. Alvarez and Jermann (2004) estimate that eliminating business cycle fluctuations costs only 0.08-0.49% of consumption.

Since in this model, endogenous technology choice yields macroeconomic variables that are sensitive to risk aversion, it appears that our welfare costs could differ from the literature.

The perturbation method used directly provides the welfare costs of business fluctuations. Given other variables at the steady state, the utility after one standard deviation of underlying shock is

\[ U(K, 0, 0, \sigma) \approx U(K, 0, 0, 0) + \frac{1}{2} U^{(0,0,0,2)} \sigma^2, \]  

(E.43)

where \( K \) is the steady state capital stock. Here, \( \frac{1}{2} U^{(0,0,0,2)} \sigma^2 \) measures the welfare costs from business cycles. This computation is accurate up to the third-order. To compute the equivalent percentage consumption decrease from a business cycle shock, we convert the above utility into certainty equivalent of consumption. Suppose the representative agent is indifferent between consuming the above consumption stream and \((1 - \tau)C\) deterministic consumption, i.e.,

\[ U(K, 0, 0, 0) + \frac{1}{2} U^{(0,0,0,2)} \sigma^2 = (1 - \tau)C, \]  

(E.44)

where \( C \) denotes steady state consumption. Then, the welfare costs can be expressed as a percentage decrease in consumption, as follows

\[ \tau = - \frac{U^{(0,0,0,2)} \sigma^2}{2C}. \]  

(E.45)

Table E.3 reports the welfare costs from business cycles for various preferences. Because of the amplification mechanism, we only need a small risk aversion to fit the equity premium. Hence, disciplined by the asset prices, we see welfare costs of 0.97% consumption in the main model (recursive preferences with EIS=0.03 and \( \gamma = 7.5 \)). This value is an order
of magnitude larger than Lucas’ estimate but smaller than Tallarini’s estimate; it is close
to the estimates in Alvarez and Jermann (2004) and Barro (2009). Table E.3 shows that
welfare costs increase with risk aversion, but decrease when technology choice is allowed for
because technology choice serves as smoothing device when faced with uncertainty. Also,
the welfare costs of economic fluctuations decrease with EIS because agents are less averse
to intertemporal variations when EIS is high. The welfare costs under the CRRA utility
and external habit are 0.83% of consumption, which are smaller than those with recursive
preferences. Overall, various specifications demonstrate that the welfare costs of business
cycles are roughly 1% of consumption.
Figure C.1: The Euler Equation Error

Figure (a) compares the Euler equation errors of different order perturbation for capital $K_t$ over $[20, 40]$, assuming all other variables are in the steady state and shock=0. Figure (b) displays the Euler equation errors of a second-order perturbation for capital $K_t$ over $[20, 40]$ and shock $\theta_t$ over $[-3\sigma, 3\sigma]$, assuming all other variables are in the steady state.
Table D.1: Calibrations: Recursive Preferences with a Large EIS

This table summarizes key moments of macroeconomic quantities (Panel A) and asset prices (Panel B) from calibrations of recursive preferences models with and without technology choice, under a large ($\psi = 1.5$) elasticity of intertemporal substitution. This table also includes the case of a small EIS ($\psi = 0.03$) for comparison. The empirical data are from the NIPA tables and the annual Fama-French factor data file over 1964-2011. Unlevered returns are computed as assuming debt/equity=0.5. The macroeconomic quantities are reported as quarterly, while the asset prices are annualized. The volatilities of output, consumption, and investment are computed from the Hodrick-Prescott filter. All moments are reported in percentages, except the Sharpe ratio.

<table>
<thead>
<tr>
<th></th>
<th>U.S. Data (1964-2011)</th>
<th>Unlevered Returns</th>
<th>Technology Choice</th>
<th>No Technology Choice</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Model (1)</td>
<td>Model (2)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\psi = 0.03$</td>
<td>$\psi = 1.5$</td>
</tr>
<tr>
<td><strong>Panel A Macroeconomic quantities</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Volatility of output</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma(Y)$</td>
<td>1.57</td>
<td>1.58</td>
<td>3.84</td>
<td>0.97</td>
</tr>
<tr>
<td>Volatility of consumption</td>
<td>0.84</td>
<td>0.72</td>
<td>3.87</td>
<td>0.81</td>
</tr>
<tr>
<td>Volatility of investment</td>
<td>7.56</td>
<td>6.00</td>
<td>3.72</td>
<td>1.47</td>
</tr>
<tr>
<td><strong>Panel B Asset prices</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean and volatility of the equity returns</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E[R]$</td>
<td>7.27</td>
<td>5.37</td>
<td>5.40</td>
<td>5.14</td>
</tr>
<tr>
<td>$\sigma(R)$</td>
<td>18.21</td>
<td>12.16</td>
<td>10.84</td>
<td>7.37</td>
</tr>
<tr>
<td>Mean and volatility of the risk-free rate</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E[R_f]$</td>
<td>1.58</td>
<td>2.05</td>
<td>2.89</td>
<td>4.92</td>
</tr>
<tr>
<td>$\sigma(R_f)$</td>
<td>2.14</td>
<td>1.38</td>
<td>1.45</td>
<td>0.23</td>
</tr>
<tr>
<td>Equity premium</td>
<td>5.69</td>
<td>3.79</td>
<td>3.35</td>
<td>2.25</td>
</tr>
<tr>
<td>Sharpe ratio</td>
<td>0.31</td>
<td>0.31</td>
<td>0.31</td>
<td>0.30</td>
</tr>
</tbody>
</table>

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Table D.2: **Calibrations: External Habit**

This table summarizes key moments of macroeconomic quantities (Panel A) and asset prices (Panel B) from calibrations of external habit models with and without technology choice, under different risk aversion (γ). The empirical data are from the NIPA tables and the annual Fama-French factor data file over 1964-2011. Unlevered returns are computed as assuming debt/equity=0.5. The macroeconomic quantities are reported as quarterly, while the asset prices are annualized. The volatilities of output, consumption, and investment are computed from the Hodrick-Prescott filter. All moments are reported in percentages, except the Sharpe ratio.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Volatility of output</td>
<td>γ = 7.5</td>
<td>γ = 2</td>
<td>Model (1)</td>
<td>Model (2)</td>
</tr>
<tr>
<td>σ(Y)</td>
<td>1.57</td>
<td>1.77</td>
<td>0.98</td>
<td>0.98</td>
</tr>
<tr>
<td>Volatility of consumption</td>
<td>γ = 7.5</td>
<td>γ = 2</td>
<td>Model (3)</td>
<td>Model (4)</td>
</tr>
<tr>
<td>σ(C)</td>
<td>0.84</td>
<td>0.73</td>
<td>0.75</td>
<td>0.24</td>
</tr>
<tr>
<td>Volatility of investment</td>
<td>γ = 7.5</td>
<td>γ = 2</td>
<td>Model (3)</td>
<td>Model (4)</td>
</tr>
<tr>
<td>σ(I)</td>
<td>7.56</td>
<td>5.54</td>
<td>1.71</td>
<td>3.17</td>
</tr>
</tbody>
</table>

| Panel B Asset prices           |                        |                   |                   |                      |
| Mean and volatility of the equity returns | E[R] | σ(R)                  | E[R_f]            | σ(R_f)               |
| E[R]                            | 7.27                   | 18.21             | 1.58              | 2.14                 |
| σ(R)                            | 5.37                   | 12.16             | 4.53              | 4.50                 |
| Mean and volatility of the risk-free rate | E[R_f] | σ(R_f)               | E[R − R_f]        |                      |
| E[R_f]                          | 5.69                   | 2.18              | 2.97              | 0.34                 |
| σ(R_f)                          | 3.79                   | 4.53              | 0.34              | 1.23                 |
| Equity premium                 |                        |                   |                   |                      |
| E[R − R_f]                      | 5.69                   | 3.79              | 2.97              | 0.34                 |
| Sharpe ratio                    | E[R − R_f]/σ(R)        | 0.31              | 0.29              | 0.15                 |

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This table reports the welfare costs of business cycles as the percentage consumption loss from calibrations of various preferences, under different risk aversion. The preferences considered here include the constant relative risk aversion (CRRA), recursive preferences with a small (0.03) or large EIS (1.5), and the external habit.

<table>
<thead>
<tr>
<th>Model</th>
<th>Technology Choice</th>
<th>No Technology Choice</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\gamma = 7.5$</td>
<td>$\gamma = 2$</td>
</tr>
<tr>
<td>CRRA</td>
<td>0.83</td>
<td>0.14</td>
</tr>
<tr>
<td>Recursive preferences (EIS=0.03)</td>
<td>0.97</td>
<td>0.32</td>
</tr>
<tr>
<td>Recursive preferences (EIS=1.5)</td>
<td>0.72</td>
<td>0.12</td>
</tr>
<tr>
<td>External habit</td>
<td>0.83</td>
<td>0.14</td>
</tr>
</tbody>
</table>