Disclosure, Betas and Information Quality*

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Abstract:

We consider managers' disclosures in an exchange economy where the risk premium of stocks' expected returns is the product of two factors: the firm-specific beta and the equity premium (the expected return on the market portfolio in excess of the return on risk-free bonds). First, we show how managers' disclosures affect both the betas and the equity premium. Specifically, we establish that disclosure by one firm's manager affects the betas of other firms. Second, we find that the information quality of the managers' disclosures, as measured by the variance of the measurement error, also affects the betas and the equity premium. Finally, while the standard representation of the betas arises in the absence of disclosure as well as under mandatory full disclosure, we establish that this representation does not extend to discretionary disclosure. In its place we provide an appropriate "disclosure adjusted" representation for the betas when disclosure is discretionary.

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1. DISCLOSURES, BETAS AND INFORMATION QUALITY - INTRODUCTION

We analyze the effect of public disclosures on two aspects of the risk premium in stock returns. First, we consider the equity premium, the expected return on the market portfolio in excess of the return on risk-free bonds. Second, we consider the individual stock's risk premiums as measured by beta in the Capital Asset Pricing Model (CAPM), that is, $firm's\, risk\, premium = beta \times equity\, premium$. We establish how managers' public disclosures affect both the equity premium and the firms' betas. We also study information quality represented by the measurement error in the managers' disclosures and find that an exogenous change in information quality affects both the equity premium and the betas. We consider the relation from disclosure and information quality to the equity premium and betas in two settings. In our first setting, disclosure is mandatory. Consistent with prior literature, we view this setting as a stylized representation of firms' required disclosures to the SEC, such as earnings reports. We view our second setting where disclosure is discretionary as a representation of managers' voluntary earnings forecasts.

Currently, no formal theory links discretionary disclosures to the risk premiums in stock returns, as measured by beta.[1] Consequently, prior empirical studies of the consequences of disclosures interpret beta as a control variable. In contrast, our findings suggest that discretionary disclosures can directly affect both the beta and the market portfolio. We want to emphasize the distinction we make among three types of disclosure strategies: mandatory, voluntary and discretionary, especially as the latter two are sometimes conflated. For mandatory, we can have mandatory full, where the manager fully discloses his information, or mandatory no, where the manager is prohibited from disclosing his information. For voluntary, the manager chooses the variable of interest, usually the precision or variance of his cash flows, but sometimes other information of interest. Under voluntary disclosure, we derive prices analogous to how we derive prices

[1] The difficulty in creating a formal connection between discretionary disclosure and the cost of capital has been pointed out in the theoretical literature, see Verrecchia (2001). Leuz and Verrecchia (2000) note that "a relation between the firm's disclosure and its beta factor has little support in theory."

under mandatory disclosures, as the information that is chose is usually revealed. This changes for discretionary disclosure.

For discretionary disclosure, we have the situations where the manager may or may not have information that he may or may not disclose at a cost. These are situations where the manager may refrain from disclosing but, if the investor sees no disclosure, then she knows that the manager refrained from disclosing. Hence deriving prices with discretionary disclosure is qualitatively different than deriving prices under either mandatory or voluntary disclosures as the distribution are no longer full, as is true for mandatory or voluntary disclosure regimes. For discretionary disclosure regime, we have truncated distributions, making deriving the prices a much more complicated endeavor. Deriving discretionary disclosure prices and their associated betas are a main contribution of our study.

In our model, the firm's beta is, as usual, the scalar that, when multiplied with the excess return on the market portfolio, characterized the excess return of a stock within the framework of the CAPM. When disclosure is mandatory, a firm's beta can be characterized as the covariance between the return on the stock and the return on the market portfolio divided by the variance of the return on the market portfolio. We establish that when the disclosure decision is discretionary, left to the discretion of the manager, this characterization is incorrect, provided that at least one firm manager opts to not disclose. In this paper, we provide the correct, alternative representation of beta. These results have implications for the design of event studies of discretionary disclosures in empirical capital market research that typically use abnormal stock returns resulting from an adjustment of individual stock returns for the performance of the market portfolio.[2]

To summarize, our model provides a framework for the traditional application of event study methodologies to both mandatory and discretionary management disclosures, such as mandatory earnings announcements and discretionary management forecasts.

[2] See, among many others, Baginski (1987).

Further, our results provide a theoretical foundation for empirical studies of the association between management disclosures and firms' betas or implied cost of capital.

2. BACKGROUND AND LITERATURE REVIEW

Empirical evidence is consistent with reduced cost of capital resulting from increased disclosures regarding future firm profitability, such as management forecasts and other communications with analysts.[3] However, prior theoretical analyses of the link between cost of capital and asymmetric information do not consider managers' strategic disclosure decisions. Consider two papers by Barry and Brown (1985) and Merton (1987) that are often cited as establishing a theoretical relationship between disclosure and beta. Barry and Brown (1985) demonstrate that a firm's beta varies with the degree of parameter uncertainty about the variance of future firm value.[4] In contrast, Merton (1987) considers investors to be uninformed about the existence of some stocks and, in equilibrium, the lesser-known stocks trade at a discount. Neither of these papers allows managers to make discretionary disclosures that affect the extent of either parameter uncertainty (as in Barry and Brown) or investors' ignorance (as in Merton). As such, neither supports studying the impact of disclosure (mandatory or discretionary) on beta.

To establish a link between disclosure and beta, we require that investors are risk-averse since, otherwise, there would be no risk premia in stock returns.[5] While some papers

[3] Such studies include Lang and Lundholm (1993, 1996, 2000), Imhoff and Thomas (1994), Frankel, McNichols, and Wilson (1995), Botosan (1997), Healy, Hutton, and Palepu (1999), Botosan and Plumlee (2002), and Bushee and Noe (2000). For example, Lang and Lundholm (1993) study variation in AIMR analysts' rankings of firms' disclosures. They use the standard deviation of market-adjusted stock returns and the historical correlation between annual returns and annual earnings. Botosan (1997) and Botosan and Plumlee (2002) find that the estimated expected cost of capital vary cross-sectionally with disclosure and the betas estimated from the market model. Beardsley and O'Brien (2003a,b) document how regulations regarding disclosures have a discernable effect on the risk-neutralized distribution of stock market returns.

[4] In a similar vein, see Coles and Loewenstein (1988) and Coles, Loewenstein, and Suay (1995).

[5] Most prior work on discretionary disclosures assumes either that investors are risk neutral and solves for stock prices (including Dye (1985) and Verrecchia (1990)) or allows for risk-averse investors but exogenously imposes how risk is reflected in stock prices (including Verrecchia (1983)). Other than Jorgensen and Kirschenheiter (2015), studies of discretionary disclosure, as we define it, include Hughes and Pae (2004), Bertomeu, Beyer and Dye (2011), Cheynel (2013) and Clinch and Verrechia (2015). These other studies, which we discuss in detail below, all assume managers may not be informed.

on disclosures do allow for risk-averse investors, their models differ from the current model in various respects. First, Dye (1990) and Jorgensen and Kirschenheiter (2003) address disclosures that mitigate parameter uncertainty regarding future firm cash flows' mean and variance, respectively. Dye (1990) maintains that disclosures about the unknown mean are mandatory and characterizes managers voluntarily choice of disclosure precision.[6] Jorgensen and Kirschenheiter (2003) investigate managers' discretionary disclosure decisions about the unknown variance, but do not allow that managers disclose earnings or voluntary forecasts regarding future firm values. Second, Hakansson, Kunkel and Ohlson (1982) address mandatory disclosures but do not formalize the link to betas. Third, Jorgensen and Kirschenheiter (2012 and 2015) extend Verrecchia's (1983, 1990) discretionary disclosure setting to allow, on the one hand, for two firm managers and then, on the other hand, for risk-averse investors. While Jorgensen and Kirschenheiter (2015) endogenously derives market-clearing stock prices when investors are risk-averse and firm managers can choose whether to disclose, that study does not consider betas and the equity premium.

More recently, we have numerous articles on disclosure and the cost of capital, many that look to the role of disclosure with risk averse investors and multiple firms. Bertomeu and Cheynel (2016) provide a survey article of the theoretical research on this area that discusses the role of discretionary disclosure, heterogenous beliefs, investor base, liquidity shocks, earnings management, and agency problems as determinants of the cost of capital. Many studies include multiple firms with disclosure priced by risk averse investor, but do not include analysis of discretionary disclosure.[7] Other studies do model the impact of discretionary disclosure strategies on cost capital, including Hughes and Pae (2004), Bertomeu, Beyer and Dye (2011), Cheynel (2013) and Clinch and Verrechia

[6] Other models where the "voluntary" nature of disclosure is the manager's choice of precision include Diamond and Verrecchia (1991), Penno (1996) and Admati and Pfleiderer (2000).

[7] Models of multi-firm economies analyzing the various impact of disclosure on cost of capital without discretionary disclosure include Hughes, Liu, and Liu (2007) Gao (2010), Johnston (2016), Lambert, Leuz, and Verrecchia (2007, 2012), and Lambert and Verrecchia (2015).

(2015). In each of these, discretionary disclosure arising based on the model of Dye (1985) where managers have discretion because there is a positive probability that they are not informed. Hughes, Liu and Liu (2007) model the disclosure of precision and show that managers disclose higher or lower levels of discretionary depending on the value of the underlying cash flows, with higher (or lower) discretion being disclosed when cash flows are better (or worse). Bertomeu, Beyer and Dye (2011) model discretionary disclosure of signals about the cash flows of the firm in an economy with differentially informed investors. They analyze the impact of the manager's disclosure decision on cost of capital, where this is defined as the difference between the unconditional expected value less the ex-anted expected value of the firm's security. They find that the volatility of the cash flows and interaction of mandatory and voluntary disclosure decisions affect the informational advantage of the informed trader and alter the capital structure choices and the cost of capital. Clinch and Verrechia (2015) define the price discount from risk aversion as the difference between prior (unconditional) expected price and the price after disclosures. They study the relation between discretionary disclosures and this discount and find that underlying model parameters affect this relation and also study the impact of changing both exogenous and endogenous model parameters on this relation.

Cheynel (2013) is probably closes to our study, as she studies the impact of systematic risk on cost of capital with discretionary disclosure. While we differ from Cheynel (2013) in how we measure discretionary disclosure (we look at costly discretionary disclosure as derived in Jorgensen and Kirschenheiter, 2015), more importantly, we derive the cost of capital as a result of prices derived from solving the investors portfolio optimization problem. This enables us to analyze the differential impact of mandatory and discretionary disclosure on the cost of capital of the different firms.

With this final point in mind, and before continuing, we wish to stress that we derive betas using a slightly unusual approach. The usual approach is to begin with assumptions about the distribution of returns and derive betas from these. However, we cannot adopt this approach, as we need to determine how disclosure affects returns. Hence, we start by deriving the equilibrium prices and then use these prices to determine both returns and betas. The approach may be unorthodox, but it leads to analysis that is exactly analogous to that usually followed. In particular, we show that the CAPM betas are identical to those derived using the usual approach, when we assume either full or no disclosure.[8] However, the analysis of a discretionary disclosure regime, in particular deriving the discretionary disclosure prices, necessitates that we adopt this more complicated approach.

Despite similarities in assumptions, firms' risk premiums have not been formally connected to the managers' decision concerning a voluntary forecast of future firm value where investors correctly interpret absence of disclosure as a discretionary disclosure strategy. Hence, no theory currently links the discretionary disclosure of a forecast of firm value to that firm's beta.[9] As a caveat, this paper fails to complete the link between managers' discretionary disclosure decisions and cost of capital on a comprehensive basis. A comprehensive link requires, in our mind, a model that deals explicitly with an investment decision. In this paper, we restrict our focus to analyzing the link between discretionary disclosure and a firm's beta, deferring the additional step that links disclosure to cost of capital for future research.

The paper proceeds as follows. In section 3, we present the basic model with a single stock. In section 4, we present our results for an economy with two firms, assuming the future liquidating values of the firms are independent. We summarize and conclude with section 5. Appendix A contains the proofs while Appendix B presents an analysis for an economy with an arbitrary number of firms.

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[8] The approach for deriving the CAPM that we use is exactly analogous to that used in Lambert, Leuz, and Verrecchia (2007), except they eschew deriving prices with discretionary disclosures, focusing instead only on prices with mandatory full and mandatory no disclosures.

[9] We interpret this statement to include voluntary forecasts of earnings. More specifically, no theoretical model currently links voluntary earnings forecasts to risk where some, but not all, firms issue earnings forecasts, a situation we describe as being one of discretionary disclosure.

3. Equity Premium with a Single Risky Asset

We want to analyze how disclosure affects both the equity premium and a firm's beta. To do so, we need to construct both of these measures. However, the effects are most clearly discernable if we break the analysis into steps. We therefore start with a single risky asset. Although there is no beta on the firm, this is the simplest setting with an equity premium. First, we analyze the benchmark cases of mandatory no and mandatory full disclosure to show how mandatory disclosure of a public signal can affect the equity premium. Second, we analyze how, when left to the discretion of a manager, the manager's decision to disclose affects the equity premium.

3.1 Equity Premium with Mandatory No Disclosure: Single Risky Asset

Consider a market with a risk-free government bond and a single risky stock. This stock represents a claim on a firm with a risky investment project in place such that the future liquidating values of the firm are summarized in $\widetilde{U}$, where the tilde indicates a random variable. When the market opens, it is common knowledge that the liquidating value follows a normal distribution with mean, μ, and variance, $\sigma_U^2 > 0$. There are I investors who select their portfolios taking as given the return on the bond, $(R_f - 1)$, and the stock price, P. When possible, we impose the parameter restriction that stock prices are non-negative.[10] Each individual investor $i = 1, \ldots, I$ has initial wealth of ${W_i}^0$ and constant absolute risk-aversion, a_i. Investor i spends B_i on bonds and purchases a fraction S_i of the firm to maximize his expected utility, that is,

$$\max_{S_i, B_i} E\left[-exp\left\{-a_i \widetilde{W}_i\right\}\right]$$

$$s.t. \widetilde{W}_i = B_i R_f + S_i \widetilde{U}$$

$$B_i + S_i P \leq {W_i}^0.$$

[10] In this initial setting, a sufficient condition is that $\mu \geq a\sigma_U^2$. While implausible, we only consider unlimited liability firms to maintain tractability under mandatory disclosure.

All investors take as given the market-clearing stock prices, that is, $\sum_{i=1}^{I} \boldsymbol{S}_i^*(\boldsymbol{P}) = \mathbf{1}.$ We are interested in the *equity premium*: the expected excess return on the market portfolio (represented by a single stock) over bonds, that is, $ER = E[\tilde{R}] - R_f$ where the (gross) stock returns are $\tilde{R} = \widetilde{U}/P$.[11] In terms of expected returns and the standard deviation of returns, the upward sloping line in Figure 1 represents investors' efficient portfolios consisting of the bond (on the vertical axis) and the single risky stock. The equity premium arises as indicated on the vertical axis of Figure 1. For ease of exposition, we denote the aggregate risk tolerance by $\boldsymbol{a}^{-1} = \sum_{i=1}^{I} \boldsymbol{a_i}^{-1}$ as in Wilson (1968) and normalize the return on bonds to zero, that is, $R_f = 1$. In this setting, the market-clearing stock price is $P^{No} = E[\widetilde{U}] - aVAR[\widetilde{U}]$, where the "No" in the superscript denotes that no disclosure is mandatory. The associated stock returns are normally distributed (see Jorgensen and Kirschenheiter, 2015) in this regime and the equity premium becomes $ER^{No} = {aVar[\widetilde{U}]}/{P^{No}}$. Hence, the equity premium increases in investors' aggregate risk aversion and in the variance of the liquidating values, and decreases in the expected liquidating value.

While this stylized setting includes only a single stock, we interpret this stock's excess return as the equity premium on the market portfolio to gain initial intuition.

3.2 Equity Premium under Mandatory Full Disclosure: Single Risky Asset

Modify the above scenario such that sometime between when the market opens and the liquidation of the firm, a manager incurs a cost, c, to publicly disclose a signal, $\tilde{Y}$, reporting information about the liquidating value perturbed by measurement error, $\tilde{\varepsilon}$, such that $\tilde{Y} = \widetilde{U} + \tilde{\varepsilon}$. The measurement error, $\tilde{\varepsilon}$, is independent of firm value and normally distributed with mean zero and variance, σ_ε^2. Consequently, the public signal is also normally distributed, that is, $\tilde{Y} \sim N(\mu, \sigma^2)$, where $\sigma^2 = \sigma_U^2 + \sigma_\varepsilon^2$. Subsequent to the disclosure of the public signal, each investor's decision problem entails selection of the

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[11] As usual, we use E for expected values, VAR for variances, and COV for covariance.

portfolio that maximizes expected utility given the observed signal, $\tilde{Y} = y$. For any signal, the market price ensures that the demand for shares equals the supply for shares, that is, $\sum_{i=1}^{I} \boldsymbol{S}_i^* \left(\boldsymbol{P}(\tilde{\boldsymbol{Y}} = \boldsymbol{y})\right) = \mathbf{1}$. The resulting market-clearing stock price is:

$$P^{Full}(\tilde{Y} = y) = E[\tilde{U}|\tilde{Y} = y] - c - aVAR[\tilde{U}|\tilde{Y} = y] \qquad (1.a).$$

"Full" denotes that full disclosure is mandatory. For any disclosed signal $\tilde{Y} = y$, the associated stock returns, $R^{Full} = U/P^{Full}(\tilde{Y} = y)$, are normally distributed. Since the variance and covariance for conditional random variables do not depend on the realization, we simplify the subsequent notation by suppressing the realization. Here, the equity premium would be

$$ER^{Full} = E[R^{Full}(\tilde{Y} = y)|\tilde{Y} = y] - R_f = \frac{E[\tilde{U}|\tilde{Y}=y]-c}{P^{Full}(\tilde{Y}=y)} - R_f = \frac{aVar[\tilde{U}|\tilde{Y}]}{P^{Full}(\tilde{Y}=y)}.$$

We ignore realizations of the signal for which the equity premium becomes negative since in that case, after observing the signal, all investors would prefer to invest in the bond since bond returns second order stochastically dominate the returns on the stock. This possibility becomes remote as the parameter for the expected payoff, μ, increases. With this caveat, the equity premium continues to increase in the investors' aggregate risk aversion, and decrease in the expected liquidating value. Furthermore, we can make the following observation concerning the impact of the disclosed signal, the variance of the liquidating value and the quality of the disclosure in a mandatory disclosure regime.

Observation: *The equity premium decreases in the signal $\tilde{Y} = y$. The equity premium increases (decreases) in the variance of the cash flow, σ_U^2, for $\mu > c$ ($\mu < c$). The equity premium decreases (increases) in information quality, $1/\sigma_\varepsilon^2$, for $y > c$ ($y < c$).*

The intuition for the comparative static results summarized in the Observation can best be understood by relating the equity premium to price. The equity premium here is simply the excess expected return to the single risky asset and the expected return varies directly with the ratios of the conditional variance of the terminal cash flow over the price of the asset. The first result is immediate: the conditional variance does not change with the signal while the price increases in the signal, as a higher signal indicates higher terminal cash flow. The next two results are more complicated.

The impact on the equity premium of increasing either information quality or the cash flow variance is complicated by the impact of increasing either of these parameters on price. Consider first an increase the cash flow variance, σ_U^2. Increasing the cash flow variance increases the conditional variance, but the net effect may to be increase price, since it may increase the mean more than it increases the conditional cash flow variance. This occurs if the prior mean is sufficiently high relative to the cost of disclosure. [12] Next, considering an increase in information quality, the analysis is exactly analogous. The conditional variance decreases in information quality, but the price may decrease if the signal is sufficiently low, with the net effect being to increase the equity premium.

The preceding observation establishes that public disclosures can affect the equity premium in future stock returns in a non-trivial matter. Further, the information quality of the public disclosure affects the equity premium. Consequently, the design of empirical may be affected. For example, it is not *a priori* clear whether event studies should, or should not, use returns or market-adjusted returns.

3.3 Equity Premium under Discretionary Disclosure: Single Risky Asset

Expanding on the previous section, consider next the scenario where the provision of public information to the capital market is left to the manager's discretion. Assume the manager maximizes firm value, net of any disclosure cost, by her choice to either (i) disclosure publicly and truthfully or (ii) withhold the noisy signal taking as given the inferred disclosure strategies of the investors and her information, $\tilde{Y} = y$. Thus, the manager's disclosure strategy is characterized by a disclosure threshold, x, such that the

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[12] The cut-off where the sign switches depends on our assumption that $R_F = 1$. Relaxing this assumption changes the cut-off, but the thrust of the result remains the same.

manager discloses when $y \geq x$, and otherwise withholds. Investors hold the common prior belief that the manager's disclosure threshold is $\hat{x}$. In equilibrium, investors correctly anticipate the manager's disclosure strategy, which implies that $x = \hat{x}$. As above, investors set the prices of each firm to clear the market for their shares. When the manager chooses to disclose, i.e., when the information is such that $y \geq x$, market clearing arises when $\sum_{i=1}^{I} S_i^* \left(P(\tilde{Y} = y) \right) = 1$; otherwise $\sum_{i=1}^{I} S_i^* \left(P(\tilde{Y} \leq x) \right) = 1$.

Jorgensen and Kirschenheiter (2015) show that when a manager voluntarily discloses, the price is as described in (1.a) in the previous section. Consequently, for any disclosed signal, $\tilde{Y} = y$, the equity premium is the same when (i) disclosure is mandatory and (ii) disclosure is discretionary and then the manager chooses to disclose. The effect of signal quality on the equity premium is also the same in these two cases. We therefore direct our discussion to the case in which the manager does not disclose. Given no disclosure, rational investors correctly infer that $\tilde{Y} \leq x$ and update their priors accordingly. Subsequent to investors' updating of their beliefs, neither the future liquidating dividends nor the future stock return, $R(\tilde{Y} \leq x)$, are normally distributed. In this case, expected cash flow, conditional on no disclosure, can be expressed as

$$E[\tilde{U} | \tilde{Y} \leq x] = \mu - \sigma_U^2 \alpha(x),$$

where $\alpha(y) \equiv f(y)/F(y)$ is the anti-hazard rate of signal, the ratio of the probability density function of the signal divided by the cumulative density function of the signal. Jorgensen and Kirschenheiter (2015) establish that the equilibrium price of the firm is

$$P^{DD}(\tilde{Y} \leq x) = \mu - a\sigma_U^2 - \sigma_U^2 \alpha(x + a\sigma_U^2) \qquad (1.b).$$

Superscript "DD" indicates disclosure is discretionary. Then, given no disclosure, the equity premium required by risk-averse investors is

$$ER^{DD} = E[R^{DD}(\tilde{Y} \leq x)] - R_f = \frac{E[\tilde{U} | \tilde{Y} \leq x]}{P^{DD}(\tilde{Y} \leq x)} - R_f = \frac{\upsilon(x)}{P^{DD}(\tilde{Y} \leq x)},$$

where we define the function $\upsilon(x)$ as $\upsilon(x) \equiv E[\tilde{U} | \tilde{Y} \leq x] - P^{DD}(\tilde{Y} \leq x)$. This representation suggests that the equity premium continues to depend on information quality, $1/\sigma_\varepsilon^2$, when the manager voluntarily refrains from disclosing.

We have now documented how the equity premium is affected by the public signal. In the next section, we explore whether disclosure affects individual firms' betas.

4. Beta and Informational Quality with Two Risky Assets

In this section, we expand our analysis to allow investors to invest in two risky stocks indexed by $j = 1{,}2$. Except for subscripts for the firm, all other aspects of the model remain the same as in the last section. In particular, we assume that the cash flows of the two firms are independently distributed. In this way, we facilitate the analysis of the impact of disclosures and information quality on firm betas in as simple a setting as possible. However, our results are robust to the introduction of multiple stocks and correlation between liquidating dividends. To see this, consider how our model would accommodate expansion.

In general when there are multiple $(J > 1)$ stocks, the market portfolio is the sum of all stocks and its initial value is $P_m = \sum_{j=1}^{J} P_j$. The return on the market portfolio (gross of disclosure costs incurred, if any) is therefore $\tilde{R}_m = \left(\sum_{k=1}^{J} \tilde{U}_k\right)/P_m$. Two observations apply to all disclosure settings that we consider.

First, two-fund separation continues to hold, that is, $S_{ij}^{*} = a_i^{-1} a$ where $a^{-1} = \sum_{i=1}^{I} {a_i}^{-1}$ denotes the aggregate risk tolerance as in Wilson (1968). This implies that even when there are multiple stocks, two funds suffice for characterizing the efficient portfolio set. In standard portfolio theory this implies that the efficient frontier can be spanned by two (well-chosen) funds. These funds represent a line, or parable, in mean-variance space depending on whether risk-free bonds are, or are not, available to investors. In any equilibrium with two-fund separation, all investors hold the same fraction of all stocks. Furthermore, investors' demand for stocks and bonds vary based only on what fraction of their wealth is invested in bonds or in the market portfolio.

Second, the CAPM Pricing relation holds for each stock j. This means there is some constant, β_j, such that expected excess return for firm j can be expressed as $ER_j = \beta_j ER_m$, where ER_j is the excess return on stock j and ER_m is the market equity premium or the expected excess return on the market portfolio. In each disclosure regime, we denote the value weight of each stock in the market portfolio by $\omega_j = P_j/P_m$, and use superscripts "No", "Full" and "DD" to denote the regime. Since the prices will vary according to the disclosure regime, so will these weights. Clearly, these portfolio weights add to one by construction: $\sum_{j=1}^{J} \omega_j = 1$. In addition, recall that the value-weighted average of betas sum to one, that is, $\sum_{j=1}^{J} \omega_j \beta_j = 1$. While we restrict attention to an economy with two risky assets, it should be clear that the extension to an economy with more than two risky assets does not affect our results. (See Appendix B for more details.)

4.1 Beta and Informational Quality under Mandatory No Disclosure

We start our analysis by deriving the beta and equity premium with disclosure prohibited. After observing the prices of both stocks, each investor i demands $S_{ij}(P_1, P_2)$ shares of firm j's stock and spends $B_i(P_1, P_2)$ on bonds to maximize the expected utility of his terminal wealth, $\widetilde{W}_i = B_i R_f + S_{i1}\widetilde{U}_1 + S_{i2}\widetilde{U}_2$. In this setting, the market clearing stock prices are $P_j^{No} = \mu_j - a\sigma_{Uj}^2$ for $j = 1,2$. When there are two (or more) stocks, the investors' efficient portfolios are still spanned by the risk-free bond and the risky market portfolio. As in section 3.1, all efficient portfolios lie on an upward sloping line whose origin is the return on the risk free bond intersecting the market portfolio, as shown in Figure 2.[13] Hence, a straightforward derivation generates the betas that result in the mandatory no disclosure regime, as the following Theorem clarifies.

Theorem 1 *With mandatory no disclosure, the beta can be calculated in the usual manner as*

[13] Alternatively, instead of standard deviation of returns, we could have measured betas on the horizontal axis. In such a graph, the market portfolio would have a beta of one and the equity premium would have been the slope of the upwards sloping (capital market) line.

$$\beta_j^{No} = \frac{COV[\tilde{R}_j^{No}, \tilde{R}_m^{No}]}{VAR[\tilde{R}_m^{No}]} = \left(\omega_j^{No}\right)^{-1}\left(COV\left[\tilde{U}_j, \sum_{k=1}^{2} \tilde{U}_k\right] \Big/ VAR\left[\sum_{k=1}^{J} \tilde{U}_k\right]\right)$$

where the superscript "No" indicates that there was no disclosure prior to investors' trading and as earlier defined, $\omega_j^{No} = \frac{P_j^{No}}{P_1^{No}+P_2^{No}}$.

The first equality in Theorem 1 simply restates the standard equation for the CAPM beta, while the second equality confirms that the value-weighted sum of the betas equals one. In this setting, the variance of stock returns and their betas depends on the initial uncertainty as we show in the following Corollary to Theorem 1.

Corollary 1 *With mandatory no disclosure, the following comparative static results hold:*

a. *the variance of the return on the stock of firm* j *is increasing in the variance of firm* j*'s cash flow,*
b. *the beta for firm* j *is increasing in the variance of firm* j*'s cash flow, and*
c. *the beta for firm* j *is decreasing in the variance of firm* k*'s cash flow, for* $k \neq j$.

The intuition for the first result is straightforward: increasing the initial uncertainty flows through to increase the uncertainty over the return on the stock. The intuition for the next two comparative static results then follow immediately from this result and the expression for beta. The beta for firm *j* can be expressed as the covariance of the return on firm *j* with the market over the variance on the market return. Increasing the initial uncertainty on the cash flow of the firm increases it's own return variance (equal to the numerator) more than it increases the market return variance. Increasing the initial uncertainty on the cash flow of the competing firm increases the market return variance, while leaving the covariance term unaffected.

4.2 Beta and Information Quality under Mandatory Full Disclosure

Next, we analyze the betas and equity premium when disclosure is mandatory. In this setting, a public signal of its liquidating dividend is disclosed for each stock. We assume that the signals are independent, so that the price is given as follows for

$$P_j^{Full}(\tilde{Y}_1 = y_1, \tilde{Y}_2 = y_2) = E[\tilde{U}_j | \tilde{Y}_j = y_j] - c_j - aVAR[\tilde{U}_j | \tilde{Y}_j] \quad \text{(2.a).}$$

Since the variance and covariance for conditional normal random variables do not depend on the realization, we simply the notation by omitting this realization. Clearly equation (2.a) is analogous to the price for a single risky asset under mandatory disclosure from equation (1.a). We next use these prices from (2.a) to represent the betas.

Theorem 2 *Under mandatory disclosure, suppose that* $(\tilde{Y}_1 = y_1, \tilde{Y}_2 = y_2)$ *has been disclosed. Then systematic risk,* β_j^{Full}*, is priced as follows:*

$$\beta_j^{Full}(y_1, y_2) = \frac{COV[\tilde{R}_j^{Full}, \tilde{R}_m^{Full} | \tilde{Y}_1 = y_1, \tilde{Y}_2 = y_2]}{VAR[\tilde{R}_m^{Full} | \tilde{Y}_1 = y_1, \tilde{Y}_2 = y_2]} = \left(\omega_1^{Full}(y_1, y_2)\right)^{-1} \left(\frac{COV[\tilde{U}_j, \sum_{k=1}^{J} \tilde{U}_k | \tilde{Y}_1, \tilde{Y}_2]}{VAR[\sum_{k=1}^{J} \tilde{U}_k | \tilde{Y}_1, \tilde{Y}_2]}\right),$$

where, in this case, $\omega_j^{Full}(y_1, y_2) = \frac{P_j^{Full}(y_1, y_2)}{P_1^{Full}(y_1, y_2) + P_2^{Full}(y_1, y_2)}$.

First, Theorem 2 states that under mandatory disclosure, the beta is calculated using conditional distributions taking into account the disclosed signals. In concert, Theorems 1 and 2 confirm that when there is either no information or full information available, each beta can be calculated as the covariance between the return for the individual firm's stock return and the market return divided by the variance of the market return. The betas provided by Theorems 1 and 2, in the absence of disclosure and under mandatory disclosure, differ only in the relevant distribution: CAPM in Theorem 1 is based on the unconditional distribution, while CAPM in Theorem 2 is based on the conditional distribution given the two signals.

Second, Theorem 2 also provides a characterization of the betas under mandatory disclosure exploiting that prices are market clearing. This alternative representation does not use the ratio of the covariance over the variance of the cash flows. Instead, the final term in this expression, is the ratio of (i) the covariance of the firm j's cash flows with the market portfolio's cash flows over (ii) the variance of the total cash flows from the market portfolio. Theorem 2 shows that the beta for firm j can be expressed as the product of this ratio times the inverse weight of asset j in the market portfolio, ω_1^{-1}. This alternative characterization proves useful in conveying how the quality of disclosure and the beta are related in the results that are shown below.

One might suspect that the results of Theorems 1 and 2 extend to a setting with discretionary disclosure. In the next section, however, we establish that these results do not generalize to discretionary disclosure. However, before proceeding to that analysis, we wish to clarify how changes in the model parameters affect the betas.

First, suppose the two firms are symmetric *a priori*, so that they have face the same disclosure costs and have the same distributional parameters, then the firm with the higher (lower) signal trades at a higher (lower) price and has a beta below (above) one. These relations hold because the betas are increasing and decreasing in their own disclosure cost and signal, respectively, while they are decreasing and increasing in the disclosure cost and signal, respectively, of the competing firm. These relationships are summarized in the following Corollary.

Corollary 2.1 *Under mandatory full disclosure, firm j has a beta that*

a. *increases in firm $j's$ disclosure cost, c_j, and decreases in firm$j's$ signal, y_j , and*

b. *decreases in firm $k's$ disclosure cost, c_k, and increases in firm $k's$ signal, y_k, for $k \neq j$.*

Corollary 2.1 shows how the beta of each firm depends on disclosure costs and signal of that firm and the competing firm. The intuition for these results can be seen by reference to the characterization of the betas as the product of the inverse market portfolio weight, times the ratio of the covariance to the variance of the cash flows. Neither the disclosure costs nor the realized signal affect the covariance or the variance of the cash flows. Hence, disclosure costs and signals affect the betas only through price, that is, only through the weight of the asset in the market portfolio. Increasing the disclosure cost or decreasing the signal for firm j will decrease price of firm j. This raises the inverse weight of firm j in the market portfolio, denoted as ω_j^{-1}, in turn increasing $\beta_j^{Full}(y_1, y_2)$. An analogous argument applies to changes in the disclosure cost and signal of the other firm $k \neq j$, causing it to have the reverse impact on the beta of firm j.

Next, we consider information quality. More specifically, we wish to know how beta changes when information quality increases. We expect that price decreases as a result of increased uncertainty with a lower disclosure quality. Lower price will lead to a higher return and hence a higher beta in a manner analogous to the impact of a decrease in the signal. This intuition in fact holds, as the next Corollary indicates.

Corollary 2.2 *Under mandatory full disclosure, firm j has a beta that*

a. *decreases as the quality of firm $j's$ signal, (as measured by $1/\sigma_{\varepsilon j}^2$) increases if and only if y_j is sufficiently large, and that*
b. *increases as the quality of firm $k's$ signal (as measured by $1/\sigma_{\varepsilon k}^2$) increases if and only if y_k is sufficiently large, for $j \neq k$.*

Corollary 2.2 formalizes the intuition that higher disclosure quality (or lower $\sigma_{\varepsilon j}^2$) increases stock price and hence, decreases stock returns which, in turn, decreases beta. This intuition is simplistic since, at the same time, lower disclosure quality raises the ratio of the covariance over the variance of the cash flows. However, these effects reinforce each other as long as the signal is large enough. For extremely negative values of the signal, the effect on the price is so pronounced that it overwhelms the effect on the cash flows. In such extreme cases, the derivative reverses, and a decrease in quality will actually decrease the beta. This non-monotonic effect of public disclosures on beta could explain the paucity of empirical evidence on this relationship.

A final point is warranted regarding the pricing of the idiosyncratic risk under mandatory disclosure. In a finite economy, firm specific cash flow risk is not fully diversifiable since the market portfolio is still affected by the investors' beliefs about the residual risk for all firms. In the limit, as the number of firms (J) goes to infinity, this effect disappears and all betas are independent of the discretionary disclosure decision. In the limit, the demand for discretionary disclosures should also fade. However, as long as the economy is finite, the discretionary disclosure decision is driven by the idiosyncratic risk. This is true even when this risk has a negligible impact on the pricing of the risky stocks, relative to the systematic risk.

4.3 Beta and Informational Quality under Discretionary Disclosure

When the disclosure decision is discretionary then, as in section 3.3., each manager maximizes firm value, net of any disclosure cost, by her choice of either disclosing or withholding her signal.[14] There are four cases to consider: (i) neither manager discloses, (ii) only manager 1 discloses, (iii) only manager 2 discloses, and (iv) both managers disclose. In parallel with the discussion in section 3.3, case (iv) where both managers disclose results in the same prices, the same betas, and the same effect of information quality on beta as under mandatory disclosure. Further, cases (ii) and (iii) are symmetric. Hence our discussion of betas initially is confined to the remaining two cases.

Theorem 3 *Let x_j be the inferred disclosure threshold for firm j and define the function $\upsilon_j(x_j)$ as*

$$\upsilon_j(x_j) \equiv E[\tilde{U}_j | \tilde{Y}_j \leq x_j] - P_j[\tilde{Y}_j \leq x_j].$$

[14] While one might worry about the sequence of the two managers' disclosure decisions, the results remain robust to sequential disclosure decisions, see Jorgensen and Kirschenheiter (2012).

Then the betas in a discretionary disclosure regime can be written as follows.

(a) If neither firm manager discloses, so that investors infer that $\tilde{Y}_1 \leq x_1$ *and* $\tilde{Y}_2 \leq x_2$*, then*

$$\beta_j^{DD}(x_1, x_2) = \omega_j^{-1}\, v_j(x_j) / \big(v_1(x_1) + v_2(x_2)\big) \textit{ for } j = 1,2.$$

(b) Assume that only firm manager 1 discloses, that is, investors observe that $\tilde{Y}_1 = x_1$ *and that manager 2 chose to not disclose which led investors to infer that* $\tilde{Y}_2 \leq x_2$*, then*

$$\beta_1^{DD}(y_1, x_2) = \omega_j^{-1}\, Var\big[\tilde{U}_1 | Y_1\big] / \Big(Var\big[\tilde{U}_1 | Y_1\big] + v_2(x_2)\Big)$$

$$\beta_2^{DD}(y_1, x_2) = \omega_j^{-1}\, v_2(x_2) / \Big(Var\big[\tilde{U}_1 | Y_1\big] + v_2(x_2)\Big).$$

Casual inspection of Theorem 3 suggests that all betas depend on the managers' disclosure decisions – as captured by the disclosure thresholds – in a non-trivial matter.[15] Theorem 3 (b) reveals that when one firm manager elects to not disclose, both betas still depend on the information voluntarily provided by the other manager. The betas varying with the manager's disclosure is consistent with our results presented when disclosure is mandatory. We formalize this below.

Corollary 3.1 *Assume that, under discretionary disclosure, only firm manager 1 discloses. Then the beta of each firm will vary in the disclosed signal,* y_1*.*

Two observations are in place. First, betas continue to vary with the disclosures. Second, how the beta of a disclosing firm varies with that firm's disclosure, say y_1, depends on whether the other firm manager chooses to either disclose or withhold her information. We therefore predict that the empirical association from publicly disclosed information to the firms' betas is modified by whether the disclosure decision is mandatory or discretionary.

[15] To simplify the representation in Theorem 3, we suppress that the value weights, ω_j, also depend on the disclosure decisions. Our proofs and discussion take this into account. Further, the disclosure thresholds are defined independently when future firm values are independent.

As mentioned earlier, one might extrapolate from the results of Theorems 1 and 2 that the betas of Theorem 3 can also be calculated in the usual manner. Corollary 3.2 states that this would be incorrect.

Corollary 3.2 *When disclosure is discretionary and at least one firm manager refrains from disclosing, then the CAPM beta is* <u>*not*</u> *calculated as the ratio of the covariance between a stock's return and the return on the market portfolio over the variance of the return on the market portfolio.*

Under mandatory disclosure, the stock returns subsequent to the public disclosures by all firm managers are normally distributed with conditional variance, $VAR\left[\tilde{U}_j|\tilde{Y}_j\right]$. In contrast, the return on a stock after the rational investors observe that the manager made no discretionary disclosure reflects the unfavorable (undisclosed) news inferred by investors. This truncation in investors' updating of their beliefs introduces non-normality in the stock's returns even though investors' initial priors are that future firm values are normally distributed. This implies that the returns on the market portfolio cannot be normally distributed after investors observe that some firm manager did not disclose. Since the beta could be viewed as arising from an imaginary regression of individual stock returns on the return on the market portfolio, it is not surprising that truncation from investors' beliefs rule out the usual beta representation. One can think of the betas in Theorem 3 as being risk-adjusted since they represent investors' updated expectations defined over risk adjusted variables.[16]

[16] The intuition for these risk adjustments is analogous to the adjustments required for stock options in Rubinstein (1976), see Jorgensen and Kirschenheiter (2003) for more details. Cumulative abnormal returns calculated from standard betas, without using our correction for non-disclosure, would not rinse returns of the effect of the market risk. The problem arises because, in equilibrium, betas reflect the incremental risk from non-disclosure by some firm managers. This mis-specification of the beta applies to firms that disclose voluntarily as well as to firms that do not disclose.

With regards to comparison of disclosure regimes, our work indicates that introducing mandatory disclosures of information that had previously been voluntarily disclosed will increase at least some of the betas. This is an application of the classic argument regarding the introduction of mandatory disclosures, beyond the disclosures that would naturally arise in the absence of regulation. When managers choose their disclosure policy to maximize the current market value of their firm, the consequence of mandating disclosure is to increase the information in future stock returns causing a net (expected) loss to current shareholders. Empirical tests documenting value relevance of a particular disclosure do not suffice to support requiring that disclosure be mandatory. This highlights the necessity for additional research on discretionary disclosure, especially directed at the cost of disclosures.

5. SUMMARY

The contribution of this paper is two-fold. First, we consider a setting in which public disclosure is mandatory for all firms, such as earnings announcements. We establish how managerial disclosures and their quality affect the betas of individual stock returns and the expected excess return on the market portfolio. Second, we consider a setting in which public disclosure is discretionary, left to the discretion of firm managers, such as management earnings forecasts. We find a non-trivial relation between public disclosures and their quality on excess returns and betas. This relation is complex when at least one firm manager chooses to strategically withhold her information (i.e., chooses to not disclose). We establish that CAPM pricing -- the proportional relationship between the excess return of each firm and the equity premium -- does hold for all firms in the settings that we consider. We establish two results.

First, beta is traditionally expressed as the covariance between the individual stock's return and the return on the market portfolio divided by the variance of the return of the market portfolio. Intuitively, this seems consistent with estimating betas empirically by a regression. We show here that this representation of betas is appropriate when all firms are prohibited from providing information and when disclosure is mandatory. When disclosure is discretionary, however, betas no longer allow this representation, provided at least one firm manager chooses not to disclose. The intuition is that selective non-disclosure introduces a truncation in investors' perception of the distribution of stock returns that undermines the regression analogy.

Second, using the return on all stocks as the market portfolio, we find that the beta of a firm is affected by disclosures of other firms. Alternatively, we could have used the return on all stocks and all bonds as the market portfolio and calculated different betas accordingly. These alternative betas would be less subject to spillover effects of one firm's disclosure to other firms' betas.[17]

For ease of exposition, our results are presented in an economy where two firms' liquidating cash flows are assumed independent. Given two fund separation, it is not surprising that our results extend to any finite number of firms. In addition, we have established that the results reported here are robust to the introduction of correlated liquidating values (we omit the details of this extension). In conclusion, our results provide additional support for the empirical evidence on the association between management disclosures and beta. However, our results also suggest care must be exercised in the construction of empirical tests to reflect selective non-disclosures by firm management.

[17] The resulting value-weighted betas would still add up to one. We support these claims by the results (and their proofs) reported in Appendix B.

6. REFERENCES

Admati, Anat R. and Paul Pfleiderer. (2000). "Forcing Firms to Talk: Financial Disclosure Regulation and Externalities." *Review of Financial Studies*, 13 (3), 479-519.

Baginski, Stephen P. (1987). "Intra-industry Information Transfers Associated with Management Forecasts of Earnings." *Journal of Accounting Research*, 25 (2), 196-216.

Barry, Christopher B., and Stephen J. Brown. (1985). "Differential Information and Security Market Equilibrium." *Journal of Financial and Quantitative Analysis*, 13 (December), 407-422.

Beardsley, Colin, and John R. O'Brien. (2003a). "Detecting Shifts in the Voluntary Disclosure Threshold: A Reexamination of the Dispersion Results Associated with the 1933 SEC Act." Working paper, Carnegie Mellon University.

Beardsley, Colin, and John R. O'Brien. (2003b). "Measuring the Impact of New Regulation on Market Stability: Evidence from the U.S., U.K. and Australian Markets." Working paper, Carnegie Mellon University.

Bertomeu, Jeremy, Anne Beyer and Ronald Dye, (2011). "Capital Structure, Cost of Capital and Voluntary Disclosure." *The Accounting Review*, 86 (3), 857–886.

Bertomeu, Jeremy and Edwige Cheynel, 2016. "Disclosure and the Cost of Capital: A Survey of the Theoretical Literature". *Abacus*, 52 (2), 221 – 258.

Botosan, Christine A. (1997). "Disclosure Level and the Cost of Equity Capital." *The Accounting Review*, 72 (July), 323-349.

Botosan, Christine A., and Marlene Plumlee. (2002). "A Re-examination of Disclosure Level and Expected Cost of Equity Capital." *Journal of Accounting Research*, 40 (March), 21-40.

Bushee, Brian J., and Christopher F. Noe. (2000). "Corporate Disclosure Practices, Institutional Investors, and Stock Return Volatility." *Journal of Accounting Research*, 38 (Supplement), 171-202.

Cheynel, Edwige. 2013. "A Theory of Voluntary Disclosure and Cost of Capital." *Review of Accounting Studies*, 18 (4), 987-1020.

Clinch, Greg, and Robert E. Verrecchia. (2015). "Voluntary Disclosure and the Cost of capital." *Australian Journal of Management*, 40 (2), 201-223.

Coles, Jeffrey L., and Uri Loewenstein. (1988). "Equilibrium Pricing and Portfolio Composition in the Presence of Uncertain Parameters." *Journal of Financial Economics* 22 (December), 279-303.

Coles, Jeffrey L., Uri Loewenstein, and Jose Suay. (1995). "On Equilibrium Pricing Under Parameter Uncertainty." *Journal of Financial and Quantitative Analysis* 30 (3), 347-364.

Diamond, Douglas W., and Robert E. Verrecchia. (1991). "Disclosure, Liquidity, and the Cost of Capital." *The Journal of Finance* 66 (September), 1325-1355.

Dye, Ronald A. (1985). "Disclosure of Nonproprietary Information." *Journal of Accounting Research*, 23 (1), 123-145.

Dye, Ronald A. (1990). "Mandatory versus Voluntary Disclosures: The Case of Financial and Real Externalities." *The Accounting Review*, 65 (January), 1-24.

Frankel, Richard M., Maureen McNichols, and G. Peter Wilson. (1995). "Discretionary Disclosure and External Financing." *The Accounting Review*, 70 (January), 135-150.

Gao, Pingyang. 2010. "Disclosure Quality, Cost of Capital, and Investor Welfare." *The Accounting Review*, 85 (1), 1-29.

Hakansson, Nils, J. Gregory Kunkel, and James A. Ohlson (1982). "Sufficient and Necessary Conditions for Information to Have Social Value in Pure Exchange." *The Journal of Finance*, 37 (December), 1169-1181.

Healy, Paul, Amy Hutton, and Krishna Palepu. (1999). "Stock Performance and Intermediation Changes Surrounding Sustained Increases in Disclosure." *Contemporary Accounting Research*, 16 (Fall), 485-520.

Hope, Ole-Kristian. (2003). "Disclosure Practices, Enforcement of Accounting Standards and Analysts' Forecast Accuracy: An International Study." *Journal of Accounting Research*, 41 (2), 235-272.

Hughes, John S., Jing Liu, and Jun Liu. (2007). "Information Asymmetry, Diversification, and Cost of Capital." *The Accounting Review*, 82 (3), 705–729.

Hughes, John S., and Suil Pae. (2004). "Voluntary Disclosure of Precision Information." *Journal of Accounting and Economics*, 37 (2), 261–289.

Imhoff, Jr., Eugene A., and Jacob K. Thomas. (1994). Accounting Quality. In Stephen A. Butler (ed.), *Asset Valuation*. Norman: The University of Oklahoma.

Johnston, David, (2016). "The Effect of Information on Uncertainty and the Cost of Capital. *Contemporary Accounting Research*, 33 (Summer), 752–774.

Jorgensen, Bjorn N., and Michael T. Kirschenheiter. (2003). "Discretionary Risk Disclosures." *The Accounting Review*, 78 (April), 449-469.

Jorgensen, Bjorn N., and Michael T. Kirschenheiter. (2012). "Interactive Discretionary Disclosures." Journal of *Contemporary Accounting Research*, 29 (Summer), 382-397.

Jorgensen, Bjorn N., and Michael T. Kirschenheiter. (2015). "Discretionary Disclosures with Risk-Averse Traders: A Research Note." Journal of *Contemporary Accounting Research*, 32 (Fall), 1224-1235.

Lambert, Richard A., Christian Leuz, and Robert E. Verrecchia. (2007). "Accounting Information, Disclosure, and the Cost of Capital." *Journal of Accounting Research*, 36 (2), 385–420.

Lambert, Richard A., Christian Leuz, and Robert E. Verrecchia. (2012). "Information Asymmetry, Information Precision, and the Cost of Capital." *Review of Finance*, 16 (1), 1-29.

Lambert, Richard A., and Robert E. Verrecchia. (2015)." Information, Illiquidity, and Cost of Capital." *Contemporary Accounting Research*, 32 (2), 438-454.

Lang, Mark H., and Russell J. Lundholm. (1993). "Cross-sectional Determinants of Analyst Ratings of Corporate Disclosures." *Journal of Accounting Research*, 31 (2), 246-271.

Lang, Mark H., and Russell J. Lundholm. (1996). "Corporate Disclosure Policy and Analyst Behavior." *The Accounting Review*, 71 (4), 467-492.

Lang, Mark H., and Russell J. Lundholm. (2000). "Voluntary Disclosure During Equity Offerings: Reducing Information Asymmetry or Hyping the Stock." *Contemporary Accounting Review*, 17 (4), 623-662.

Leuz, Christian, and Robert E. Verrecchia. (2000). "The Economic Consequences of Increased Disclosure." *Journal of Accounting Research* 38 (Supplement), 91-124.

Merton, Robert C. (1987). "A Simple Model of Capital Market Equilibrium with Incomplete Information." *The Journal of Finance*, 42 (3), 483-510.

Penno, Mark. (1996). "Unobservable Precision Choices in Financial Reporting." *Journal of Accounting Research*, 34 (1), 141-149.

Rubinstein, Mark. (1976). "The Valuation of Uncertain Income Streams and the Pricing of Options." *The Bell Journal of Economics*, 7 (2), 407-425.

Verrecchia, Robert E. (1983). "Discretionary Disclosure." *Journal of Accounting and Economics*, 5 (December), 179-194.

Verrecchia, Robert E. (1990). "Discretionary Disclosure and Information Quality." *Journal of Accounting and Economics*, 12 (January), 179-145.

Verrecchia, Robert E. (2001). "Essays on Disclosure." *Journal of Accounting and Economics*, 32 (1-3), 97-180.

7. APPENDIX A Proofs of Results

We begin by summarizing some of the equations in the text. For example, the market clearing price in an economy with a single risky asset under a mandatory no disclosure regime is written as follows.

$P = E\left[\widetilde{U}\right] - aVAR\left[\widetilde{U}\right] = \mu - a\sigma_U^2.$

So the equity premium (based on the gross stock return $\tilde{R} = \widetilde{U}/P$) is given as follows.

$ER = E[\tilde{R}] - R_f = a\sigma_U^2/(\mu - a\sigma_U^2).$

Recall that we normalize the return on bonds to zero, that is, $R_f = 1$.

Next, we introduce a signal, denoted as $\tilde{Y}$, that represents reporting information about the liquidating value perturbed by an independent, normally distributed error term, $\tilde{\varepsilon}$, such that $\tilde{Y} = \widetilde{U} + \tilde{\varepsilon}$ and $\tilde{Y} \sim N(\mu, \sigma^2)$, where $\sigma^2 = \sigma_U^2 + \sigma_\varepsilon^2$. The resulting market-clearing stock price with mandatory full disclosure is written as follows.

$$P(\tilde{Y} = y) = E\left[\widetilde{U}|\tilde{Y} = y\right] - c - aVAR\left[\widetilde{U}|\tilde{Y} = y\right] = \frac{(y-c)\sigma_U^2 + (\mu - c - a\sigma_U^2)\sigma_\varepsilon^2}{(\sigma_U^2 + \sigma_\varepsilon^2)} \qquad \text{(1.a).}$$

The associated equity premium would be

$ER = E\left[R(\tilde{Y} = y)|\tilde{Y} = y\right] - R_f = \frac{E[U|\tilde{Y}=y]-c}{P(\tilde{Y}=y)} - R_f = a\{(y-c)\sigma_\varepsilon^{-2} + (\mu - c)\sigma_U^{-2} - a\}^{-1}.$

The preceding observations establish that public disclosures can affect the equity premium in future stock returns in a non-trivial matter. Consequently, it is not *a priori* clear whether event studies should, or should not, use returns or market-adjusted returns. Further, the information quality of the public disclosure affects the equity premium.

Next we consider the discretionary disclosure regime. In a discretionary regime, if the manager discloses if and only if the signal exceeds a threshold denoted as x, then the expected cash flow, conditional on no disclosure, can be expressed as

$$E\left[\widetilde{U}|\tilde{Y} \leq x\right] = \mu - \sigma_U^2\alpha(x),$$

where $\alpha(y) \equiv f(y)/F(y)$ is the anti-hazard rate of signal, the ratio of the probability density function of the signal divided by the cumulative density function of the signal. Jorgensen and Kirschenheiter (2015) establish that the price of the firm with nondisclosure is

$$P(\tilde{Y} \leq x) = \mu - a\sigma_U^2 - \sigma_U^2\alpha(x + a\sigma_U^2) \qquad \text{(1.b).}$$

Further, they show that the manager's choice of disclosure threshold is characterized as follows:

$$x = \mu - a\sigma_U^2 + \frac{\sigma^2}{\sigma_U^2}\left(c - \sigma_U^2\alpha(x + a\sigma_U^2)\right) \qquad (1.c).$$

Hence, given no disclosure, the equity premium required by risk-averse investors is

$$ER = E\left[R\left(\tilde{Y} \le x\right)\right] - R_f = \frac{E[\tilde{U}|\tilde{Y}\le x]}{P(\tilde{Y}\le x)} - R_f = \frac{\sigma_U^2\{a+\alpha\left(x+a\sigma_U^2\right)-\alpha(x)\}}{\mu-a\sigma_U^2-\sigma_U^2\alpha\left(x+a\sigma_U^2\right)}.$$

Having provided the detailed equations, we are now in position to begin the proofs, starting with the proof of the Observation.

Observation: *The equity premium decreases in the signal* $\tilde{Y} = y$. *The equity premium increases (decreases) in the variance of the cash flow,* σ_U^2, *for* $\mu > c$ ($\mu < c$). *The equity premium decreases (increases) in information quality,* $1/\sigma_\varepsilon^2$, *for* $y > c$ ($y < c$).

Proof of Observation: First notice that the following equations hold.

$$E\left[\tilde{U}\middle|\tilde{Y} = y\right] = \mu + (y-\mu)\frac{\sigma_U^2}{(\sigma_U^2+\sigma_\varepsilon^2)}, \text{ and}$$

$$VAR\left[\tilde{U}\middle|\tilde{Y} = y\right] = \sigma_{U|Y}^2 = \sigma_U^2\left\{1 - \frac{\sigma_U^2}{(\sigma_U^2+\sigma_\varepsilon^2)}\right\} = \sigma_U^2 - \frac{\sigma_U^4}{(\sigma_U^2+\sigma_\varepsilon^2)}.$$

Using these expressions and rearranging, we can rewrite the price and equity premium when disclosure is mandatory as follows:

$$P\left(\tilde{Y} = y\right) = E\left[\tilde{U}\middle|\tilde{Y} = y\right] - c - aVAR\left[\tilde{U}\middle|\tilde{Y} = y\right] = \frac{(y-c)\sigma_U^2 + (\mu - c - a\sigma_U^2)\sigma_\varepsilon^2}{(\sigma_U^2 + \sigma_\varepsilon^2)}$$

and the equity premium as

$$ER = E\left[R\left(\tilde{Y} = y\right)\middle|\tilde{Y} = y\right] - R_f = \frac{E[\tilde{U}|\tilde{Y}=y]-c}{P(\tilde{Y}=y)} - R_f = \frac{a\sigma_U^2\sigma_\varepsilon^2}{(y-c)\sigma_U^2+\left(\mu-c-a\sigma_U^2\right)\sigma_\varepsilon^2},$$

both as shown in the text of the paper. Then, the observation follows directly from by taking the derivative of the equity premium with respect to the signal, the variance of the cash flows and the quality of the disclosure, respectively. This completes the proof of the observation.

Theorem 1 *In the absence of disclosure, the beta is calculated in the usual manner as*

$$\beta_j^{No} = \frac{COV\left[\tilde{R}_j^{No}, \tilde{R}_m^{No}\right]}{VAR\left[\tilde{R}_m^{No}\right]} = \left(\left(\frac{\mu_1+\mu_2}{\sigma_{U1}^2+\sigma_{U2}^2}\right) - a\right) \Big/ \left(\left(\frac{\mu_j}{\sigma_{Uj}^2}\right) - a\right).$$

Proof of Theorem 1: First note that the following equations hold.

$$COV\left[\tilde{R}_j^{No}, \tilde{R}_m^{No}\right] = COV\left[\frac{\tilde{U}_j}{P_j}, \frac{\sum_{k=1}^{J}\tilde{U}_k}{P_m}\right] = \frac{COV\left[\tilde{U}_j, \sum_{k=1}^{J}\tilde{U}_k\right]}{P_j P_m} = \frac{VAR\left[\tilde{U}_j\right]}{P_j P_m} = \frac{\sigma_{Uj}^2}{P_j P_m},$$

and

$$VAR\left[\tilde{R}_m^{No}\right] = VAR\left[\frac{\sum_{k=1}^{J}\tilde{U}_k}{P_m}\right] = \frac{\sum_{k=1}^{J} VAR(\tilde{U}_k)}{P_m^2} = \frac{\sigma_{U1}^2+\sigma_{U2}^2}{P_m^2}.$$

Further, the prices are given as $P_1 = \mu_1 - a\sigma_{U1}^2$ and $P_m = \mu_1 + \mu_2 - a\sigma_{U1}^2 - a\sigma_{U2}^2$, so that the beta can be expressed as follows

$$\beta_1^{No} = \frac{COV\left[\tilde{R}_1^{No}, \tilde{R}_m^{No}\right]}{VAR\left[\tilde{R}_m^{No}\right]} = \left(\frac{\sigma_{U1}^2}{P_1 P_m}\right) \Big/ \left(\frac{\sigma_{U1}^2 + \sigma_{U2}^2}{P_m^2}\right) = \left(\frac{P_m}{\sigma_{U1}^2 + \sigma_{U2}^2}\right) \Big/ \left(\frac{P_1}{\sigma_{U1}^2}\right)$$

$$= \left(\frac{\mu_1+\mu_2-a\sigma_{U1}^2-a\sigma_{U2}^2}{\sigma_{U1}^2+\sigma_{U2}^2}\right) \Big/ \left(\frac{\mu_1-a\sigma_{U1}^2}{\sigma_{U1}^2}\right) = \left(\left(\frac{\mu_1+\mu_2}{\sigma_{U1}^2+\sigma_{U2}^2}\right) - a\right) \Big/ \left(\left(\frac{\mu_j}{\sigma_{Uj}^2}\right) - a\right).$$

as shown in the Theorem, and completing the proof of the Theorem.

Corollary 1 *With mandatory no disclosure,* $\frac{\partial VAR\left[\tilde{R}_j^{No}\right]}{\partial \sigma_{Uj}^2} > 0$*,* $\frac{\partial \beta_j^{No}}{\partial \sigma_{Uj}^2} > 0$*, and* $\frac{\partial \beta_j^{No}}{\partial \sigma_{Uk}^2} < 0$ *for* $k \neq j$*.*

Proof of Corollary 1: These results follow immediately from the partial differentiation of the equation in Theorem 1. As an example, consider $\frac{\partial \beta_j^{No}}{\partial \sigma_{Uj}^2} > 0$. Writing this out explicitly we get

$$\frac{\partial \beta_1^{No}}{\partial \sigma_{U1}^2} = \frac{\partial}{\partial \sigma_{U1}^2}\left(\frac{\mu_1+\mu_2}{\sigma_{U1}^2+\sigma_{U1}^2} - a\right) \Big/ \left(\frac{\mu_1}{\sigma_{U1}^2} - a\right) = \frac{-\frac{\mu_1+\mu_2}{\left(\sigma_{U1}^2+\sigma_{U1}^2\right)^2}\left(\frac{\mu_1}{\sigma_{U1}^2} - a\right) + \frac{\mu_1}{\sigma_{U1}^4}\left(\frac{\mu_1+\mu_2}{\sigma_{U1}^2+\sigma_{U1}^2} - a\right)}{\left(\frac{\mu_1}{\sigma_{U1}^2} - a\right)^2}.$$

This means that $\frac{\partial \beta_j^{No}}{\partial \sigma_{Uj}^2} > 0$ follows, since

$$-\frac{\mu_1+\mu_2}{(\sigma_{U1}^2+\sigma_{U1}^2)^2}\left(\frac{\mu_1}{\sigma_{U1}^2} - a\right) + \frac{\mu_1}{\sigma_{U1}^4}\left(\frac{\mu_1+\mu_2}{\sigma_{U1}^2+\sigma_{U1}^2} - a\right)$$

$$= \frac{a\mu_2}{(\sigma_{U1}^2 + \sigma_{U1}^2)^2} + \mu_1(\mu_1 + \mu_2 - a)\left(\frac{1}{\sigma_{U1}^4(\sigma_{U1}^2 + \sigma_{U1}^2)} - \frac{1}{\sigma_{U1}^2(\sigma_{U1}^2 + \sigma_{U1}^2)^2}\right)$$

$$=\frac{a\mu_2}{\left(\sigma_{U1}^2+\sigma_{U1}^2\right)^2} + \mu_1(\mu_1 + \mu_2 - a)\left(\frac{\sigma_{U1}^2}{\sigma_{U1}^4\left(\sigma_{U1}^2+\sigma_{U1}^2\right)^2}\right)>0.$$

Note that this derivation relied on the assumption that all stock prices are positive, i.e., that $\mu_j > a\sigma_{Uj}^2$. The other results are derived in an analogous fashion.

Theorem 2 *Under mandatory full disclosure, suppose that* $\left(\tilde{Y}_1 = y_1, \tilde{Y}_2 = y_2\right)$ *has been disclosed. Then systematic risk,* β_j^{Full}*, is priced as follows:*

$$\beta_j^{Full}(y_1, y_2) = \frac{COV\left[\tilde{R}_j^{Full}, \tilde{R}_m^{Full}\middle|\tilde{Y}_1 = y_1, \tilde{Y}_2 = y_2\right]}{VAR\left[\tilde{R}_m^{Full}\middle|\tilde{Y}_1 = y_1, \tilde{Y}_2 = y_2\right]}$$

$$= \omega_j^{-1} \frac{COV\left[\widetilde{U}_j, \sum_{k=1}^J \widetilde{U}_k\middle|\tilde{Y}_1 = y_1, \tilde{Y}_2 = y_2\right]}{VAR\left[\sum_{k=1}^J \widetilde{U}_k\middle|\tilde{Y}_1 = y_1, \tilde{Y}_2 = y_2\right]}$$

$$= \omega_j^{-1} \frac{\sigma_{Uj}^2\left(1 - \sigma_{Uj}^2/\sigma_j^2\right)}{\{\sigma_{U1}^2(1 - \sigma_{U1}^2/\sigma_1^2) + \sigma_{U2}^2(1 - \sigma_{U2}^2/\sigma_2^2)\}},$$

where

$$\omega_1^{-1} = \frac{P_m^{Full}(y_1,y_2)}{P_1(y_1,y_2)} = 1 + \frac{P_2(y_1,y_2)}{P_1(y_1,y_2)}.$$

Proof of Theorem 2: The proof of Theorem 2 follows in an analogous fashion to the proof of Theorem 1, with the difference being that now the conditional expectations and conditional distributions are used.

Assuming we are in a mandatory full disclosure regime, Corollaries 2.1 and 2.2 can be written formally as follows.

Corollary 2.1 $\frac{\partial \beta_j^{Full}(y_1,y_2)}{\partial c_j} > 0$, $\frac{\partial \beta_j^{Full}(y_1,y_2)}{\partial y_j} < 0$ *and for* $k \neq j$, $\frac{\partial \beta_j^{Full}(y_1,y_2)}{\partial c_k} < 0$, *and* $\frac{\partial \beta_j^{Full}(y_1,y_2)}{\partial y_k} > 0$.

Corollary 2.2 $\frac{\partial \beta_j^{Full}(y_1,y_2)}{\partial \sigma_{\varepsilon j}^2} > 0$ *if and only if* y_j *is sufficiently large. Analogously,* $\frac{\partial \beta_j^{Full}(y_1,y_2)}{\partial \sigma_{\varepsilon k}^2} < 0$ *if and only if* y_k *is sufficiently large for* $k \neq j$.

Proofs of Corollaries 2.1 and 2.2: The proofs of Corollaries 2.1 and 2.2 follow analogously to the proof of Corollary 1, that is, we take the partial derivative of the relevant variable with respect to beta, using the equation for beta given in Theorem 2.

Theorem 3 *Consider the setting where disclosure is discretionary and let* $\alpha_j(x_j) \equiv f_j(x_j)/F_j(x_j)$ *denote the anti-hazard rate of the signal for firm* j. *We denote the risk premium of stock* j *given no disclosure by firm manager* j *(scaled by investors' aggregate risk aversion) as*

$$v_j(x_j) \equiv E[\tilde{U}_j | \tilde{Y}_j \leq x_j] - P_j[\tilde{Y}_j \leq x_j] = \sigma_{Uj}^2 \left(1 + \frac{\alpha_j(x_j + a\sigma_{Uj}^2) - \alpha_j(x_j)}{a}\right),$$

where x_j *is the disclosure threshold specified in (1.b).*

(a) Assume that neither firm manager chose to disclose, that is, investors infer that $\tilde{Y}_1 \leq x_1$ *and* $\tilde{Y}_2 \leq x_2$, *then*

$$\beta_j^{DD}(x_1, x_2) = \omega_j^{-1} \frac{V_j(x_j)}{\{V_1(x_1) + V_2(x_2)\}} \text{ for } j = 1,2.$$

(b) Assume that only firm manager 1 discloses, that is, investors observe that $\tilde{Y}_1 = x_1$ *and that manager 2 chose to not disclose which led investors to infer that* $\tilde{Y}_2 \leq x_2$, *then*

$$\beta_1^{DD}(y_1, x_2) = \omega_1^{-1} \frac{\sigma_{U1}^2(1 - \sigma_{U1}^2/\sigma_1^2)}{\{\sigma_{U1}^2(1 - \sigma_{U1}^2/\sigma_1^2) + V_2(x_2)\}}$$

$$\beta_2^{DD}(y_1, x_2) = \omega_2^{-1} \frac{V_2(x_2)}{\{\sigma_{U1}^2(1 - \sigma_{U1}^2/\sigma_1^2) + V_2(x_2)\}}.$$

Proof of Theorem 3: We outline a shorter but indirect proof here for part (a). A complete proof proceeds along the lines of the proof of Theorem 4 in Appendix B. The proof for part (b) of Theorem 3 proceeds analogously.

When there are 2 stocks, each investor's portfolio choice problem

$$\max_{S_{i1}, S_{i2}, B_i} E[-exp\{-a_i \tilde{W}_i\}]$$

$$s.t. \widetilde{W}_i = B_i R_f + S_{i1}\widetilde{U}_1 + S_{i2}\widetilde{U}_2$$

$$B_i + S_{i1}P_1 + S_{i2}P_2 \leq W_i{}^0$$

Since the initial budget constraint is binding, rewriting yields

$$B_i = W_i{}^0 - S_{i1}P_1 - S_{i2}P_2$$

such that

$$\widetilde{W}_i = \left(W_i{}^0 - S_{i1}P_1 + S_{i2}P_2\right)R_f + S_{i1}\widetilde{U}_1 + S_{i2}\widetilde{U}_2$$

$$= W_i{}^0 R_f + S_{i1}\left(\widetilde{U}_1 - P_1R_f\right) + S_{i2}\left(\widetilde{U}_2 - P_2R_f\right)$$

The investor's objective function reduces to

$$\min_{S_{i1},S_{i2}} E\left[exp\left\{-a_i\widetilde{W}_i\right\}\right]$$

$$= exp\left\{-a_i W_i{}^0 R_f\right\} E\left[exp\left\{-a_i S_{i1}\left(\widetilde{U}_1 - P_1R_f\right) - a_i S_{i2}\left(\widetilde{U}_2 - P_2R_f\right)\right\}\right]$$

or

$$\min_{S_{i1},S_{i2}} E\left[exp\left\{-a_i S_{i1}\left(\widetilde{U}_1 - P_1R_f\right) - a_i S_{i2}\left(\widetilde{U}_2 - P_2R_f\right)\right\}\right]$$

Using the assumed independency of future liquidating cash flows,

$$\min_{S_{i1},S_{i2}} E\left[exp\left\{-a_i S_{i1}\left(\widetilde{U}_1 - P_1R_f\right)\right\}\right] E\left[exp\left\{-a_i S_{i2}\left(\widetilde{U}_2 - P_2R_f\right)\right\}\right]$$

we get the separation into two unrelated problems:

$$\min_{S_{i1}} E\left[exp\left\{-a_i S_{i1}\left(\widetilde{U}_1 - P_1R_f\right)\right\}\right]$$

$$\min_{S_{i2}} E\left[exp\left\{-a_i S_{i2}\left(\widetilde{U}_2 - P_2R_f\right)\right\}\right]$$

In case (ii) where manager 1 discloses and manager 2 does not, we get

$$\min_{S_{i1}} exp\left\{-a_i S_{i1}\left(\mu_1 - P_1R_f\right) + a_i^2 S_{i1}^2 VAR\left[\widetilde{U}_1 \middle| \widetilde{Y}_1 = y_1\right]\right\}$$

$$\min_{S_{i2}} E\left[exp\left\{-a_i S_{i2}\left(\widetilde{U}_2 - P_2R_f\right)\right\} \middle| \widetilde{Y}_2 \leq x_2\right]$$

We can use the first order conditions for an interior optimum:

$$0 = a_i\left(\mu_1 - P_1R_f\right) - a_i^2 S_{i1} VAR\left[\widetilde{U}_1 \middle| \widetilde{Y}_1 = y_1\right]$$

$$0 = E\left[\left(\widetilde{U}_2 - P_2R_f\right) exp\left\{-a_i S_{i2}\widetilde{U}_2\right\} \middle| \widetilde{Y}_2 \leq x_2\right]$$

to solve for optimal demand for shares. This is easily seen for firm 1 since

$$a_i^{-1} \frac{\left(\mu_1 - P_1R_f\right)}{VAR\left[\widetilde{U}_1 \middle| \widetilde{Y}_1 = y_1\right]} = S_{i1}$$

Applying the market clearing condition

$$\sum_{i=1}^{I} a_i^{-1} \frac{\left(\mu_1 - P_1 R_f\right)}{VAR\left[\tilde{U}_1 \middle| \tilde{Y}_1 = y_1\right]} = \sum_{i=1}^{I} S_{i1} = 1$$

yields

$$\left(\mu_1 - P_1 R_f\right) = aVAR\left[\tilde{U}_1 \middle| \tilde{Y}_1 = y_1\right]$$

or the usual market clearing price on mean-variance form

$P_1 = \frac{\mu_1 - aVAR[\tilde{U}_1 | \tilde{Y}_1 = y_1]}{R_f}$.

The return on stock 1 can now be calculated as $\tilde{R}_1 = \frac{\tilde{U}_1}{P_1}$. The calculation of the price on stock 2 proceeds as in Jorgensen and Kirschenheiter (2015). We then calculate the return as $\tilde{R}_2 = \frac{\tilde{U}_2}{P_2}$. Finally, we calculate the return on the market portfolio as $\tilde{R}_m = \frac{\tilde{U}_1 + \tilde{U}_2}{P_1 + P_2}$. For part (b), the equity premium is

$ER = E\left[\tilde{R}_m \middle| \tilde{Y}_1 = y_1, \tilde{Y}_2 \leq x_2\right] - R_f = \frac{E[\tilde{U}_1 | \tilde{Y}_1 = y_1] + E[\tilde{U}_2 | \tilde{Y}_2 \leq x_2]}{P_1 + P_2} - R_f$.

The betas now follow as $\beta_1 = \frac{E[\tilde{R}_1 | \tilde{Y}_1 = y_1]}{ER}$ and $\beta_2 = \frac{E[\tilde{R}_2 | \tilde{Y}_2 \leq x_2]}{ER}$.

Corollary 3.1 *Assume that, under discretionary disclosure, only firm manager 1 discloses. Then* $\frac{\partial \beta_j^{DD}(y_1, x_2)}{\partial y_1} \neq 0$ *for* $j = 1,2$.

Proofs of Corollaries 3.1 and 3.2: They follow immediately from Theorem 3.

8. APPENDIX B Results and Proofs for Alternative Market Portfolio

Throughout the body of the paper, we represent the market portfolio as the sum of $J = 2$ stocks. Alternatively, we could have used the sum of J stocks and bonds as the market portfolio. In that case, $\tilde{R}_m = \frac{\widetilde{W}_m}{{W_m}^0}$. Below, we develop this alternative setting and provide the proofs corresponding to Theorem 3 for this case, thereby providing support for our claims.

First, we introduce notation that allows for more than two stocks and the equilibrium prices with and without disclosure. Assuming the first J^* firms disclose and the remain $J - J^*$ firms do not, where $0 \leq J^* < J$, then let

$$y_1, \overrightarrow{y_{2,J^*}, x} \equiv \{Y_1 = y_1, \cdots, Y_{J*} = y_{J*}, Y_{J*+1} \leq x_{J*+1}, \cdots, Y_J \leq x_J\}$$

and denote the vector of disclosures when firm 1 discloses and denote the same vector when firm 1 does not disclose as

$x_1, \overrightarrow{y_{2,J^*}, x} \equiv \{Y_1 \leq x_1, Y_2 = y_2, \cdots, Y_{J*} = y_{J*}, Y_{J*+1} \leq x_{J*+1}, \cdots, Y_J \leq x_J\}$.

Consistent with A1 and A2, let $P_1(y_1, \overrightarrow{y_{2,J^*}, x} \,|\, \vec{x})$ and $P_1(x_1, \overrightarrow{y_{2,J^*}, x} \,|\, \vec{x})$ denote price of firm 1's share under strategies determined by the thresholds $\vec{x} = \{x_1, \cdots x_J\}$ if firm 1 does and does not disclose, respectively. From Jorgensen and Kirschenheiter (2015), we know that the equilibrium prices, when firm 1 does and does not disclose, respectively, are given as follows:

$$P_1\left(y_1, \overrightarrow{y_{2,J^*}, x} \middle| \vec{x}\right) = \left(\left(\mu_j - a\sigma_{Uj}^2\right) + \left(y_j - \left(\mu_j - a\sigma_{Uj}^2\right)\right)\frac{\sigma_{Uj}^2}{\sigma_j^2} - c_j\right) \Big/ R_F$$

$$= \left(\mu_j^D + \left(y_j - \mu_j^D\right)\frac{\sigma_{Uj}^2}{\sigma_j^2} - c_j\right) \Big/ R_F$$

and

$P_1(x_1, \overrightarrow{y_{2,J^*}, x} \,|\, \vec{x}) = \left(\left(\mu_j - a\sigma_{Uj}^2\right) - \sigma_{Uj}^2 \alpha_j\left(x_j + a\sigma_{Uj}^2\right)\right)/R_F =$

$\left(\mu_j^D - \sigma_{Uj}^2 \alpha_j\left(x_j + a\sigma_{Uj}^2\right)\right)/R_F$,

where we let $\mu_j^D \equiv \left(\mu_j - a\sigma_{Uj}^2\right)$ and use $\alpha_j(y)$ to denote the anti-hazard rate of the distribution for the signal, evaluated at y, that is, $\alpha_j(y) \equiv f_j(y)/F_j(y)$. We are now in position to present and prove the analogous claims.

Theorem 4 *Consider a discretionary disclosure setting where the first J^* firms' managers disclose and the last $J - J^*$ firms managers opt to not disclose, where $0 \leq J^* < J$. Let $\overrightarrow{z} = \{y_1, \ldots y_{J*} x_{J*+1}, x_J\}$ denote a disclosure vector that summarizes investors' information set. Then the following are true:*

a. While the CAPM pricing does hold, it is <u>not</u> supported by the betas calculated in the standard manner. The expression for the standard beta for firm $j = 1, \ldots, J$ is shown as follows:

$$For\ j < J^*: \beta_j^{Std}(\overrightarrow{z}) = \frac{W_m{}^0}{(\mu_j - a\sigma_{Uj}^2) + \left(y_j - (\mu_j - a\sigma_{Uj}^2)\right)\frac{\sigma_{Uj}^2}{\sigma_j^2} - c_j} \frac{VAR[\tilde{U}_j | \tilde{Y}_j = y_j]}{\Omega^{Std}};$$

$$For\ j \geq J^*: \beta_j^D(\overrightarrow{z}) = \frac{W_m{}^0}{\mu_j - \sigma_{Uj}^2\left(a + \alpha_j(x_j + a\sigma_{Uj}^2)\right)} \frac{VAR[\tilde{U}_j | \tilde{Y}_j \leq x_j]}{\Omega^{Std}};$$

where $\Omega^{Std} \equiv \sum_{k=1}^{J*} VAR[\tilde{U}_k | \tilde{Y}_k = y_k] + \sum_{j=J*+1}^{J} VAR[\tilde{U}_k | \tilde{Y}_k \leq x_k]$.

b. However, letting $\alpha_j(x_j) \equiv f_j(x_j)/F_j(x_j)$ denote the anti-hazard rate of the signal for firm j, then the price of systematic risk for firm $j = 1, \ldots, J$, that supports the CAPM pricing is given as follows:

$$For\ j < J^*: \beta_j^D(\overrightarrow{z}) = \frac{W_m{}^0}{(\mu_j - a\sigma_{Uj}^2) + \left(y_j - (\mu_j - a\sigma_{Uj}^2)\right)\frac{\sigma_{Uj}^2}{\sigma_j^2} - c_j} \frac{VAR[\tilde{U}_j | \tilde{Y}_j = y_j]}{\Omega^D};$$

$$For\ j \geq J^*: \beta_j^D(\overrightarrow{z}) = \frac{W_m{}^0}{\mu_j - \sigma_{Uj}^2\left(a + \alpha_j(x_j + a\sigma_{Uj}^2)\right)} \frac{\sigma_{Uj}^2\left(a + \alpha_j(x_j + a\sigma_{Uj}^2) - \alpha_j(x_j)\right)}{a\Omega^D};$$

where $\Omega^D \equiv \sum_{k=1}^{J*} VAR[\tilde{U}_k | \tilde{Y}_k = y_k] + \sum_{j=J*+1}^{J} \frac{\sigma_{Uj}^2}{a}\left(a + \alpha_j(x_j + a\sigma_{Uj}^2) - \alpha_j(x_j)\right)$.

Before turning to the proofs, we state corollaries that characterize the discretionary disclosure equilibrium when betas are measured relative to the total economy wealth.

Corollary 4.1 *While the market risk premium and each firm's beta depend on the disclosure decision made by all firms in the economy, the risk premium (or expected excess return) of each firm is independent of the disclosure decisions by other firms.*

Corollary 4.2 *The following are true*

a. *If all firms disclose when disclosure is discretionary, the resulting CAPM is identical to the CAPM when disclosure is mandatory.*
b. *If no firms disclose when disclosure is discretionary, the resulting CAPM is different from the CAPM in the absence of any disclosures (corresponding to Theorem 1).*
c. *The market risk premium under mandatory disclosure is weakly (strictly) lower than the market risk premium when disclosure is discretionary (if any firm fails to disclose). Further, the market risk premium is lower when disclosure is discretionary than when disclosures cannot be made.*

Corollary 4.3 *In Theorem 4 when the first* $\boldsymbol{J}^*$ *firms disclose and the next* $\boldsymbol{J}-\boldsymbol{J}^*$ *firms do not. Then the following is true for the relative levels for the expected excess returns and betas under the three disclosure scenarios.*

a. *For the* J^* *firms that do disclose, the risk premium is the same under mandatory disclosure as under discretionary disclosure while the betas are higher when disclosure is mandatory.*
b. *For each firm* j *of the* $J-J^*$ *firms that do not disclose, let* $x_j^R < x_j$ *denote the disclosure threshold chosen by the manager of that firm. Then there exists a threshold,* $x_j^R < x_j$*, such that for signals below (or above) this threshold, the risk premium for firm* j *is lower (or higher) when disclosure is discretionary. There also exists a threshold,* $x_j^\beta > x_j^R$*, such that for signals below this threshold, the beta is lower when disclosure is discretionary. Also, if* $x_j^\beta < x_J$ *holds, then the betas are higher when disclosure is discretionary for the signals in the interval bounded by these thresholds, that is, for signals* $y_j \in \left(x_j^\beta, x_J\right)$.

Corollary 4.4 *Consider two identical firms except that one firm manager discloses while the other does not. Then the risk premium and the beta are lower for the disclosing firm than for the non-disclosing firm.*

Proof of Theorem 4: To obtain the CAPM betas, we first derive the market return assuming both disclosure and non-disclosure are possible. Using this derivation, we derive the variance of the market return. Next we derive the return to the shares based on whether or not disclosure occurs, and then calculate the covariance of the return to the disclosing and non-disclosing firms with the market portfolio. Using these derivations, we show that the traditional CAPM holds if all firms disclose, but not if any firm manager fails to disclose. Finally, we derive the adjusted betas needed to form the disclosure adjusted CAPM.

To consolidate notation, assume the first J^* firms disclose where $0 \leq J^* \leq J$, and assume the last $J - J^*$ firms do not disclose. Similar to the proof of the pricing in Jorgensen and Kirschenheiter (2015), we conjecture (and then verify) that the disclosure decision is independent of the equilibrium demand for stock, S_{ij}^*. The initial wealth of the economy, including bonds, is $W_m^0 = \sum_{i=1}^{I} W_i{}^0$. Likewise, the terminal wealth of the economy is

$$\widetilde{W}_m = \sum_{i=1}^{I} \widetilde{W}_i = \sum_{i=1}^{I}\left(W_i{}^0 - \sum_{j=1}^{J} S_{ij}P_j\right)R_f + \sum_{j=1}^{J}\sum_{i=1}^{I} S_{ij}\widetilde{U}_j - \sum_{j=1}^{J*}\sum_{i=1}^{I} S_{ij}c_j$$

$$= W_m{}^0 R_f + \sum_{j=1}^{J}\left(\widetilde{U}_j - P_j R_f\right) - \sum_{j=1}^{J*} c_j$$

Substituting in the equilibrium prices for the J^* disclosing firms and the $J - J^*$ non-disclosing firms, we find that

$$\widetilde{W}_m = R_f\left(W_m{}^0 + \sum_{j=1}^{J*}\left(\frac{\widetilde{U}_j - c_j}{R_f} - \mu_j^D - \left(y_j - \mu_j^D\right)\frac{\sigma_{Uj}^2}{\sigma_j^2}\right)\right.$$
$$\left. + \sum_{j=J*+1}^{J}\left(\frac{\widetilde{U}_j}{R_f} - \mu_j^D + \sigma_{Uj}^2\alpha_j^D(x_j)\right)\right)$$
$$= R_f\left(W_m{}^0 + \sum_{j=1}^{J}\left(\frac{\widetilde{U}_j}{R_f} - \mu_j^D\right) - \sum_{j=1}^{J*}\left(\frac{c_j}{R_f} + \left(y_j - \mu_j^D\right)\frac{\sigma_{Xj}^2}{\sigma_{Yj}^2}\right)\right.$$
$$\left. + \sum_{j=J*+1}^{J}\left(\sigma_{Uj}^2\alpha_j\left(x_j + a\sigma_{Uj}^2\right)\right)\right).$$

This is the expression for the terminal wealth where trading occurs after the managers observe the signals. The return on the market portfolio is then given as

$$\tilde{R}_m = \frac{\widetilde{W}_m}{{W_m}^0}$$

$$= R_f$$

$$+ \frac{\sum_{j=1}^{J}\left(\widetilde{U}_j/R_f - \mu_j^D\right) - \sum_{j=1}^{J*}\left(c_j/R_f + \left(y_j - \mu_j^D\right)\frac{\sigma_{Uj}^2}{\sigma_j^2}\right) + \sum_{j=J*+1}^{J}\left(\sigma_{Uj}^2\alpha_j\left(x_j + a\sigma_{Uj}^2\right)\right)}{{W_m}^0} R_f$$

The excess return on the market portfolio is

$$\tilde{R}_m - R_f$$

$$= \frac{\sum_{j=1}^{J}\left(\widetilde{U}_j/R_f - \mu_j^D\right) - \sum_{j=1}^{J*}\left(c_j/R_f + \left(y_j - \mu_j^D\right)\frac{\sigma_{Uj}^2}{\sigma_j^2}\right) + \sum_{j=J*+1}^{J}\left(\sigma_{Uj}^2\alpha_j\left(x_j + a\sigma_{Uj}^2\right)\right)}{{W_m}^0} R_f$$

Using $\overrightarrow{y_{J*}, x_J} = \{\tilde{Y}_1 = y_1, \tilde{Y}_2 = y_2, \ldots, \tilde{Y}_{J*} = y_{J*}, \tilde{Y}_{J*+1} \le x_{C,J*+1}, \ldots \tilde{Y}_J \le x_{C,J}\}$ to denote the vector of disclosures, the expected excess market return or market risk premium is given as follows:

$$E\left[\tilde{R}_m \middle| \overrightarrow{y_{J*}, x_J}\right] - R_f = \left(\frac{R_f}{{W_m}^0}\right)\sum_{j=1}^{J*}\left(\frac{E\left[\widetilde{U}_j\middle|\tilde{Y}_j = y_j\right] - c_j}{R_f} - \mu_j^D - \left(y_j - \mu_j^D\right)\frac{\sigma_{Uj}^2}{\sigma_j^2}\right)$$

$$+ \left(\frac{R_f}{{W_m}^0}\right) + \sum_{j=J*+1}^{J}\left(\frac{E\left[\widetilde{U}_j\middle|\tilde{Y}_j \le x_j\right]}{R_f} - \mu_j^D + \sigma_{Uj}^2\alpha_j\left(x_j + a\sigma_{Uj}^2\right)\right).$$

The difference between the actual and expected return is then given as

$$\tilde{R}_m - E\left[r_m \middle| \overrightarrow{y_{J*}, x_J}\right] = \left({W_m}^0\right)^{-1}\left(\sum_{j=1}^{J*}\left(\widetilde{U}_j - E\left[\widetilde{U}_j\middle|\tilde{Y}_j = y_j\right]\right) + \sum_{j=J*+1}^{J}\left(\widetilde{U}_j - E\left[\widetilde{U}_j\middle|\tilde{Y}_j \le x_j\right]\right)\right).$$

Hence the difference between actual market return and the expected market return is the sum of the difference between the actual and expected discounted cash flows. For the first J^* firms, the expectation is the condition mean, or $E\left[\widetilde{U}_j\middle|\tilde{Y}_j = y_j\right] = \mu_j + \left(y_j - \mu_j\right)\sigma_{Xj}^2/\sigma_{Yj}^2$. For the next $J - J^*$ firms, the expectation is the truncated mean, or $E\left[\widetilde{U}_j\middle|\tilde{Y}_j \le x_j\right] = \mu_j - \sigma_{Uj}^2\alpha_j\left(x_j\right)$, where we let $\alpha_j(y)$ denote $\alpha_j(j) \equiv f_j(y)/F_j(y)$, that is, the anti-hazard rate of the distribution for the signal before it is adjusted for risk, evaluated at y.

Next, we derive the variance of the return on the market portfolio. By construction, the cash flows are independent. Hence, the variance of the market portfolio is the sum of the variances of the conditional and truncated cash flows, or is found as follows:

$$Var\left[\tilde{R}_m \middle| \overrightarrow{y_{J*}, x_J}\right] = E\left[\left(\tilde{R}_m - E\left[\tilde{R}_m \middle| \overrightarrow{y_{J*}, x_J}\right]\right)^2\right]$$

$$= \left(W_m{}^0\right)^{-2} E\left[\left(\sum_{j=1}^{J*}\left(\tilde{U}_j - E\left[\tilde{U}_j \middle| \tilde{Y}_j = y_j\right]\right) + \sum_{j=J*+1}^{J}\left(\tilde{U}_j - E\left[\tilde{U}_j \middle| \tilde{Y}_j \le x_j\right]\right)\right)^2\right]$$

$$= \left(W_m{}^0\right)^{-2}\left(\sum_{j=1}^{J*} E\left[\left(\left(\tilde{U}_j - E\left[\tilde{U}_j \middle| \tilde{Y}_j = y_j\right]\right)^2 \middle| \tilde{Y}_j = y_j\right)\right]\right.$$

$$\left. + \sum_{j=1}^{J*} E\left[\left(\left(\tilde{U}_j - E\left[\tilde{U}_j \middle| \tilde{Y}_j \le x_j\right]\right)^2 \middle| \tilde{Y}_j =\le x_j\right)\right]\right)$$

$$= \left(W_m{}^0\right)^{-2}\left(\sum_{j=1}^{J*} Var\left[\tilde{U}_j \middle| \tilde{Y}_j = y_j\right] + \sum_{j=1}^{J*} Var\left[\tilde{U}_j \middle| \tilde{Y}_j \le x_j\right]\right)$$

$$= \left(W_m{}^0\right)^{-2}\left(\sum_{j=1}^{J*} \sigma_{Uj}^2\left(1 - \frac{\sigma_{Uj}^2}{\sigma_j^2}\right) + \sum_{j=1}^{J*} Var\left[\tilde{U}_j \middle| \tilde{Y}_j \le x_j\right]\right) = \left(W_m{}^0\right)^{-2} \Omega^{Std}$$

We use Ω^{Std}denote the sum of the conditional variances of the disclosing firms plus the sum of the truncated variances of the non-disclosing firms, or $\Omega^{Std} \equiv \sum_{j=1}^{J*} VAR\left[\tilde{U}_j \middle| \tilde{Y}_j = y_j\right] + \sum_{j=J*+1}^{J} VAR\left[\tilde{U}_j \middle| \tilde{Y}_j \le x_j\right]$.

The return to the first J^* stocks (where the manager discloses) is $\tilde{R}_j\left(y_j\right) = \frac{\tilde{U}_j - c_j}{P_j\left(y_j\right)}$, and from above we know the price is: $P_j\left(y_j\right) = \left(\mu_j^D + \left(y_j - \mu_j^D\right)\frac{\sigma_{Uj}^2}{\sigma_j^2} - c_j\right)/R_F$. Consequently, the excess return is

$$\tilde{R}_j(y_j) - R_F = \frac{(\tilde{U}_j - C_j)R_F}{\mu_j^D + (y_j - \mu_j^D)\frac{\sigma_{Uj}^2}{\sigma_j^2} - c_j} - R_F = \left(\frac{\tilde{U}_j - \mu_j^D - (y_j - \mu_j^D)\frac{\sigma_{Uj}^2}{\sigma_j^2}}{\mu_j^D + (y_j - \mu_j^D)\frac{\sigma_{Uj}^2}{\sigma_j^2} - c_j} \right) R_F.$$

This return is based on the investors observing the signal. Since

$$E[\tilde{U}_j | \tilde{Y}_j = y_j] - \mu_j^D - (y_j - \mu_j^D)\frac{\sigma_{Uj}^2}{\sigma_j^2} = aVAR[\tilde{U}_j | \tilde{Y}_j = y_j],$$

this implies that the expected excess return or risk premium on a disclosing firm is

$$E[\tilde{R}_j(y_j) | \tilde{Y}_j = y_j] - R_F = \left(\frac{aVAR[\tilde{U}_j | \tilde{Y}_j = y_j]}{\mu_j^D + (y_j - \mu_j^D)\frac{\sigma_{Uj}^2}{\sigma_j^2} - c_j} \right) R_F = \frac{aVAR[\tilde{U}_j | \tilde{Y}_j = y_j]}{P_j(y_j)}$$

The return on the $J - J^*$ stocks where the manager does not disclose is $\tilde{R}_j(x_j) = \frac{\tilde{U}_j}{P_j(x_j)}$, and from above we know that with non-disclosure the price is

$$P_j(x_j) = \left(\mu_j^D - \sigma_{Uj}^2 \alpha_j(x_j + a\sigma_{Uj}^2)\right)/R_F$$

so the excess return on the non-disclosing firms can be represented as

$$\tilde{R}_j(x_j) - R_F = \frac{\tilde{U}_j R_F}{\mu_j^D - \sigma_{Uj}^2 \alpha_j(x_j + a\sigma_{Uj}^2)} - R_F = \left(\frac{\tilde{U}_j - \mu_j^D + \sigma_{Uj}^2 \alpha_j(x_j + a\sigma_{Uj}^2)}{\mu_j^D - \sigma_{Uj}^2 \alpha_j(x_j + a\sigma_{Uj}^2)} \right) R_F.$$

This return is based on the investors not observing the signal, but inferring the signal is below the threshold. Since

$$E[\tilde{U}_j | \tilde{Y}_j \le x_j] - \mu_j^D = \sigma_{Uj}^2(-\alpha_j(x_j) + a),$$

this implies the expected excess return, or risk premium, on a non-disclosing firm is

$$E[\tilde{R}_j(x_{C,j}) | \tilde{Y}_j \le x_j] - R_F = \left(\frac{\sigma_{Uj}^2 \left(a + \alpha_j(x_j + a\sigma_{Uj}^2) - \alpha_j(x_j)\right)}{\mu_j^D - \sigma_{Uj}^2 \left(a + \alpha_j(x_j + a\sigma_{Uj}^2)\right)} \right) R_F$$

$$= \frac{a\sigma_{Uj}^2}{P_j(x_j)} \left(1 + \frac{\left(\alpha_j(x_j + a\sigma_{Uj}^2) - \alpha_j(x_j)\right)}{a} \right)$$

Next, we derive the covariances and the betas for the disclosing and non-disclosing firms, respectively. If manager j discloses, then the covariance with the market portfolio is

$$COV\left[\tilde{R}_j(y_j), \tilde{R}_m(\overrightarrow{y_{J*}, x_J}) \mid \overrightarrow{y_{J*}, x_J}\right] = COV\left[\frac{\tilde{U}_j}{P_j(y_j)}, \frac{\sum_{k=1}^{J} \tilde{U}_k}{W_m{}^0} \middle| y_{J*}, x_J\right]$$

$$= \left(P_j(y_j) W_m{}^0\right)^{-1} E\left[\left(\tilde{X}_j - E[\tilde{U}_j | \tilde{Y}_j = y_j]\right)\left(\tilde{X}_j - E[\tilde{U}_j | \tilde{Y}_j = y_j]\right) \middle| \tilde{Y}_j = y_j\right]$$

$$= \left(P_j(y_j) W_m{}^0\right)^{-1} VAR\left[\tilde{U}_j | \tilde{Y}_j = y_j\right].$$

The returns on all the non-j firms in the market portfolio drop out due to the assumed non-correlation among all the cash flows. Consequently, the systematic risk premium for firm j if the manager discloses a forecast using the standard calculation is

$$\beta_j^{Std}(y_j) = \frac{COV\left[\tilde{R}_j(y_j), \tilde{R}_m\right]}{VAR[\tilde{r}_m]} = \frac{\left(P_j(y_j) W_m{}^0\right)^{-1} VAR\left[\tilde{U}_j | \tilde{Y}_j \leq x_j\right]}{\left(W_m{}^0\right)^{-2} \Omega^{Std}}$$

$$= \frac{W_m{}^0}{P_j(x_{C,j})} \frac{VAR\left[\tilde{U}_j | \tilde{Y}_j = y_j\right]}{\Omega^{Std}}$$

In analogous manner, if manager j does not disclose, then the covariance with the market portfolio is

$$COV\left[\tilde{R}_j(x_j), \tilde{R}_m \mid \overrightarrow{y_{J*}, x_J}\right] = \left(P_j(x_j) W_m{}^0\right)^{-1} COV\left[\tilde{U}_j, \sum_{k=1}^{J} \tilde{U}_k \mid \overrightarrow{y_{J*}, x_J}\right]$$

$$= \left(P_j(x_j) W_m{}^0\right)^{-1} E\left[\left(\tilde{U}_j - E[\tilde{U}_j | \tilde{Y}_j \leq x_j]\right)\left(\tilde{U}_j - E\tilde{U}_j | \tilde{Y}_j \leq x_j\right) \middle| \tilde{Y}_j \leq x_j\right]$$

$$= \left(P_j(x_j) W_m{}^0\right)^{-1} VAR\left[\tilde{U}_j | \tilde{Y}_j \leq x_j\right].$$

Hence, the systematic risk premium for firm j if its manager does not disclose its forecast is

$\beta_j^{Std}(x_j) = \frac{COV[\tilde{r}_j(x_j), \tilde{r}_m]}{VAR[\tilde{r}_m]} = \frac{W_m{}^0}{P_j(x_{C,j})} \frac{VAR[\tilde{U}_j | \tilde{Y}_j \leq x_j]}{\Omega^{Std}}.$

Note that the beta of the market portfolio is indeed one:

$$\sum_{j=1}^{J} \beta_j^{Std} P_j = W_m{}^0$$

However, the usual CAPM relation does not hold. To see this, note that the excess return on the market portfolio can be written as follows:

$\tilde{R}_m - R_f = \frac{\widetilde{W}_m}{{W_m}^0} - R_f = \frac{\sum_{j=1}^{J}(\widetilde{U}_j - P_j R_f) - \sum_{j=1}^{J*} c_j}{{W_m}^0}$,

and this implies that the expected excess return on the market portfolio reduces to:

$$E\left[\tilde{R}_m \mid \overrightarrow{y_{J*}, x_J}\right] - R_f$$
$$= \frac{\left(\sum_{j=1}^{J*}\left(E\left[\widetilde{U}_j \middle| \tilde{Y}_j = y_j\right] - c_j - P_j(x_j)R_f\right) + \sum_{j=J*+1}^{J}\left(E\left[\widetilde{U}_j \middle| \tilde{Y}_j \le x_j\right] - P_j(x_j)R_f\right)\right)}{{W_m}^0}$$
$$= \frac{a}{{W_m}^0}\left(\sum_{j=1}^{J*}\left(VAR\left[\widetilde{U}_j \middle| \tilde{Y}_j = y_j\right]\right) + \sigma_{Uj}^2 \sum_{j=J*+1}^{J}\left(1 + \frac{\left(\alpha_j\left(x_j + a\sigma_{Uj}^2\right) - \alpha_j(x_j)\right)}{a}\right)\right)$$
$$= \frac{a}{{W_m}^0}\Omega^D$$

We substituted using the price equations for the disclosing and non-disclosing shares from above and we used Ω^D to denote the sum of the two summations in the numerator, or

$\Omega^D \equiv \sum_{j=1}^{J*}\left(VAR\left[\widetilde{U}_j \middle| \tilde{Y}_j = y_j\right]\right) + \sigma_{Uj}^2 \sum_{j=J*+1}^{J}\left(1 + \frac{\left(\alpha_j\left(x_j + a\sigma_{Uj}^2\right) - \alpha_j(x_j)\right)}{a}\right)$.

To summarize, the excess expected return on stock j is

$$E\left[\tilde{R}_j(y_j)\right] - R_F = \frac{aVAR\left[\widetilde{U}_j \middle| \tilde{Y}_j = y_j\right]}{P_j(y_j)}$$

if manager j discloses and is

$$E\left[\tilde{R}_j(x_{C,j})\right] - R_F = \frac{a\sigma_{Uj}^2}{P_j(x_j)}\left(1 + \frac{\left(\alpha_j\left(x_j + a\sigma_{Uj}^2\right) - \alpha_j(x_j)\right)}{a}\right)$$

if the manager does not disclose. The beta on the disclosing firm j is

$$\beta_j^{Std}(y_j) = \frac{{W_m}^0}{P_j(y_j)} \frac{VAR\left[\widetilde{U}_j \middle| \tilde{Y}_j = y_j\right]}{\Omega^{Std}}$$

while on the non-disclosing firm it is

$\beta_j^{Std}(x_j) = \frac{W_m{}^0}{P_j(x_j)} \frac{VAR[\tilde{U}_j|\tilde{Y}_j \le x_j]}{\Omega^{Std}}$.

Two points follow immediately. First, the CAPM holds if all firms disclose. In this case $\Omega^D = \Omega^{Std}$ and for each disclosing firm we have

$\frac{aVAR[\tilde{U}_j|\tilde{Y}_j = y_j]}{P_j(y_j)} = E[\tilde{R}_j(y_j)] - R_F = \beta_j(E[\tilde{R}_m] - R_f) = \frac{W_m^0}{P_j(y_j)} \frac{VAR[\tilde{U}_j|\tilde{Y}_j = y_j]}{\Omega^{Std}} \left(\frac{a}{W_m^0}\Omega^D\right)$.

This is the result for mandatory disclosure, or part b) of Observation 1.

Second, if a single firm does not disclose, then the traditional CAPM fails for all firms. For the disclosing firms, the expected excess return is off from the CAPM return by a factor of Ω^D/Ω^{Std}. For the non-disclosing firms, the difference is more pronounced, since in this case we have

$$\frac{a\sigma_{Uj}^2}{P_j(x_j)}\left(1 + \frac{\left(\alpha_j^D(x_j) - \alpha_j(x_j)\right)}{a}\right) = E[\tilde{R}_j(x_j)] - R_F$$

$$\neq \beta_j(E[\tilde{R}_m] - R_f)$$

$$= \frac{W_m^0}{P_j(x_j)} \frac{VAR[\tilde{U}_j|\tilde{Y}_j \le x_j]}{\Omega^{Std}} \left(\frac{a}{W_m^0}\Omega^D\right) = \frac{aVAR[\tilde{U}_j|\tilde{Y}_j \le x_j]}{P_j(x_j)} \left(\frac{\Omega^D}{\Omega^{Std}}\right).$$

Hence the *disclosure adjusted CAPM* is defined as the set of betas which are formed as follows:

For the J^* disclosing firms, the beta is

$$\beta_j^D(y_j) = \beta_j^{Std}(y_j)\frac{\Omega^{Std}}{\Omega^D} = \frac{W_m{}^0}{P_j(y_j)} \frac{VAR[\tilde{U}_j|\tilde{Y}_j = y_j]}{\Omega^D}$$

while for the non-disclosing firms, the disclosure-adjusted beta is

$$\beta_j^D(x_j) = \beta_j^{Std}(x_j)\frac{\Omega^{Std}}{\Omega^D}\left(\frac{\sigma_{Uj}^2\left(1 + \frac{\left(\alpha_j(x_j + a\sigma_{Uj}^2) - \alpha_j(x_j)\right)}{a}\right)}{VAR[\tilde{U}_j|\tilde{Y}_j \le x_j]}\right)$$

$$= \frac{W_m{}^0}{P_j(x_j)\Omega^D}\sigma_{Uj}^2\left(1 + \frac{\left(\alpha_j(x_j + a\sigma_{Uj}^2) - \alpha_j(x_j)\right)}{a}\right)$$

To see that the CAPM linear relationship holds for the risk adjusted variables, we need to derive the excess return for each firm and the market return using these variables. From above we have the excess market return as

$E\left[\tilde{R}_m \mid \overrightarrow{y_{J*}, x_J}\right] - R_f = \frac{a}{W_m{}^0} \Omega^D.$

Substituting for the excess return and the equation for the disclosure adjusted betas for the J^* disclosing firms we get

$$E\left[\tilde{R}_j(y_j)\right] - R_F = \frac{aVAR\left[\tilde{U}_j \middle| \tilde{Y}_j = y_j\right]}{P_j(y_j)}$$

$$= \frac{W_m^0}{P_j(x_j)\Omega^D}\left(\frac{VAR\left[\tilde{U}_j \middle| \tilde{Y}_j = y_j\right]}{\Omega^D}\right)\left(\frac{a}{W_m{}^0}\Omega^D\right) = \beta_j^D(y_j)\left(E\left[\tilde{R}_m \mid \overrightarrow{y_{J*}, x_J}\right] - R_f\right)$$

and for the $J - J^*$ non-disclosing firms we get

$$E\left[\tilde{R}_j(x_j) \middle| \tilde{Y}_j \le x_j\right] - R_F = \frac{a\sigma_{Uj}^2}{P_j(x_j)}\left(1 + \frac{\left(\alpha_j(x_j + a\sigma_{Uj}^2) - \alpha_j(x_j)\right)}{a}\right)$$

$$= \frac{W_m^0\sigma_{Uj}^2}{P_j(x_j)\Omega^D}\left(1 + \frac{\left(\alpha_j(x_j + a\sigma_{Uj}^2) - \alpha_j(x_j)\right)}{a}\right)\left(\frac{a}{W_m{}^0}\Omega^D\right)$$

$$= \beta_j^D(x_j)\left(E\left[\tilde{R}_m \mid \overrightarrow{y_{J*}, x_J}\right] - R_f\right)$$

Together these equations show that the linear relationship predicted by the CAPM holds for the disclosure adjusted CAPM parameters. This completes the proof of Theorem 4.

Proof of Corollary 4.1: The results in this Corollary follow immediately from the proof of Theorem 4. In particular, the market-wide disclosure affect both the equity premium and each firm's beta through the market wide risk measure, $\boldsymbol{\Omega^D}$, entering the numerator of the equity premium and the denominator of the betas. Clearly these cancel, leaving the expected excess return unaffected by the market-wide disclosures.

Proof of Corollary 4.2:

The results of parts a and b of this Corollary follow immediately from a comparison of the CAPM parameters in Theorems 1 and 2 and the CAPM parameters derived in the proof of Theorem 4. For part c, we have from the proof of Theorem 4 that the expected excess market return is

$$E\left[R_m^D | \overrightarrow{y_{J*}, x_j}\right] - R_f = \frac{a}{{W_m}^0} \Omega^D$$

where

$$\Omega^D \equiv \sum_{j=1}^{J*} \sigma_{Uj}^2 \left(1 - \sigma_{Uj}^2/\sigma_j^2\right) + \sum_{j=J*+1}^{J} \sigma_{Uj}^2 \left(1 + \frac{\left(\alpha_j\left(x_j + a\sigma_{Uj}^2\right) - \alpha_j(x_j)\right)}{a}\right).$$

Similarly the expected excess market returns are

$$E\left[R_m^{Full} | \overrightarrow{y_j}\right] - R_f = \frac{a}{{W_m}^0} \Omega^{Full}$$

under mandatory disclosure and

$$E[R_m^{No}] - R_f = \frac{a}{{W_m}^0} \Omega^{No}$$

in the absence of any disclosures, where $\Omega^{Full} \equiv \sum_{j=1}^{J} \sigma_{Uj}^2 \left(1 - \sigma_{Uj}^2/\sigma_j^2\right)$ and $\Omega^{No} \equiv \sum_{j=1}^{J} \sigma_{Uj}^2$.

From the proof of part c of Corollary 1 in Jorgensen and Kirschenheiter (2015), we have $\alpha_j(x_j) > \alpha_j\left(x_j + a\sigma_{Uj}^2\right)$. Further, the proof of the same Corollary shows that for the anti-hazard rate for normal distributions, the first derivative of the anti-hazard rate of the signal for firm j is negative and increasing, asymptotically reaching zero at positive infinity and reaching $-1/\sigma_j^2$ at negative infinity. Since the derivative is negative and bounded by one over the variance of the signal, it follows that

$$a\sigma_{Uj}^2/\sigma_j^2 > \alpha_j(x_j) - \alpha_j\left(x_j + a\sigma_{Uj}^2\right)$$

always holds. The first inequality insures that

$$\Omega^D \equiv \sum_{j=1}^{J*} \sigma_{Uj}^2 \left(1 - \sigma_{Uj}^2/\sigma_j^2\right) + \sum_{j=J*+1}^{J} \sigma_{Uj}^2 \left(1 + \frac{\left(\alpha_j\left(x_j + a\sigma_{Uj}^2\right) - \alpha_j(x_j)\right)}{a}\right) < \sum_{j=1}^{J} \sigma_{Uj}^2 \equiv \Omega^{No}$$

always holds, implying that

$$E\left[R_m^D \mid \overrightarrow{y_{J*}, x_j}\right] - R_f = \frac{a}{{W_m}^0}\Omega^D < \frac{a}{{W_m}^0}\Omega^{No} = E[R_m^{No}] - R_f$$

always holds. The second inequality insures that

$$\Omega^D \equiv \sum_{j=1}^{J*} \sigma_{Uj}^2\left(1 - \sigma_{Uj}^2/\sigma_j^2\right) + \sum_{j=J*+1}^{J} \sigma_{Uj}^2 \left(1 + \frac{\left(\alpha_j\left(x_j + a\sigma_{Uj}^2\right) - \alpha_j\left(x_j\right)\right)}{a}\right)$$

$$\geq \sum_{j=1}^{J} \sigma_{Uj}^2\left(1 - \sigma_{Uj}^2/\sigma_j^2\right) \equiv \Omega^{Full}$$

always holds, with the inequality strict as long as one firm fails to disclose (i.e., as long as $J^* < J$). This in turn implies that

$$E\left[R_m^D \mid \overrightarrow{y_{J*}, x_j}\right] - R_f = \frac{a}{{W_m}^0}\Omega^D \geq \frac{a}{{W_m}^0}\Omega^{Full} = E\left[R_m^{Full} \mid \overrightarrow{y_j}\right] - R_f$$

always holds, where again, non-disclosure by a single firm insures that the inequality is strict. This completes the proof of part c and of Corollary 7.

Proof of Corollary 4.3:

As in the proof of Theorem 4, let $\tilde{R}_j(y_j) - R_F$ denote the excess return for firm j under disclosure and let $\tilde{R}_j(x_j) - R_F$ denote the excess return under non-disclosure when disclosure is discretionary. From the proof of theorem 1, under disclosure we have

$$E\left[\tilde{R}_j(y_j)\right] - R_F = \left(\frac{aVAR[\tilde{U}_j|\tilde{Y}_j=y_j]}{E[\tilde{U}_j|\tilde{Y}_j=y_j]-c_j-aVAR[\tilde{U}_j|\tilde{Y}_j=y_j]}\right)R_F = \frac{a\sigma_{Uj}^2\left(1-\sigma_{Uj}^2/\sigma_j^2\right)}{P_j(y_j)}.$$

It follows immediately that this is the excess return for a disclosing firm when disclosure is either mandatory or discretionary. From Theorem 4, the betas arising when disclosure is discretionary, assuming disclosure, can be written as follows:

$$\beta_j^D(y_j) = \frac{E[\tilde{R}_j(y_j)]-R_F}{\left(E[\tilde{R}_m^D|\overrightarrow{y_{J*},x_j}]-R_f\right)}.$$

Similarly the betas under mandatory disclosure are given as follows:

$$\beta_j^{Full}(y_j) = \frac{E[\tilde{R}_j(y_j)]-R_F}{\left(E[\tilde{R}_m^{Full}|\overrightarrow{y_j}]-R_f\right)}.$$

From Corollary 7, we have that the equity premium is lower under mandatory disclosure than when disclosure is discretionary. Since the expected excess share returns are equal, it follows that the mandatory disclosure betas are higher, completing the proof of part a.

For part b, we begin by noting that, from the proof of Theorem 1, the expected excess return under discretionary disclosure, given non-disclosure, can be expressed as follows:

$$E[\tilde{R}_j(x_j)] - R_F = \frac{a\sigma_{Uj}^2}{P_j(x_j)}\left(1 + \frac{\left(\alpha_j(x_j + a\sigma_{Uj}^2) - \alpha_j(x_j)\right)}{a}\right).$$

By construction, the threshold is chosen so that $P_j(x_j) = P_j(y_j)$ for $y_j = x_j$. From the proof of Corollary 7 we have that $a\sigma_{Uj}^2/\sigma_j^2 > \alpha_j(x_j) - \alpha_j(x_j + a\sigma_{Uj}^2)$ always holds, so at the threshold, we have

$$E[\tilde{R}_j(y_j)] - R_F = \frac{a\sigma_{Uj}^2(1 - \sigma_{Uj}^2/\sigma_j^2)}{P_j(y_j)} < \frac{a\sigma_{Uj}^2}{P_j(x_j)}\left(1 + \frac{\left(\alpha_j(x_j + a\sigma_{Uj}^2) - \alpha_j(x_j)\right)}{a}\right) = E[\tilde{R}_j(x_j)] - R_F.$$

Hence, at the threshold, the expected excess return under discretionary disclosure is higher than in the mandatory regime. However, as the signal falls, the disclosure price, $P_j(y_j)$, falls, but nothing else changes in the preceding equation. Hence the expected excess return in the mandatory regime increases monotonically, so that the threshold $x_j^R < x_j$ is found as the signal that solves the following equation:

$$\frac{a\sigma_{Uj}^2(1 - \sigma_{Uj}^2/\sigma_j^2)}{P_j(x_j^R)} = \frac{a\sigma_{Uj}^2}{P_j(x_j)}\left(1 + \frac{\left(\alpha_j(x_j + a\sigma_{Uj}^2) - \alpha_j(x_j)\right)}{a}\right).$$

Such a threshold exists, since the left-hand side goes to infinity as $P_j(y_j)$ goes to zero. This completes the proof of the results on the expected excess returns.

The results on the betas follow by noting that the beta for firm j can be written as the expected excess return on the shares of firm j divided by the excess expected market return. The results then follow by combining the results on the market returns from Corollary 7 with the results on the share returns just completed. The mandatory market return is always lower than the discretionary market return. Since the expected excess return on the shares is higher under the mandatory than discretionary regime for signals below the threshold, that is, for all $y_j < x_j^R$, this implies the mandatory beta is higher than the discretionary beta for all these signals. For signals above this threshold, the mandatory betas are initially higher but are falling, since the expected excess return to the stock is monotonically decreasing in the signal. At some point, the betas in the two regimes will equal one another, and this is how the threshold $x_j^{\beta} > x_j^{R}$ is found. This completes the proof of part b, and of Corollary 8.

Proof of Corollary 4.4:

We can show that the beta is decreasing in the level of disclosure, given disclosure, in a manner analogous to Corollary 2, so it suffices to show that $\beta_j^D(x_j) > \beta_j^D(y_j)$ holds when $x_j = y_j$. Since the manager is indifferent between disclosing and not disclosing at the threshold, when $x_j = y_j$ we have $P_j(x_j) = P_j(y_j)$. Writing the expression for beta with and without disclosure, as

$$\beta_j^D(y_j) = \frac{W_m^0 \sigma_{Uj}^2}{P_j(y_j)\Omega^D}\left(1 - \frac{\sigma_{Uj}^2}{\sigma_j^2}\right)$$

and

$$\beta_j^D(x_j) = \frac{W_m^0 \sigma_{Uj}^2}{P_j(x_j)\Omega^D}\left(1 + \frac{\left(\alpha_j(x_j + a\sigma_{Uj}^2) - \alpha_j(x_j)\right)}{a}\right)$$

respectively. Hence, canceling terms, we have $\beta_j^D(x_j) > \beta_j^D(y_j)$ if and only if
$a\sigma_{Uj}^2/\sigma_j^2 > \alpha_j(x_j) - \alpha_j(x_j + a\sigma_{Uj}^2)$.

But this inequality was shown to hold in the proof of Corollary 3.2 above, so this completes the proof of Corollary 9.

Figure 1

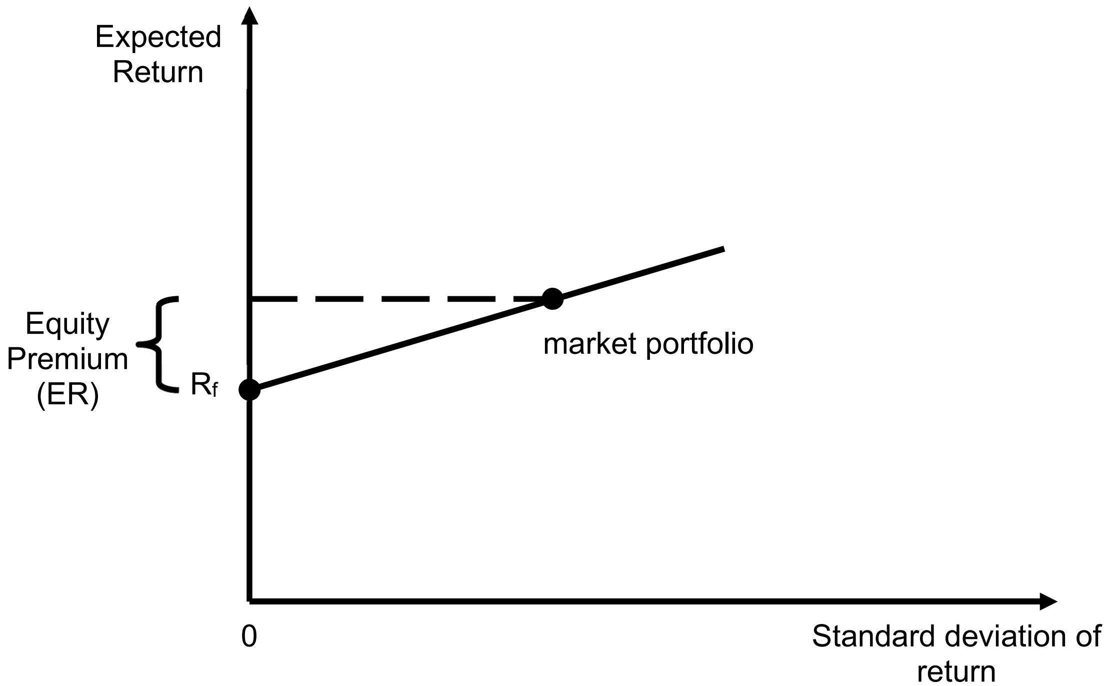

Figure 2