# Bank Risk Taking and Creditors Bargaining Power *

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**Abstract**

We analyze risk taking in a bank with uninsured subordinated debt (subdebt) as part of its capital, while considering the strategic interaction between the bank's stakeholders and assuming that bank assets are risky debt claims. We show: (1) when creditors control risk level, risk shifting occurs in undercapitalized bank, but to a lesser extent than in Gorton and Santomero [1990], since debt value is maximized in an interior risk level. (2) The efficiency of subdebt in limiting risk shifting, relative to a stock only capital structure, depends on the stockholder's bargaining power. (3) Counterintuitively, risk shifting can increase with a tighter regulatory control on asset risk.

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# 1 Introduction

Bank depositors tend to be small and dispersed and have limited ability and will to monitor the bank due to implicit or explicit government guarantee. Therefore, governments have strong incentive to monitor banks in order to prevent excessive risk taking, which can lead to costly bailouts and spillovers to the rest of the financial system and to the real economy (Allen and Gale [2000]; Leitner [2005]; Gai and Kapadia [2010]; and Gofman [2017]). However, as the size and complexity of financial institutions increases, regulators' ability to control bank asset risk using traditional supervisory techniques, such as minimum capital requirements and supervisory review, has been eroding (Basel Committee on Banking Supervision [1999]; Morgan and Stiroh [2001]; Berger et al. [2000]; DeYoung et al. [2001]).

In response, banks were encouraged to issue uninsured subordinated debt (hereafter: subdebt), in addition to stock. The subdebt serves as a buffer against declines in asset value and consequently protects depositors from costly failures (Calomiris [1999]; Evanoff and Wall [2000]). Moreover, subdebtholders are considered sophisticated creditors that increase bank transparency. Therefore, they may be able to affect bank behavior ("direct discipline") in a way that is believed to be aligned with the deposit insurer's incentive (Flannery [2001]; and Vashishtha et al. [2018]). In addition, they can indirectly discipline banks, by providing a signal of bank risk to the regulator and other market participants who can discipline the bank (Gorton and Santomero [1990]; and Dewatripont and Tirole [1993]).

Following the 20072009 financial crisis, the use of subdebt as a monitoring tool was questioned by researchers who found mixed evidence of its effectiveness . On the one hand, several empirical works did find that subdebt reduced bank risk taking, both during the financial crisis and the period following it (Danisewicz et al. [2017]; Nguyen [2013]; John et al. [2010]; and Belkhir [2013]). On the other hand, many financial institutions with subdebt as part of their capital structure defaulted or were bailed-out using taxpayers' money. The financial literature explains subdebt's ineffectiveness in mitigating risk either by low ability of the subdebtholders to control bank risk (for example due to low bank transparency that left subdebtholders uninformed about risk shifting events), or by subdebtholders' low motivation to control bank asset risk since they are better off with the higher level of risk (consistent with the early work of Gorton and Santomero [1990]).

In response to the crisis, a new international regulatory framework, Basel III (Basel Committee on Banking Supervision [2010]), was adopted. Key elements of the new accord include strengthening bank capital adequacy and its supervision and increasing bank transparency. The first is implemented through higher capital-ratios (Dermine [2015]) and the second by requiring banks to participate in periodic stress tests, which involve disclosure of unprecedented details (Goldstein and Sapra [2014]; and Goldstein and Leitner [2015]).

While the separate effect of each of these two components is heavily studied the joint effect is almost unexplored. Exceptions are Vashishtha et al. [2018], who using data from a large sample of U.S. banks from 1994-2013 find a significant positive relation between bank transparency and the sensitivity of uninsured deposit flows to bank performance. Still, no theoretical model investigates the relationship between the bargaining power of subdebtholders, which depends on transparency, and the effectiveness of subdebt at mitigating risk taking or its interaction with capital adequacy and with supervisory processes.

Motivated by this gap in the literature and the inconsistent assessments of subdebt's ability to mitigate risk taking during the 2007-2009 financial crisis, our goal is to enhance the understanding of subdebt's effect on bank risk taking and its stability . To this end, we begin by finding the fair value of the bank's different liabilities using a framework where bank assets are risky debt claims with limited upside (Nagel and Purnanandam [2015], Gornall and Strebulaev [2013], Dermine and Lajeri [2001] and Peleg Lazar and Raviv [2017]). Next, we apply a game theory approach, where we study the strategic bargaining interaction between the bank's claimholders and its borrowers, to find the equilibrium level of asset risk. Bank risk taking depends on (1) the position of each of the claimholders who affect the level of asset risk; (2) the bargaining power of the subdebtholders, which reflects the information available to them and their ability to credibly threat to "punish" stockholders for excessive risk taking (3) and the corrective measures that are taken by the regulator when information on risk taking is disclosed. Finally, we analyze the effect of risk shifting on the liabilities of the bank and of its borrowers and on the cost of deposit insurance.

Consistent with the corporate finance literature, we assume that the asset value of the bank's borrower follows a geometric Brownian motion (Merton [1974]). Due to the legal limited liability of the stockholders, the borrower's stock value is a convex function of the borrower's asset value and can be analyzed as a call option on the borrower's asset value (Merton [1977]), meaning that the borrower's stockholders always wish to increase risk. Our analysis differs from traditional banking models in assuming that bank assets are risky debt claims whose value is contingent on the borrowers' asset value and whose risk is determined by the borrower's asset risk. This means that the value of the bank's assets is limited from above by the face value of the borrower's loan. Consequently, the values of the bank's stock and subdebt are also limited from above and the stock value cannot exceed the difference between the face value of the borrower's loan and the total face value of the bank's debt (deposits and subdebt). Finally, the payoff to subdebtholders is capped from above by the face value of subdebt (Black and Cox [1976]).

We assume that a regulator conducts periodic audits, at which time the bank's asset risk is set according to the regulator's policy (Ronn and Verma [1986]; and Marcus and Shaked [1984]). However, between regulatory audits, risk shifting might occur.[1] Consistent with the literature we assume that depositors are incapable of monitoring their bank's risk as it is difficult and costly (Morgan [2000]; Caprio and Levine [2002] and Flannery et al. [2013]). In addition, since banks are efficient at monitoring and limiting the risk of their borrowers the borrower can only increase the level of asset risk above the initial level with the consent of the bank.

We leave the question of the bank's subdebtholders' ability to control the bank's risk taking open, and study the outcomes under various assumptions, where the level of bank asset risk between regulatory audits is determined in a bargaining process between the bank's stockholders and the subdebtholders, with the outcome depending on the relative power of the claimholders. First, we analyze the case in which the subdebtholders cannot affect the level of asset risk and therefore, the equilibrium level of risk is the result of the strategic interaction between the stockholders of the bank and of the borrower. This case is consistent with the criticism made during the 2007-2009 financial crisis, that subdebtholders are very limited in their ability to enforce market discipline (Flannery et al. [2013] and Calomiris and Herring [2013]), possibly due to asymmetric information between the subdebtholders and the stockholders. We prove that in equilibrium, risk shifting occurs only when the borrower is in financial distress and its asset value is below a threshold equal to the discounted geometric mean of the face value of the borrower's debt and the face value of the bank's total debt.

Next, we analyze the scenario where the subdebtholders can choose the level of asset risk. We prove analytically that risk shifting occurs only when the bank is in financial distress and the value of its assets is below the discounted geometric mean of the face value of deposits and the bank's total debt. Therefore, in equilibrium, risk shifting occurs for a smaller range

[1] Several papers suggest that since the regulator conducts on-site examinations of banks, it is better than other claimants at uncovering negative information, however the additional information which is reveled, becomes stale within a few months (Berger and Davies [1998]; Dahl et al. [1995]; Flannery and Houston [1999]; Cole and Gunther [1998]; DeYoung et al. [1998]).

of the borrower's asset value than when the equityholders are in control, and when it occurs, the risk shifting is to a lower level. Meaning that even in this extreme case, we find that risk shifting is not avoided completely. This result is similar to the analysis in Gorton and Santomero [1990] who show that subdebtholders are motivated to shift risk when asset value is below the threshold discussed above. However, Gorton and Santomero [1990] claim that the value of the subdebt increases with asset risk, while we prove that the value of subdebt is bell shaped with respect to asset risk, and that as a result, there is an interior solution that maximizes the value of subdebt, where we find a closed form for this level. This means that the increase in risk we predict in the event of financial distress is more moderate and limited than the one predicted by Gorton and Santomero [1990].

Following these two extreme cases, where only one of the claim holders chooses the equilibrium level of asset risk, we explore the more general case, where risk is determined jointly by the two claimholders as the outcome of a bargaining process. In this case, the subdebtholders are informed about the chosen level of asset risk, i.e., they have the ability to identify a shift in asset risk taken by the stockholders and to disclose this information to the regulator. In response, the regulator can take corrective measures and to restore the level of risk to its initial level. We solve this case applying the concept of asymmetric Nash bargaining solution (Nash [1950]) and find that the range of asset values for which risk shifting occurs is identical to the case where the level of asset risk is chosen to maximize the value of subdebt. In addition, when risk shifting does occur in equilibrium, it is to a level of risk between the levels that maximize the value of the stock and the level that maximizes the value of subdebt, and this level of risk is decreasing with the subdebtholders' bargaining power. In general, the analysis of the above three cases shows that risk shifting decreases with the subdebtholders' control and bargaining power. In addition, we show that replacing bank stock with subdebt leads to less asset values in which risk shifting can take place. However, when the barraging power of stockholders is relatively high, replacing stock with subdebt can lead to a higher level of risk in equilibrium.

Since the 2007-2009 financial crisis, the issue of bank transparency has been heavily debated. The international regulatory framework Basel III (Basel Committee on Banking Supervision [2010]) calls to strengthen bank transparency by conducting periodic stress tests which involve an unprecedented amount of disclosure by financial institutions. These stress tests increase transparency and consequently, can prevent excessive risk shifting in banks. However, it is often argued that transparency entails significant costs to financial institutions given their role in liquidity provision and risk sharing, and the fact that they are prone to runs. The overall tradeoff is still being evaluated (Goldstein and Leitner [2015]). Our contribution to this debate is the counterintuitive result that when the subdebtholders' bargaining power is low, suggesting low transparency, more restrictive corrective measures imposed by the regulator can motivate claimholders to agree on a higher level of risk in the bargaining process. Thus, the efficiency of the subdebt as a disciplinary tool declines with the enforcement of the traditional regulatory tools, such as capital adequacy and on-site supervision. We show that the two measures can be substitutes rather than complementaries.

Finally, we extend our model to address a concern raised in previous literature, by allowing side payments between the stockholders and the subdebtholders.[2] We solve this case in two steps. First, the stockholders and the subdebtholders jointly choose the risk level that maximizes the sum of their payoffs. Next, the side payment is chosen as the unique Nash solution to the bargaining problem of dividing the joint payoff between the two claimholders. We find that risk shifting takes place for a larger range of borrower asset values than when the subdebtholders determine asset risk, but smaller than when the stockholders do. In addition, when risk shifting occurs in equilibrium, it is to a level between the asset risk that maximizes the value of subdebt and the level that maximizes the stock value. The above threshold for risk shifting and equilibrium level of risk are identical to those found for a bank funded by just stock and deposits, with no subdebt. Therefore, when concerned by the possibility of side payments, to determine whether requiring a bank to issue subdebt, to replace a portion of stock, will decrease or increase risk taking, one must determine how much information and bargaining power subdebtholders will poses.

While the financial crisis was a turning point in the assessment of subdebt's effectiveness as a monitoring tool, there is a large literature, both empirical and theoretical, from the pre-crisis period, on the effect of subdebt on risk taking that our paper is directly linked to. This literature presents mixed results, where some works find that subdebt spreads are informative about the issuing banks' financial condition (Flannery and Sorescu [1996], Evanoff and Wall [2001], Hancock and Kwast [2001], Morgan and Stiroh [2001] and Sironi [2003]), while others do not (Balasubramnian and Cyree [2011]; Krishnan et al. [2005]; Avery et al. [1988] and Gorton and Santomero [1990]). Decamps et al. [2004] is probably the closest

[2]Furlong et al. [1987] show that equityholders could compensate the uninsured debt holders for risk in the form of higher promised interest rates. Calomiris [1999] and Chen and Hasan [2011] discuss the need to regulate the design of subordinated debt including its maturity and maximum allowable yield in order to assure that subdebtholders are motivated to control stockholder's risk taking.

paper to ours, as it also analyzes the interaction between the three pillars of the Basel accord: capital adequacy, supervisory review and market discipline. However, while the focus of their paper is on the effect of supervisory review and market discipline on capital requirement, our analysis focuses on the effect of capital adequacy and supervisory review on market discipline. Decamps et al. find that direct market discipline reduces the preferred level of regulatory capital when risk shifting is limited, but increases when large increases in risk are possible. In addition, the authors find that indirect market discipline reduces the need for regulatory audits and the need for minimal capital requirement, when subdebtholders are informed.

The rest of the paper is organized as follow. Section 2 describes the liability structure of the bank and its borrower and expresses the values of their different claims. Section 3 discusses these valuations' sensitivity to the level of the borrower's asset risk. In section 4 we analyze the extent to which risk-shifting occurs under different scenarios. Section 5 presents a numerical example of results, and Section 6 concludes.

# 2 Liability Structure and Valuations

In this section we describe the liability structure of the bank and the borrowing corporation and express the value of their different claims. In addition, we define the cost of deposit insurance, which is used in section 3 to find the preferred level of risk of each claimholders. These levels of risk are used to analyze equilibrium risk shifting in Section 4. For convenience, all the notations are summarized in Appendix A.

## 2.1 The borrower's liability structure

A corporation is funded by stock with market value $S_C$, and a single loan with face value $F_C$, and market value $B_C$. The loan is a zero-coupon loan maturing at time $T$ and the bank is its single creditor. The value of the corporation's assets, $V_C$, under the risk neutral measure follows a Geometric Brownian Motion according to the following equation:

$$dV_{C,t} = rV_{C,t}dt + \sigma V_{C,t}dW,$$

where $r$ is the instantaneous risk free rate of return, $\sigma$ is the instantaneous volatility of the corporation's assets, and $dW$ is a standard Wiener process under the risk-neutral probability measure.

A default event occurs at debt maturity, $T$, if the corporation's asset value, $V_{C,T}$, is lower than its face value of debt. If default occurs, the creditor takes over the corporation and realizes the residual assets of the corporation, $V_{C,T}$. Otherwise, debt is fully paid and the creditor, the bank, receives the entire face value of debt, $F_C$. Therefore, the corporation's payoff at debt maturity is equal to $B_{C,T} = \min[V_{C,T}, F_C]$. This expression can be rearranged and expressed as $B_{C,T} = F_C - \max[F_C - V_{C,T}, 0]$. As discussed in Merton [1974], this payoff is equivalent to the payoff of a risk-free debt with a face value of $F^C$ and a short position in a European put option. Therefore, the present value of the corporation's debt is given by:

$$B_{C,t} = F_C \cdot e^{-r(T-t)} - Put_t(V_{C,t}, F_C, \sigma, T-t, r),$$

where $Put_t(V_{C,t}, F_C, \sigma, T-t, r)$ is the value of a European put option on the corporation's asset value at time $t$, with a strike price equal to the face value of debt $F_C$, asset risk of $\sigma$, and time to maturity of $(T-t)$. Under the above described Geometric Brownian Motion the value of the option can be found using the Black and Scholes [1973] equation.

Since the stock is the residual claim, its payoff at debt maturity is $S_{C,T} = \max[V_{C,T} - F_C, 0]$. This payoff can be replicated by a European call option on the value of the corporation's assets, with a strike price equal to its face value of debt (Galai and Masulis [1976]). Therefore the value of stock at time $t$ is:

$$S_{C,t} = Call_t(V_{C,t}, F_C, \sigma, T-t, r), \tag{1}$$

where $Call_t(V_{C,t}, F_C, \sigma, T-t, r)$ is the value of a European call option according to the Black and Scholes [1973] equation.

## 2.2 The bank's liability structure

We consider a bank funded by stock with market value $S_B$, deposits with a total face value $F_{Dep}$ and market value of $B_{Dep}$, and zero-coupon subdebt with face value $F_{Sub}$ and market value $B_{Sub}$. Deposits are insured by the government or a government agency. We follow the classical papers Marcus and Shaked [1984] and Ronn and Verma [1986] in defining deposits as debt claims that mature at the time of the following regulatory audit - $T$. This is also consistent with newer finding of Berger et al. [2000], who show that supervisory assessments following an on-site inspection - audit - are more accurate than the market in predicting changes in bank performance. Following similar logic, we assume the subdebt matures at time $T$ as well. Since the bank's asset is the loan which funds the activity of the corporation, as described in Section 2.1, we can express the payoff of the bank's assets at maturity as:

$$V_{B,T} = B_{C,T} = F_C - \max[F_C - V_{C,T}, 0]. \tag{2}$$

Thus, the asset value of the bank prior to maturity is:

$$V_{B,t} = B_{C,t} = F_C \cdot e^{-r(T-t)} - Put_t(V_{C,t}, F_C, \sigma, T-t, r).$$

The depositors are the senior claimholders, and therefore the bank pays at time $T$ the minimum between the value of the bank's assets and the face value of deposits, $F_{Dep}$, which can be expressed as:

$$\begin{aligned} B_{Dep,T} &= \min\left[V_{B,T}, F_{Dep}\right] = F_{Dep} - \max\left[F_{Dep} - V_{B,T}, 0\right] \\ &= F_{Dep} - \max\left[F_{Dep} - F_C + \max\left[F_C - V_{C,T}, 0\right], 0\right]. \end{aligned}$$

As the bank funds the borrower's loan using both stock and debt, the face value of the corporation's debt, $F_C$, must be higher than the face value of the bank's own debt, $(F_{Dep} + F_{Sub})$ and therefore, also higher than $F_{Dep}$. Under this assumption, we can express the depositor's payoff at maturity as $B_{Dep,T} = F_{Dep} - \max[F_{Dep} - V_{C,T}, 0]$. This payoff can be replicated by a long position in a risk-free debt with a face value $F_{Dep}$ and a short position in a European put option on the borrower's assets, with a strike price equal to the face value of the bank's deposits. Therefore, the value of the bank's deposits at any time $t$ prior to debt maturity can be expressed as:

$$B_{Dep,t} = F_{Dep} \cdot e^{-r(T-t)} - Put_t(V_{C,t}, F_{Dep}, \sigma, T-t, r).$$

The subdebtholders receive at debt's maturity a face value of $F_{Dep}$ if the value of the borrower's assets is above the bank's total face value of debt. Otherwise, the payoff is the maximum between zero and the difference between the value of the bank assets and the face value of the deposits. This payoff can be rearranged and expressed as:

$$B_{Sub,T} = \max\left[V_{C,T} - F_{Dep}, 0\right] - \max\left[V_{C,T} - (F_{Dep} + F_{Sub}), 0\right],$$

which is equivalent to a long position in a European call option with a strike price equal the face value of deposits, $F_{Dep}$, and a short position in a European call option with a strike price equal to the face value of the bank's total debt, $(F_{Dep} + F_{Sub})$. Therefore, the value of the subdebt prior to debt maturity is:

$$B_{Sub,t} = Call_t(V_{C,t}, F_{Dep}, \sigma, T - t, r) - Call_t(V_{C,t}, F_{Dep} + F_{Sub}, \sigma, T - t, r). \qquad (3)$$

Since the bank's stockholders are the residual claimholders, their payoff at maturity is $S_{B,T} = \max\left[V_{B,T} - (F_{Dep} + F_{Sub}), 0\right]$. If the bank is solvent at maturity, the stockholders receives payoff of $F_C - (F_{Dep} + F_{Sub})$, which is the maximum payoff that the bank's stockholders can receive. This differs from the basic structural approach in which the stockholders' payoff is unbounded. When we expand this payoff it can be expressed as:

$$S_{B,T} = \max[V_{C,T} - (F_{Dep} + F_{Sub}), 0] - \max[V_{C,T} - F_C, 0].$$

This payoff can be replicated by a long position in a European call option, with a strike price equal to the face value of the bank's total debt, $(F_{Dep} + F_{Sub})$, and a short position in a European call option, with a strike price equal to the face value of the corporation's debt, $F_C$. Therefore, the value of the bank's stock prior to debt maturity is:

$$S_{B,t} = Call_t(V_{C,t}, F_{Dep} + F_{Sub}, \sigma, T - t, r) - Call_t(V_{C,t}, F_C, \sigma, T - t, r). \qquad (4)$$

The value of the bank's assets and the payoff to each of the bank's claimholders at debt maturity is described in Figure 1.

## 2.3 The cost of deposit insurance

A regulator conducts periodic audits to assess the bank's asset risk which is then used to estimate the bank's deposit insurance premium, i.e., at the time of audit the bank pays a fair insurance premium reflecting its capital structure and risk. Since, the premium is not adjusted between audits to reflect changes in the quality of the bank's asset, it does not affect the bank's risk taking motivation between audits.[3]

[3]The literature on deposit insurance shows that regulators lack the resources, information or incentive to correctly assess bank risk and to charge deposit insurance premium accordingly (Allen et al. [2015], Anginer et al. [2014], Chan et al. [1992] and Freixas and Rochet [1998]).

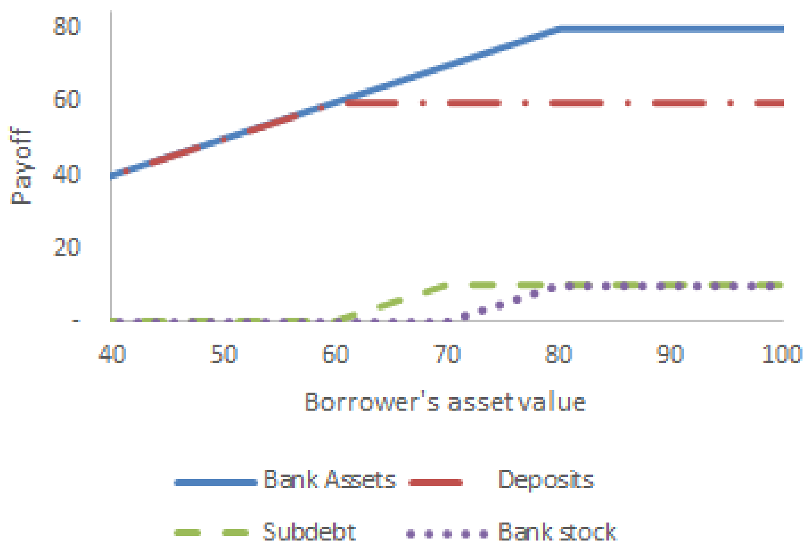



**Figure 1: The bank's asset value and the payoff of the depositors, subdebtholders and stockholders at debt maturity.** The face value of the corporation's debt is $F_C = 80$. The face value of the bank's deposits is $F_{Dep} = 60$ and of the subdebt is $F_{Sub} = 10$.

If at debt's maturity the value of the bank's assets is below the face value of its deposits, the insurer compensates the depositors with the difference between the two. Thus, the cost of deposit insurance equals the maximum between zero and the difference between the face value of the secured deposits and the value of the bank's assets: $DI_T = \max[F_{Dep} - V_{B,T}, 0]$. Replacing $V_{B,T}$ above by Equation 2 we find that $DI_T = \max[F_{Dep} - V_{C,T}, 0]$. As discussed in Merton [1977] and Crouhy and Galai [1991], this payoff is equivalent to a long put option on the corporation's assets with a strike price equal to the face value of the bank's deposits. Following convention, we normalize and use the value of deposit insurance per dollar (DIPD) of insured deposits is defined as:

$$DIPD_t = \frac{Put_t(V_{C,t}, F_{Dep}, \sigma, T-t, r)}{F_{Dep}}. \tag{5}$$

The value of deposit insurance increases with the borrowers's asset risk, and decreases with its asset value.

# 3 Preferences Regarding the Asset's Risk Level

In this section, we build on the pricing equations for the different securities developed in Section 2, and find the different claimholders' preferences for asset risk. Throughout the paper we assume that the sole objective of each claimholder is to maximize the market value of its claim. We define $\sigma_0$ as the borrower's asset risk at the time of the last regulatory audit. In what follows we study the preferences of the different claimholders at some time $t \in (0, T)$ after the loan contract is set but before the time of the next audit, which is also the time of debt maturity.

## 3.1 Preferences of the borrower's Stockholders

In line with classical agency theory (Jensen and Meckling [1976]), the borrower's stockholders are always better off increasing the level of asset risk (under the assumption that their goal is to maximize the market value of their own holding).

**Claim 1.** *The value of the stock of the bank's borrower is increasing with asset's risk.*

*Proof.* Recall that the market value of the corporation's stock is equal to the value of a European call option (Eq. 1): $S_{C,t} = Call_t(V_{C,t}, F_C, \sigma, T-t, r)$. It is well-known that the value of a call option is strictly increasing in the option's risk level $\sigma$ (see, e.g., Jensen and Meckling [1976]). $\square$

## 3.2 Preferences of the Bank's Stockholders

The payoff of the stock is similar to that of a subordinated debt when the value of assets is uncapped and therefore the asset value threshold for risk shifting is identical to the one developed by Gorton and Santomero [1990] for the case of subdebt of a corporation with uncapped assets. Specifically, the characterization of the bank stockholders' preference regarding the corporation' asset risk crucially depend on the threshold (Appendix B):

$$V^*_{S_B} \equiv e^{-r(T-t)} \sqrt{(F_{Dep} + F_{Sub}) \cdot F_C}, \tag{6}$$

which is a function of the geometric mean of the face value of the borrower's debt and the face value of the bank's total debt. In situations where the borrower's asset value is above this threshold, the value of the bank's stock decreases with risk.

By contrast, in setups where the value of the borrower's assets is below the threshold, the relationship between the market value of the bank's stock and asset risk is hump-shaped (unimodal), and value of bank stock reaches its maximum when the borrower's asset risk is (as proven in Appendix B):

$$\sigma_{S_B}^{max} \equiv \arg\max_{\sigma} S_B = \sqrt{\frac{1}{T-t}\ln\left(\frac{(F_{Dep}+F_{sub})\cdot F_C}{(V_{C,t})^2}\right) - 2r}. \tag{7}$$

The level of asset risk preferred by the bank stockholders, $\sigma_{S_B}^{max}$, is an increasing function of the borrower's leverage, ($F_C$ to $V_{C,t}$) and the bank's leverage defined as the face value of bank debt to the value of its asset ($F_{Dep} + F_{sub}$ to $V_{C,t}$). Thus, the level of asset risk preferred by the bank's stockholders depends on both the financial risk of the bank and of its borrower.

We find that $\sigma_{S_B}^{max} > \sigma_0$ if and only if the corporation's asset value is below a second threshold defined as (Appendix B):

$$V_{S_B}^{**} \equiv e^{-\left(r+\frac{\sigma_0^2}{2}\right)(T-t)}\sqrt{(F_{Dep}+F_{Sub})\cdot F_C}. \tag{8}$$

Note that $V_{S_B}^{*}$ is higher than $V_{S_B}^{**}$ for any positive initial risk $\sigma_0 > 0$. Further note that both $V_{S_B}^{*}$ and $V_{S_B}^{**}$ depend on the geometric mean of the face values of the bank's total debt and the corporation's debt. Since the face value of the bank's debt is lower than that of the corporation's debt, both thresholds are lower than the face value of the corporation's debt. Thus, when the value of the borrower's assets crosses these thresholds the borrower is already in financial distress. The following result summarizes the above analysis of the value of the bank stock as a function of its borrower's asset risk.

**Proposition 1.** *The value of the bank's stock is: (1) decreasing with the value of its borrower's asset risk if $V_{S_B}^{*} < V_{C,t}$, (2) hump-shaped (unimodal) in the asset risk if $V_{S_B}^{*} > V_{C,t}$, and, in this case, its maximum is obtained for risk level of $\sigma_{S_B}^{max}$. Moreover, the risk level that maximizes the value of the bank's stock is larger than the initial risk (i.e., $\sigma_{S_B}^{max} > \sigma_0$) if and only if $V_{C,t} < V_{S_B}^{**}$.*

### 3.3 Preferences of the Subdebtholders

The subdebt payoff has a similar structure to the payoff of the bank's stock. Both receive nothing if the value of asset is below some lower level and both are upward sloping above this level till some upper level is reached where the payoff is capped. However, while the value of the stock is capped when the value of the borrower's assets is above the face value of its debt, $F_C$, the subdebt is capped when the value of assets is above the total face value of the bank's debt, $F_{Sub}+F_{Dep}$, which is actually the level in which the payoff of the stockholders is changed from zero to positive. The asset value where above it the subdebt's payoff is positive is equals to the face value of the bank's deposits, $F_{Dep}$. Therefore, we can simply replace the threshold in Equation (6) and to receive the threshold which determines the sensitivity of the subdebt to asset risk:

$$V^*_{B_{sub}} \equiv e^{-r(T-t)}\sqrt{F_{Dep}\cdot(F_{Dep}+F_{sub})}, \tag{9}$$

This threshold is a function of the geometric mean of the face value of deposits and of the bank's total debt. Observe that: (1) the threshold $V^*_{B_{sub}}$ does not depend on the borrower's level of debt, and (2) the subdebtholders' threshold for risk shifting is strictly smaller than the stockholders' threshold for risk shifting (i.e., $V^*_{B_{sub}} < V^*_{S_B}$). In situations in which the corporation's asset value is above this threshold, the market value of the subdebt claim decreases with risk. In realizations where the value of the borrower's assets is below the threshold, the relationship between the market value of the subdebt claim and asset risk is hump-shaped, and the maximum value of the subdebt claim is reached at the level of asset risk defined as:

$$\sigma^{max}_{B_{sub}} \equiv \arg\max_{\sigma} B_{Sub,t}(\sigma) = \sqrt{\frac{1}{T-t}\ln\left(\frac{(F_{Dep}+F_{sub})\cdot F_{Dep}}{(V_{C,t})^2}\right) - 2r}. \tag{10}$$

It is immediate that $\sigma^{max}_{B_{sub}} < \sigma^{max}_{S_B}$ (since $F_{Dep} < F_{Dep}+F_{Sub} < F_C$), which implies that subdebtholders always prefer a lower level of asset's risk than the bank shareholders. Finally, the value of subdebt is maximized at a positive level of asset risk, $\sigma^{max}_{B_{Sub}} > \sigma_0$, if and only if the corporation's asset value is below a second threshold defined as:

$$V^{**}_{B_{sub}} \equiv e^{-\left(r+\frac{\sigma_0^2}{2}\right)(T-t)}\sqrt{(F_{Dep}+F_{Sub})\cdot F_{Dep}}. \tag{11}$$

Note that $V^*_{B_{sub}}$ is above $V^{**}_{B_{sub}}$ for each positive level of initial risk $\sigma_0 > 0$. In addition, both $V^*_{B_{sub}}$ and $V^{**}_{B_{sub}}$ depend on the geometric mean of the face values of the bank's total debt and of deposits. Meaning that both thresholds are bellow the face value of bank debt. Thus, when the value of the borrower's assets crosses these thresholds the bank is already in financial distress.

The following result summarizes the above analysis of the value of the bank stock as a function the asset's risk.

**Proposition 2.** *The market value of the subdebt claim is: (1) decreasing with the borrower's asset risk if $V^*_{B_{Sub}} < V_{C,t}$, (2) hump-shaped (unimodal) in the asset risk if $V^*_{B_{Sub}} > V_{C,t}$, and, in this case, its maximum is obtained for risk level $\sigma^{max}_{B_{Sub}}$. Moreover, the risk level that maximizes the value of subdebt is higher than the initial risk (i.e., $\sigma^{max}_{B_{Sub}} > \sigma_0$) if and only if $V_{C,t} < V^{**}_{B_{Sub}}$.*

It is important to note that risk shifting in our model can occur as in Gorton and Santomero [1990] when the value of the asset is below the discounted geometric mean of the face value of the deposits and the total debt of the bank. However, we show that when risk shifting may occur the subdebtholders are motivated to increase asset risk to some interior level, since the value of the subdebt is hump-shaped with respect to asset risk. In contrast, Gorton and Santomero [1990] claim that the value of the subdebt is increasing with asset risk below the lower threshold for risk shifting.[4] This is proven to be wrong in our paper.

## 3.4 Illustration for the Preferences of the Various Claim Holders

The risk preferences of the bank's subdebtholders and stockholders and their dependence on asset value are illustrated in this section and demonstrated in Figure 2. Each panel illustrates the values of stock and subdebt as a function of the borrower's asset risk for a different asset value, as well as the level of risk that maximizes the value of each claim. The figure also illustrates the stockholders' preference for a higher level of asset risk than the subdebtholders, as discussed above. The face value of the corporation's loan is 80, the face value of subdebt is 10, and the face value of deposits is 60. The risk-free rate is 1% and time to maturity is one year.

[4] Gorton and Santomero [1990], page 122: "Unlike senior debt, the default risk premium on junior debt is a decreasing function of $\sigma^2$ when $V$ is less than $\hat{V}$ and an increasing function Of $\sigma^2$ when $V$ is greater than $\hat{V}$."

Panel 2a illustrates a high asset value where $V_{C,t} = 100$, above the threshold $V^*_{S_B} = 74.1$ (Eq. 6). In this case, the value of both the stock and subdebt are decreasing in the risk, meaning that the value of both claims is maximized with zero risk. Panel 2b is an example of an intermediate asset value where $V_{C,t} = 70$, which is below $V^*_{S_B} = 74.1$ but still above $V^*_{B_{Sub}} = 64.2$ (Eq. 9). In this case, the stock value is hump-shaped with respect to asset risk and is maximized with asset risk of 33.7%. However, the value of subdebt is decreasing in risk and therefore is maximized with zero risk. Panel 2c illustrates a low asset value of $V_{C,t} = 62$, which is below $V^*_{B_{Sub}} = 64.2$. In this case, both the value of stock and subdebt are hump-shaped with respect to asset risk but the value of stock is maximized at a higher level $\sigma^{max}_{S_B} = 59.7\%$ than the subdebt $\sigma^{max}_{B_{Sub}} = 26.2\%$. In addition, the value of deposit insurance per dollar of insured deposits (Eq. 5) with asset risk $\sigma^{max}_{B_{Sub}} = 26.2\%$ is $DIPD_t = 8.5\%$ while with the higher asset risk of $\sigma^{max}_{S_B} = 59.7\%$ deposit insurance would increase to $DIPD_t = 21.6\%$.

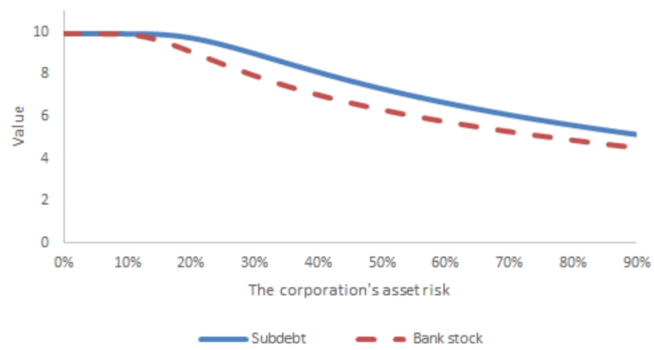
# 4 Analysis of Risk Shifting

In this section we build on the expressions developed in Section 3 to to analyze the extent to which risk shifting occurs under different scenarios regarding how the control over the bank is divided between the shareholders and the subdebtholders.

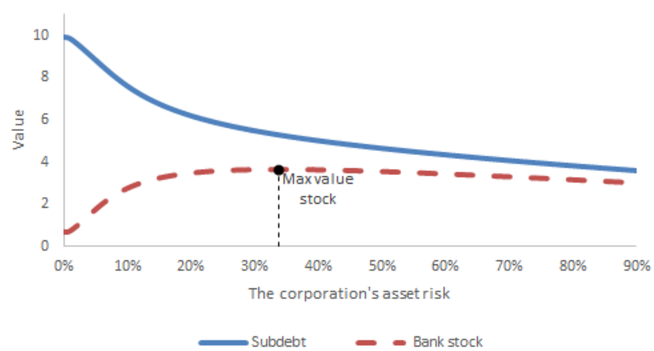
## 4.1 Framework of Analysis

We assume a regulator conducts periodic audits to assess and align the level of bank asset risk with the regulatory policy.[5] In accordance, the bank sets the face value of the borrower's loan, $F_C$, to account for the borrower's initial asset risk, $\sigma_0$. Thus, at the time of an audit, the bank's liabilities, stock, subdebt and secured deposits, are fairly priced according to the borrower's asset risk and leverage. Although at the time of an audit the level of risk is set to the initial level, at all times between the periodic audits, the bank can shift its level of asset risk by allowing the borrower to change its level of asset risk. In our model, we focus on such points in time and inquire into the conditions in which risk shifting would occur.

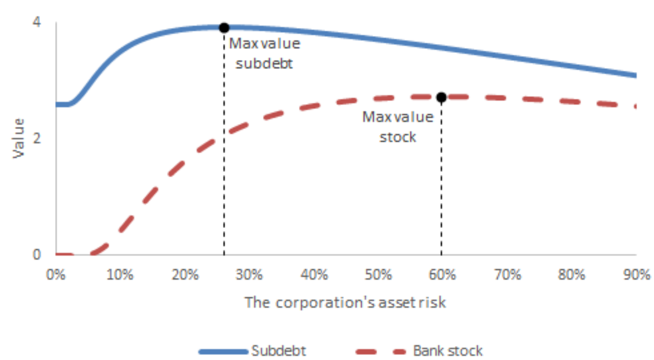
[5] The question of what is the optimal level of asset risk which should be set at the time of regulatory audit as well as the relation between the optimal level and the actual level which the regulator chooses is beyond the scope of our analysis. However, it is clear that at the time of an audit the regulator can force some level of risk which we call the initial level of risk.




**(a)** The borrower's asset value: $V_C = 100$




**(b)** The borrower's asset value: $V_C = 70$




**(c)** The borrower's asset value: $V_C = 62$

**Figure 2: The value of the stock of the bank and of the subordinated debt as a function of the borrower's asset risk.** The face value of the debtor's debt is $F_C = 80$. The face value of the bank's deposits is $F_{Dep} = 60$ and of the subordinated debt is $F_{Sub} = 10$. These data yield for the bank a book leverage of 87.5%. In addition, the time to maturity is one year and the risk free rate is $r = 1\%$.

Risk shifting may occur if the borrower's claimholders, stockholders and creditors, agree on a specific level of risk, i.e, they are all better off with risk shifting. Since the creditor is the bank, the bank's own claimholders must reach an agreement regarding the borrower's level of risk. The bank's control rights remain an open issue, specifically the ability of subdebtholders to affect the chosen level of asset risk. This ability is a function of the information available to the subdebtholders, their ability to disclose such information to the regulator and the corrective measures the regulator can take upon receiving such information. Therefore, in our analysis we cover a wide spectrum of possible states and study the effect of subdebtholders' relative power on risk shifting. The chosen cases either mark the upper and lower boundaries for risk shifting or refer to intermediate cases which are a result of different claimholders' bargaining power.

First, we consider the extreme case where the bank's stockholders hold all control rights, and therefore, can choose any level of asset risk accepted by the bank's borrower. Second, we examine the other extreme case, where the subdebtholders determine the level of asset risk while the stockholders have no say. Although this extreme case is highly improbable in reality, it serves as an informative benchmark, marking the upper bound on creditor's rights. Next we move to an intermediate bargaining framework where any change in asset risk requires the joint consent of the bank's stockholders and subdebtholders, and the joint decision is the result of a bargaining process. Under this framework, risk shifting can occur only if both the bank's stockholders and subdebtholders are better off with respect to the initial level of asset risk set at the previous audit. Finally, we allow the stockholders and the subdebtholders to exchange side payments, while jointly deciding the level of asset risk. All results presented in this section are summarized in Table 1.

Claim 1 in Section 3.1 implies that the borrower's stockholders always prefer the highest level of asset risk allowed by the bank and would never agree to decrease risk. Therefore, in the rest of our analysis we focus on the bank's controller's choice of the maximum allowed level of borrower asset risk, which must be weakly larger than the initial level of risk in order for the borrower's stockholders to agree. We assume this maximum allowed level of risk is feasible given the technological limitations governing the borrower's assets, and that the borrower's shareholders always shift asset risk to it.

## 4.2 Bank Asset Risk is Determined by the Stockholders

In this section we assume that the bank's stockholders hold all control rights, while the subdebtholders are unable to restrict the stockholders' risk taking. This extreme case is consistent with a scenario where subdebtholders are uninformed on risk shifting events or do not have the ability to disclose such information to a regulator who could have used corrective measures to affect the level of asset risk. This scenario is in line with the view that subdebtholders did not restrict excessive risk taking during the 2007-2009 financial crisis because they did not have the ability to do so and not due to lack of motivation (Gorton and Santomero [1990] and Calomiris and Herring [2013]).

Proposition 1 in Section 3.2 characterized the value of the bank stock for different asset values. An immediate corollary is that risk shifting occurs in this case if and only if the value of the borrower's asset is below the threshold, $V^{**}_{S_B}$.

**Corollary 3.** *When the bank's stockholders hold the bank's control rights, risk shifting occurs if and only if $V_{C,t} < V^{**}_{S_B}$. When there is risk shifting, the risk increases to $\sigma^{max}_{S_B}$.*

This case highlights that even when the subdebtholders are unable to restrict the level of asset risk, risk shifting is limited to states where the bank's borrower is in financial distress and the level of risk is limited by $\sigma^{max}_{S_B}$, and is not unbounded as suggested by Gorton and Santomero [1990].[6]

## 4.3 Asset Risk is Determined by the Subdebtholders

In the second case we assume that the subdebtholders possess the bank's control rights. This is an extreme case of the view that market discipline can be used to restrict bank risk and in particular that the subdebtholders possess "direct influence" on bank risk taking (as defined in Flannery [2001]).

Proposition 2 in Section 3.3 characterized the payoff to the bank's stockholders for different asset values. An immediate corollary is that risk shifting occurs in Case 2 if and only if the value of the corporation's asset is below the threshold, $V^{**}_{B_{Sub}}$

**Corollary 4.** *When the subdebtholders control the bank's decisions, risk shifting occurs if and only if $V_{C,t} < V^{**}_{B_{Sub}}$. When there is risk shifting, the risk increases to $\sigma^{max}_{B_{Sub}}$.*

[6] Since the banks stockhoders payoff is limited from above, their utility from asset risk is concave. This would not be the case if the banks assets value was unbounded, in which case the equilibrium level of asset risk would be unbounded as well.

**Table 1: Summary of Risk-shifting Analysis**

<table>
<tr><th rowspan="2">Control & bargening framework</th><th colspan="4">Corporation's asset value ($V_C$)</th></tr>
<tr><th>$V_C > V^{**}_{S_B}$</th><th>$V_C \in (V^{**}_{S_B+B_{Sub}}, V^{**}_{S_B})$</th><th>$V_C \in (V^{**}_{B_{Sub}}, V^{**}_{S_B+B_{Sub}})$</th><th>$V_C < V^{**}_{B_{Sub}}$</th></tr>
<tr><td>Stockholders control the bank</td><td rowspan="4">No risk shifting</td><td colspan="3">Risk shifting to $\sigma^{max}_{S_B}$</td></tr>
<tr><td>Joint control with side payments</td><td rowspan="3">No risk shifting</td><td colspan="2">Risk shifting to $\sigma^{max}_{S_B+B_{sub}} \in (\sigma^{max}_{B_{Sub}}, \sigma^{max}_{S_B})$</td></tr>
<tr><td>Joint control with out side payments</td><td rowspan="2">No risk shifting</td><td>Risk shifting to $\sigma^{max}_{\alpha} \in (\sigma^{max}_{B_{Sub}}, \sigma^{max}_{S_B})$</td></tr>
<tr><td>Subdebtholders control the bank</td><td>Risk shifting to $\sigma^{max}_{B_{Sub}}$</td></tr>
</table>

Observe, that when the bank is controlled by subdebtholders, risk shifting is smaller than in the case in which the bank is controlled by the shareholders in the following two ways: (1) when the corporation's asset value is between $V^{**}_{B_{sub}}$ and $V^{**}_{S_B}$, risk shifting occurs only when bank is controlled by the stockholders and does not occur when it is controlled by the subdebtholders (consistent with the example in Figure 2b); and (2) when the corporation's asset value is below $V^{**}_{B_{sub}}$, there is risk shifting in both cases, but the risk shifting is smaller when the bank is controlled by the subdebtholders, i.e., $\sigma^{max}_{B_{sub}} < \sigma^{max}_{S_B}$ (consistent with the example in Figure 2c). A comparison of the two cases is presented in Table 1.

## 4.4 Asset Risk is Determined in a Joint Decision

The previous two cases are extreme ones where the bank is solely controlled by either the stockholders or by the subdebtholders. The analysis in this section, however, refers to the rest of the spectrum, where any change in asset risk requires the joint consent of both stockholders and subdebtholders. Specifically, we assume that allowing the borrower to increase its asset's risk requires the joint agreement of the bank's stockholders and subdebtholders, and when a disagreement arises, the asset's risk remains at its initial level, $\sigma_0$. This assumption may reflect a situation were a side that does not approve of an increase in asset risk, approaches the regulator, disclosing information regarding the increase in risk, and where upon receiving this information the regulator instructs the bank to decrease the level of asset risk back to its initial level, i.e., the level that was set at the last audit and used to price the bank's deposit insurance premium. Alternatively, this assumption may represent the idea that subdebt issues are a repeated-game were the bank must issue debt in the market periodically and any abuse of subdebtholders may cause an increase in cost of debt in later iterations.

A necessary condition for the described bargaining process, is that subdebtholders can observe the level of asset risk. Thus, a regime shift from the case where the stockholders determine the level of risk alone to the case with bargaining, can be a result of a change in investor sophistication or an increase in asset transparency, such as the one brought about by the disclosure rules which followed Basel III (Vashishtha et al. [2018]).

### 4.4.1 Analysis of the Bargaining Game

As common in the game theory literature we model this strategic interaction as a bargaining situation (see, e.g., Osborne and Rubinstein [1990] Chapter 7, for a textbook introduction). Specifically, we assume that an exogenous parameter $\alpha \in [0,1]$ describes the bargaining power of stockholders relative to subdebtholders. The case of $\alpha = 1$ ($\alpha = 0$) corresponds to a state in which stockholders (subdebtholders) hold all bargaining power, i.e., the stockholders (subdebtholders) present a "take-it-or-leave-it" offer regarding the maximal allowed level of asset risk, and subdebtholders (stockholders) can either accept or reject this offer, without an opportunity to present a counter-offer. The case where $\alpha$=0.5, corresponds to a symmetric state, where both claimholders possess the same bargaining power.

The solution concept we apply to capture the joint decision of stockholders and subdebtholders is the asymmetric Nash bargaining solution (Nash [1950]; Kalai [1977]), according to which the maximal risk level that is chosen at the end of the bargaining process is:

$$\sigma_{\alpha}^{max} = \arg\max_{\substack{\sigma \geq \sigma_0 \\ S_{B,t}(\sigma) \geq S_{B,t}(\sigma_0) \\ B_{Sub,t}(\sigma) \geq B_{Sub,t}(\sigma_0)}} \left( \left(S_{B,t}\left(\sigma\right) - S_{B,t}\left(\sigma_0\right)\right)^{\alpha} \cdot \left(B_{Sub,t}\left(\sigma\right) - B_{Sub,t}\left(\sigma_0\right)\right)^{1-\alpha} \right) \quad (12)$$

### 4.4.2 Solution of the Bargaining Game

The following proposition characterizes the maximal risk level $\sigma_{\alpha}^{max}$ jointly chosen by stockholders and subdebtholders. Specifically, it shows that (1) the condition for risk shifting is the same as in the case where the bank is controlled by subdebtholders, and (2) if there is risk shifting, the level of risk is increasing in $\alpha$, and is between the levels of risk found in the two previous cases (a comparison of the solution of this case with the two previous cases is presented in Table 1).

**Proposition 5.** *When the bank is jointly controlled by subdebtholders and stockholders, risk shifting occurs if and only if $V_{C,t} < V^{**}_{B_{Sub}}$. When there is risk shifting, it is to the level of risk $\sigma^{max}_{\alpha}$ which is (1) increasing in $\alpha$, and (2) between the risk level decided upon in the case of subdebtholders sole control and the level decided upon in the case of sole control by the stockholders (i.e., $\sigma^{max}_{B_{Sub}} \leq \sigma^{max}_{\alpha} \leq \sigma^{max}_{S_B}$).*

*Proof.* We begin by showing that risk shifting occurs if and only if $V_{C,t} < V^{**}_{B_{Sub}}$. Proposition 2 immediately implies that there exists a higher risk level $\sigma > \sigma_0$ that induces a higher value for the subdebt relative to the initial level of risk if and only if $V_{C,t} < V^{**}_{B_{Sub}}$. Moreover, due to Proposition 1 and the inequality $V^{**}_{B_{Sub}} < V^{**}_{S_B}$, if the subdebtholders achieve a higher value (relative to its value given the initial risk) so do the stockholders. Thus, due to Eq. 12, there exist a higher level of risk $\sigma > \sigma_0$ that provides higher value for both the stock and the subdebt (relative to their value induced by the initial risk) if and only if $V_{C,t} < V^{**}_{B_{Sub}}$.

Next, we focus on the case in which there is risk shifting (i.e., $V_{C,t} < V^{**}_{B_{Sub}}$). Observe that due to Propositions 1 – 2 (1) both the expressions $(S_{B,t}(\sigma) - S_{B,t}(\sigma_0))$ and $(B_{Sub,t}(\sigma) - B_{Sub,t}(\sigma_0))$ are increasing in $\sigma$ for low risk levels satisfying $\sigma < \sigma^{max}_{B_{Sub}}$, (2) $(S_{B,t}(\sigma) - S_{B,t}(\sigma_0))$ is increasing in $\sigma$, while $(B_{Sub,t}(\sigma) - B_{Sub,t}(\sigma_0))$ is decreasing in $\sigma$ for intermediate risk levels satisfying $\sigma^{max}_{B_{Sub}} < \sigma < \sigma^{max}_{S_B}$, and (3) both expressions are decreasing for large values of risk satisfying $\sigma > \sigma^{max}_{S_B}$. These observations and the definition of $\sigma^{max}_{\alpha}$ in Eq. 12 imply that $\sigma^{max}_{\alpha}$ is increasing in $\alpha$, and that $\sigma^{max}_{B_{Sub}} \leq \sigma^{max}_{\alpha} \leq \sigma^{max}_{S_B}$. □

### 4.4.3 Illustration

We demonstrate the motivation of the bank's subdebtholders and stockholders in Figure 3 using the parameters discussed in Section 3.4. When the borrower's asset value is intermediate, $V_C = 70$, the value of subdebt decreases with asset risk. Therefore, subdebtholders will not agree to increase risk and in equilibrium asset risk will not exceed its initial level, set at the previous audit, $\sigma_0 = 10\%$ (Figure 3a). When the borrower's asset value is low, $V_C = 62$, the value of subdebt is higher for any level of risk between $\sigma_0 = 10\%$ and $\overline{\sigma_0} = 64\%$ than it is with the initial level of risk (Figure 3b). Therefore, when stockholders suggest an increase in asset risk to a level within this range, the subdebtholders agree to the offer instead of turning to the regulator. For example, when $\alpha = 0.8$ the equilibrium level of risk is $\sigma_{0.8}^{max}(10\%) = 40.6\%$ which is between the risk preferred by the stockholders, 59.7%, and by the subdebtholders, 26.2%. The value of deposit insurance per dollar of insured deposits (Eq. 5) with asset risk $\sigma_{0.8}^{max}(10\%) = 40.6\%$ is $DIPD_t = 14.2\%$, higher than its value using the risk preferred by subdebtholders and lower than its value using the risk preferred by stockholders (Section 3.4).

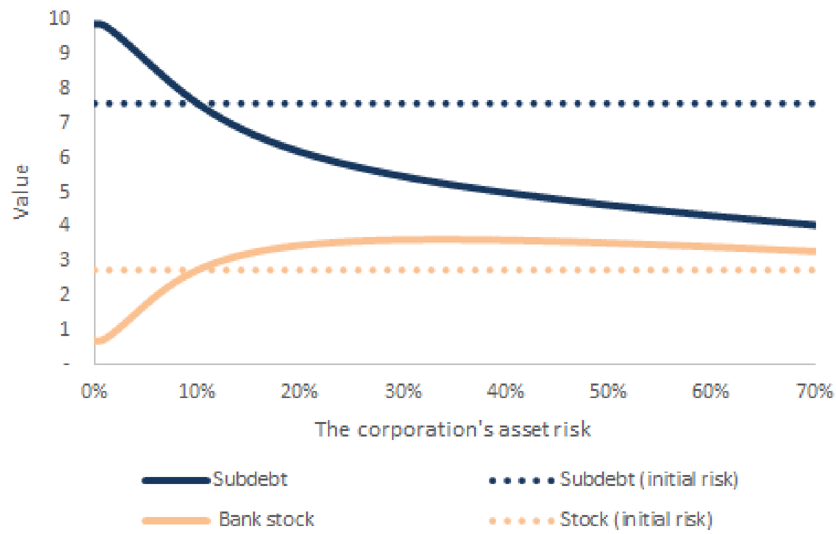
#### 4.4.4 Impact of Initial Risk Level

In the previous section we interpreted $\sigma_0$ in Equation 12 as the initial level of the borrower's asset risk set in the contract between the borrower and the bank. We assumed that if the bank's claimholders cannot agree on a higher level of risk then risk remains unchanged at its initial level. However, we can also interpret $\sigma_0$ as the level of risk that is set by the regulator when it learns from the subdebtholders that the bank took part in risk shifting. A higher $\sigma_0$ is consistent with regulator forbearance while a lower value is consistent with a regulator that is intolerant of risk shifting. The extreme case where $\sigma_0 = 0\%$ is consistent with the regulator closing any bank that she finds has indulged in risk shifting. Under this interpretation, we find that more severe regulatory corrective measures may have the opposite effect then intended and actually increase equilibrium risk. This result is even more striking if we interpret $\sigma_0$ as the level of risk that the claim holders believe the regulator will set. Meaning if claim holders' believe that the regulator will be more intolerant of risk shifting equilibrium risk would increase when the bank is jointly controlled and side payments are not possible.

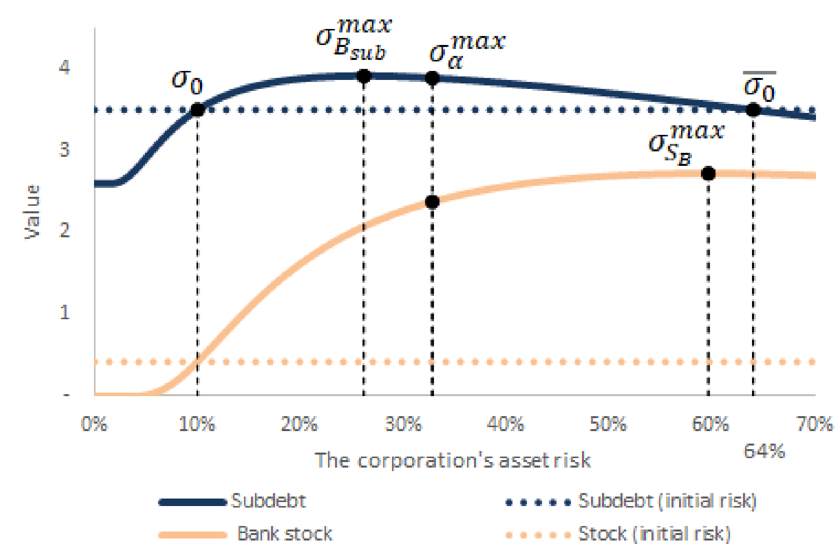
In what follows we characterize conditions under which we get the surprising comparative statics with respect to the initial risk level, namely, that reducing the initial risk level, which is set at an audit period, increases the equilibrium risk level induced by the bargaining process.

The sufficient conditions for this counterintuitive result is that: (1) subdebtholders are willing to increase risk above its initial level, i.e., $V_{C,t} < V_{B_{Sub}}^{**}$; (2) but they will not agree to increase risk to the level that maximizes the value of stock, i.e., $B_{sub,t}(\sigma_{S_B}^{max}) < B_{sub,t}(\sigma_0)$; and (3) the bargaining power of the stockholders $\alpha$ is sufficiently high. Formally:

**Proposition 6.** *Assume that the bank is jointly controlled by the subdebtholders and the stockholders and that $V_{C,t} < V_{B_{Sub}}^{**}$. Further assume that $B_{sub,t}(\sigma_{S_B}^{max}) < B_{sub,t}(\sigma_0)$. Then for*




**(a)** The corporation's asset value: $V_C = 70$




**(b)** The corporation's asset value: $V_C = 62$

**Figure 3: The value of bank stock and subordinated debt as a function of the asset risk of its borrower.** The dashed lines represent the values of subdebt and stock with the initial level of asset risk of $\sigma_0 = 10\%$. The face value of the corporation's debt is $F_C = 80$. The face value of the bank's deposits is $F_{Dep} = 60$ and of the subordinated debt is $F_{Sub} = 10$. These face values are associated with bank leverage of 87.5%. In addition, the time to maturity is one year and the risk free rate is $r = 1\%$ and $\alpha = 0.8$.

*each $\sigma' < \sigma_0$, there exists $\overline{\alpha} < 1$ such that the equilibrium level of risk shifting is larger when the initial risk level decreases from $\sigma_0$ to $\sigma'$, i.e., $\sigma_{\alpha}^{max}(\sigma_0) < \sigma_{\alpha}^{max}(\sigma')$, for each $\alpha > \overline{\alpha}$.*

*Proof.* The assumption that $V_{C,t} < V_{B_{Sub}}^{**}$ implies due to Proposition 5 that there is risk shifting, and that the payoff functions of both the stockholders and the subdebtholders are hump-shaped, and the risk that maximizes the value of the stock and the subdebt satisfy: $\sigma_0 < \sigma_{B_{Sub}}^{max} < \sigma_{S_B}^{max}$. Let $\overline{\sigma_0} > \sigma_0$ (resp., $\overline{\sigma'} > \sigma'$) be the risk level that induces the subdebtholders with the same payoff as the one induced by the initial risk level, i.e., $B_{sub,t}(\overline{\sigma_0}) = B_{sub,t}(\sigma_0)$ (resp., $B_{sub,t}\left(\overline{\sigma'}\right) = B_{sub,t}(\sigma')$). The inequality $B_{sub,t}(\sigma_{S_B}^{max}) < B_{sub,t}(\sigma_0)$ and the definition of the Nash bargaining solution imply that $\sigma_{\alpha}^{max}(\sigma_0) < \overline{\sigma_0} < \sigma_{S_B}^{max}$ for each bargaining power $\alpha$. Next, observe that $\sigma_{\alpha}^{max}(\sigma')$ converges to $min(\overline{\sigma'}, \sigma_{S_B}^{max}) > \overline{\sigma_0}$ as $\alpha$ converges to 1. This implies that there exists $\overline{\alpha} < 1$ such that $\sigma_{\alpha}^{max}(\sigma_0) < \overline{\sigma_0} < \sigma_{\alpha}^{max}(\sigma')$ for each $\alpha > \overline{\alpha}$. □

In the example in Section 4.4.3 we show that when $\sigma' = 10\%$, the equilibrium level of risk is $\sigma_{0.8}^{max}(10\%) = 40.6\%$. If the risk level dictated by the regulator when approached by the subdebtholders increased to $\sigma_0 = 15\%$, equilibrium risk would decrease to $\sigma_{0.8}^{max}(15\%) = 35.3\%$, as illustrated in Figure 4.

This result can also be seen in Figure 5 which presents the equilibrium level of risk when initial risk is 10% (in blue) versus when initial risk is 20% (in red) and for $\alpha = 0.8$ (dotted) and $\alpha = 0.2$ (dashed). In contrast to the cases of complete control of one of the claimholders, the initial level of asset risk has a reverse effect on the equilibrium level of risk.

Another interesting interpretation of this result arises if we interpret $\sigma_0$ in Equation 12 as the level of risk that is set by the regulator when it learns from the subdebtholders that the bank took part in risk shifting. A higher $\sigma_0$ is consistent with regulator forbearance while a lower value is consistent with a regulator that is intolerant of risk shifting. The extreme case where $\sigma_0 = 0\%$ is consistent with the regulator closing any bank that she finds has indulged in risk shifting. Under this interpretation, we find that more severe regulatory corrective measures may have the opposite effect then intended and actually increase equilibrium risk. This result is even more striking if we interpret $\sigma_0$ as the level of risk that the claim holders believe the regulator will set. Meaning if claim holders' believe that the regulator will be more intolerant of risk shifting equilibrium risk would increase when the bank is jointly controlled and side payments are not possible.

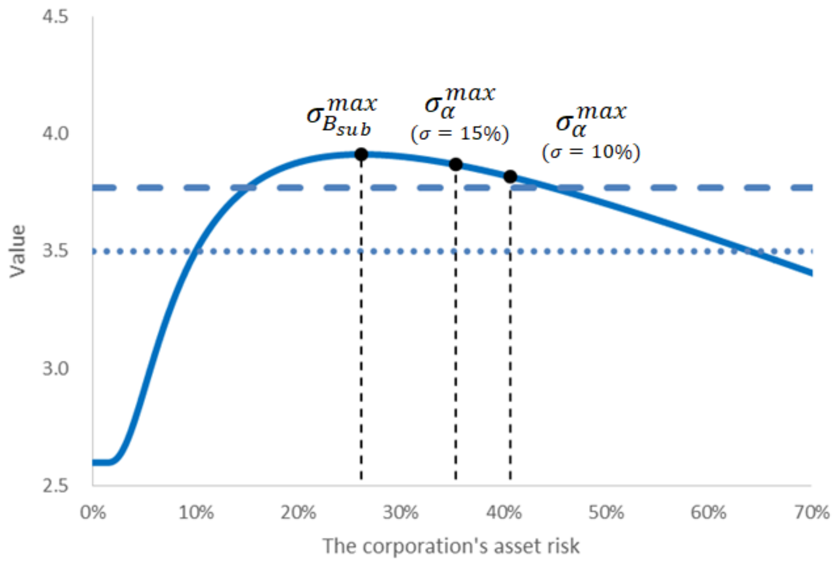



**Figure 4: The value of subdebt versus the bank's borrower asset risk for different initial level of asset risk** The face value of the bank's borrower debt is $F_C = 80$. The face value of the bank's deposits is $F_{Dep} = 60$ and of the subordinated debt is $F_{Sub} = 10$. In addition, the time to maturity is one year and the risk free rate is $r = 1\%$ and $\alpha = 0.8$. The dotted line represents the value of subdebt with asset risk of 10% and the dashed line with 15%.

## 4.5 Joint Control with Side Payments

Finally, we consider an extension where the stockholders and subdebtholders jointly control the bank, and stockholders relative bargaining power is $\alpha \in (0, 1)$, but now we introduce the option of side payments, payed by the stockholders to the subdebtholders or vice versa. Side payments allow the paying side to directly influence the receiving side's risk preferences by altering their payoff function.

Payments from stockholders to subdebtholders, can be implemented by increasing the interest rate of the subdebt claim above the rate which subdebtholders would request without side payments. Several papers expressed concerns regarding this possibility and suggested regulators cap the interest payments on subdebt (Furlong et al. [1987], Calomiris [1999] and Chen and Hasan [2011]). Payments from the subdebtholders to the stockholders can be implemented through deviations or threat of deviation from the absolute priority rule (Weiss [1990]).

Now, the two sides have to jointly decide on both (1) whether they allow the borrower to increase asset risk, and if so, to what level, and (2) a side payment of $X_\alpha$ dollars (which

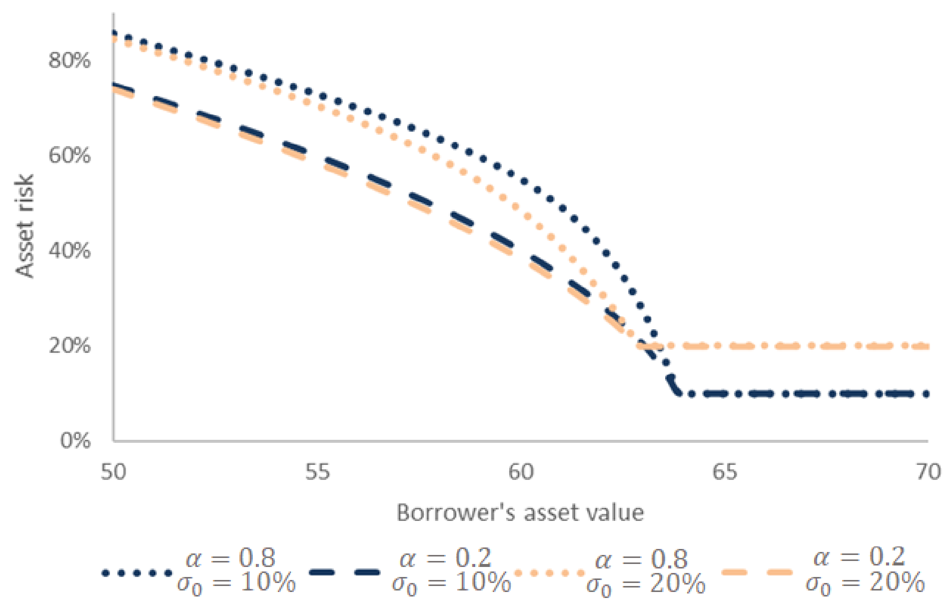



**Figure 5: The bank's borrower asset risk for different initial levels of asset risk and alphas.** The initial risk, $\sigma_0$, is 10% for the darker lines and 20% for the lighter lines. The face value of the bank's borrower debt is $F_C = 80$. The face value of the bank's deposits is $F_{Dep} = 60$ and of the subordinated debt is $F_{Sub} = 10$. In addition, the time to maturity is one year and the risk free rate is $r = 1\%$.

might be either positive or negative) that the stockholders pay the subdebtholders. As is standard in the bargaining literature, we find the solution to each one of these decisions separately in a two-step solution as follows: (1) the two sides jointly choose the risk level that maximizes the sum of their claims, and (2) the side payment is chosen as the unique Nash solution to the bargaining problem of dividing the joint value of the claims between the two parties. Therefore, an analyses of this case requires first to characterize how the total value of the subdebt and stock depend on the level of risk, which is done in Section 4.5.1, and than to apply the analysis in the solution of the bargaining situation with side payments which is done in Section 4.5.2.

### 4.5.1 Analysis of the sum of claims - Stock and subdebt

The joint payoff of stockholders and subdebtholders is the sum of payoffs from Eq. 3 and Eq. 4:

$$S_{B,t} + B_{Sub,t} = Call_t(V_{C,t}, F_{Dep}, \sigma, T-t, r) - Call_t(V_{C,t}, F_C, \sigma, T-t, r). \tag{13}$$

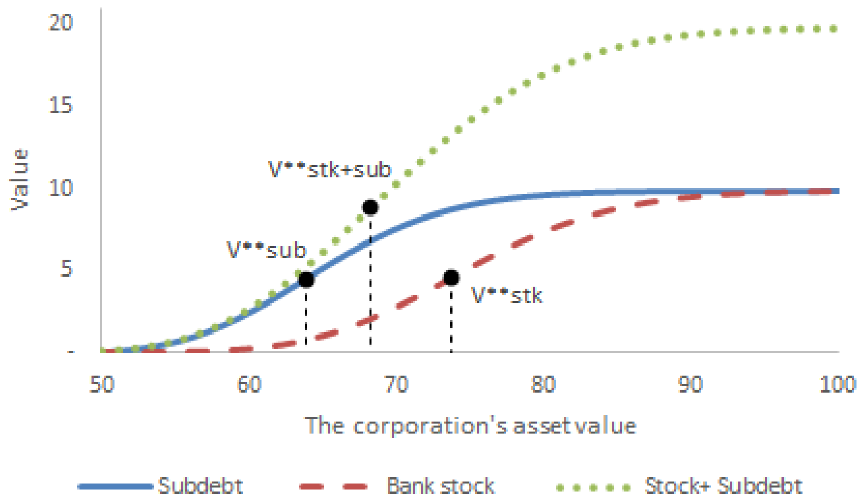



**Figure 6: The Payoff to the bank's stockholders, subdebtholders and the sum of both at debt maturity.** The face value of the borrower's debt is $F_C = 80$. The face value of the bank's deposits is $F_{Dep} = 60$ and of the subordinated debt is $F_{sub} = 10$. The initial level of the borrower's asset risk is $\sigma_0 = 10\%$. The risk free rate is $r = 1\%$ and time to maturity is one year.

Since the sum of payoffs is in itself a portfolio of two call options analogous arguments to those presented in Propositions $1-2$ show that the characterization of the sum of the value of the the two claims - the bank's stock and the subdebt, crucially depends on the threshold (see also analysis in Appendix B):

$$V^*_{S_B+B_{sub}} \equiv e^{-r(T-t)}\sqrt{F_{Dep} \cdot F_C}, \tag{14}$$

which is a function of the geometric mean of the face value of the deposits and of the borrower's loan. Observe that: (1) the threshold $V^*_{S_B+B_{sub}}$ does not depend on the size of the subdebt or on the relative bargaining power $\alpha$, and (2) this threshold is strictly between the thresholds of risk shifting for the subdebtholders and for the stockholders, i.e., $V^*_{B_{sub}} < V^*_{S_B+B_{sub}} < V^*_{S_B}$, as demonstrated in Figure 6.

In situations in which the borrower's asset value is above this threshold, the total value of stock and subdebt decreases with risk. By contrast, in setups in which the value of the borrower's assets is below the threshold, the relationship between sum of values and asset risk is hump-shaped, and the maximum value of stock and subdebt is achieved with the risk level of (due to analogous arguments to 1 – 2):

$$\sigma^{max}_{S_B+B_{sub}} \equiv \arg\max_{\sigma}(S_B + B_{sub})(\sigma) \equiv \sqrt{\frac{1}{T-t}\ln\left(\frac{F_C \cdot F_{Dep}}{(V_{C,t})^2}\right) - 2r} \tag{15}$$

It is immediate that $\sigma^{max}_{B_{sub}} < \sigma^{max}_{S_B+B_{sub}} < \sigma^{max}_{S_B}$ (due to $F_{Dep} < F_{Dep} + F_{sub} < F_C$), which implies that the level that maximizes the total value of the two claims is between the level that maximizes the value of the subdebt and the level that maximizes the stock value.

Finally, an analogous argument to 1 – 2 shows that $\sigma^{max}_{S_B+B_{sub}} > \sigma_0$ if and only if the bank's borrower asset value is below a second threshold defined as:

$$V^{**}_{S_B+B_{sub}} \equiv e^{-\left(r+\frac{\sigma_0^2}{2}\right)(T-t)}\sqrt{F_C \cdot F_{Dep}}. \tag{16}$$

Note that: (1) $V^{**}_{S_B+B_{sub}}$ is above $V^{*}_{S_B+B_{sub}}$ whenever assets are risky, i.e., $\sigma \neq 0$; (2) $V^{**}_{S_B+B_{sub}}$ is between the thresholds for risk shifting for the subdebt and for the stock, i.e., $V^{**}_{B_{Sub}} < V^{**}_{S_B+B_{sub}} < V^{**}_{S_B}$, and, (3) this threshold as well does not depend on the relative bargaining power $\alpha$.

The following result summarizes the above analysis regarding the value of the bank stock as a function the asset's risk.

**Proposition 7.** *The total value of the bank's stock and subdebt $S_{B,t}+B_{Sub,t}$ is: (1) decreasing in asset risk if $V_{C,t} > V^{*}_{S_B+B_{sub}}$, (2) hump-shaped (unimodal) in asset risk if $V_{C,t} > V^{*}_{S_B+B_{sub}}$, and, in this case, its maximum is obtained for risk level $\sigma^{max}_{S_B+B_{sub}}$. Moreover, the risk level that maximizes the sum of payoffs is larger then the initial risk (i.e., $\sigma^{max}_{S_B+B_{sub}} > \sigma_0$) if and only if $V_{C,t} < V^{**}_{S_B+B_{sub}}$.*

It is important to note, that the problem faced by the two claimholders in this case is the same as the one faced by stockholders in a bank with no subdebt. In both cases, the party in control can increase its payoff by extracting value only from depositors. This means that if side payments are possible, depositors and the deposit insurer may not benefit from regulation requiring banks to issue subdebt to replace stock, in fact their state will not change at all.

### 4.5.2 Solution of Bargaining with Side Payments

Proposition 7 implies the following characterization of risk shifting in the Nash bargaining solution in the case of joint control of the bank with side payments.

**Corollary 8.** *When the bank is jointly controlled by the subdebtholders and the stockholders (with relative bargaining power of $\alpha$ for the stockholders), and side payments are feasible, risk shifting occurs if and only if $V_{C,t} < V^{**}_{S_B+B_{sub}}$. When there is risk shifting, the level of risk shifting (is independent of $\alpha$) is $\sigma^{max}_{S_B+B_{sub}} \in (\sigma^{max}_{B_{sub}}, \sigma^{max}_{S_B})$.*

Observe that introduction of side payments increases the range of market values of the asset for which risk shifting occurs. Specifically, without side payments risk shifting occurs if and only if $V_{C,t} < V^{**}_{B_{Sub}}$, while with side payments risk shifting also occurs in the interval $V^{**}_{B_{Sub}} < V_{C,t} < V^{**}_{S_B+B_{sub}}$. A comparison of the different cases is presented in Table 1.

Next we characterize the amount paid as part of the bargaining solution.

**Proposition 9.** *When the bank is jointly controlled by the subdebtholders and the stockholders (with relative bargaining power of $\alpha$ for the stockholders), and side payments are feasible, then side payments are used if and only if there is risk shifting (i.e., if $V_{C,t} < V^{**}_{S_B+B_{sub}}$). In this case, the (possibly negative) amount $x_\alpha$ that the stockholders pay to the subdebtholders is:*

$$x_\alpha =$$
$$\arg\max_{x \in R} \left( \left( S_{B,t}\left(\sigma^{max}_{S_B+B_{sub}}\right) - S_{B,t}\left(\sigma_0\right) - x_\alpha \right)^{\alpha} \cdot \left( B_{Sub,t}\left(\sigma^{max}_{S_B+B_{sub}}\right) - B_{Sub,t}\left(\sigma_0\right) + x_\alpha \right)^{1-\alpha} \right).$$

Observe that the side payment $x_\alpha$ is decreasing in $\alpha$, i.e., the stockholders have to increase their side payment the lower is their bargaining power $\alpha$.

# 5 A Numerical Analysis: Changes in Capital Structure and in Regulatory Policy

The equilibrium level of risk for different control rights and different bargaining rules is analyzed in Section 4. However, the analyzes of each case is conduct separately. The aim of this section is to summarize and to compare the results of all cases by presenting a numerical analysis. The analysis is focused on the effect of changes in the bank's capital structure as well as changes of the regulator forbearance policy and their effects on the equilibrium level of risk and on the cost of default.

The analysis is conducted using our base case parameters, discussed in section 3.4. The face value of the corporation's loan is 80, the face value of subdebt is 10, and the face value of deposits is 60. These face values are associated with bank leverage of 87.5%. In addition, the time to maturity is one year and the risk free rate is 1%. We consider an initial level of asset volatility of 10%, similar to the risk of investment grade bonds (Huang and Huang [2012]).

When bank assets are risky debt claim, as in our framework of analysis, the highest asset value for which risk shifting occurs is the discounted geometric average of the face value of the borrower's debt and the total face value of the bank's debt, equal to 73.7. However, risk shifting at this point occurs only if the bank's stockholders possess the control rights, as presented in Table 2. In fact, when the stockholders control the bank, risk shifting is possible for any asset value below 73.7. For example, if the corporation's asset value is 70, risk shifting will occur only if stockholders control the bank, in which case equilibrium risk is 33.7% and the value of deposit insurance per dollar of insured deposits is 7.3%.

If the bank is jointly controlled and side payments are possible, risk shifting can occur for any asset value below the discounted geometric average of the face value of the borrower's debt and the face value of the bank's deposits, equal to 68.3. For example, if the corporation's asset value decreases to 65, risk shifting takes place either when stockholders are in control, in which case equilibrium risk is 51.2% corresponding to a cost if deposit insurance per dollar (DIPD) of 16.6% or if the bank is jointly controlled and side payments are possible, in which case subdebtholders are able to restrict the increase in equilibrium risk to 32.8% corresponding to DIPD of 9.3%. This case is important, since as shown in the previous section, a capital structure with subdebt where side payments are possible yields identical results to the case where subdebt is swapped with stock. Thus, if the subdebtholder cannot affect the level of risk as in the first case, a capital structure with stock only is superior. However, as can be seen from the table, side payments clearly decrease market monitoring, since if side payments were restricted, subdebtholders would not agree to an increase in risk at all. As discussed above, while the level of equilibrium risk is unaffected by stockholders' relative bargaining power, the payment they must make to subdebtholders to prevent them from approaching the regulator is decreasing with stockholders' relative bargaining power.

The last threshold which effect risk shifting is the discounted geometric average between

**Table 2: Numerical Analysis: Equilibrium level of risk and the cost of deposit insurance**

<table>
<tr><td rowspan="2">Control and bargening<br>framework</td><td colspan="4">Corporation's asset value ($V_C$)</td></tr>
<tr><td>$V_C > 73.7$</td><td>$V_C = 70$</td><td>$V_C = 65$</td><td>$V_C = 62$</td></tr>
<tr><td>Stockholders control</td><td rowspan="4">No risk<br>shifting</td><td>Risk shifting to 33.7%<br>DIPD=7.3%</td><td>Risk shifting to 51.2%<br>DIPD=16.6%</td><td>Risk shifting to 59.7%<br>DIPD=21.6%</td></tr>
<tr><td>Joint control with<br>side payments<br>$\alpha = 0.2/\alpha = 0.5/\alpha = 0.8$</td><td rowspan="3">No risk shifting</td><td>Risk shifting to 32.8%<br>Side payment: 1.62/1.28/0.95<br>DIPD=9.3%</td><td>Risk shifting to 45.0%<br>Side payment: 1.72/0.98/0.23<br>DIPD=15.9%</td></tr>
<tr><td>Joint control without<br>side payments<br>$\alpha = 0.2/\alpha = 0.5/\alpha = 0.8$</td><td rowspan="2">No risk shifting</td><td>Risk shifting to<br>28.5%/32.9%/40.6%<br>DIPD: 9.4%/11.2%/14.2%</td></tr>
<tr><td>Subdebtholders control</td><td>Risk shifting to 26.2%<br>DIPD=8.5%</td></tr>
</table>

The numerical analysis refers to the case where the face value of the bank's borrower debt is $F_C = 80$. The face value of the bank's deposits is $F_{Dep} = 60$ and of the subordinated debt is $F_{sub} = 10$. The initial level of risk is $\sigma_0 = 10\%$. The risk free rate is $r = 1\%$ and the time to maturity is one year. The table presents the equlibrium level of asset risk as well as the cost of deposit insurance, in percentage of the face value of deposits, for different levels of assets value and subdebtholder's bargaining power.

the face value of the deposits and the total face value of the bank's debt. Under this level risk shifting can occur if the subdebtholder has a complete control on the chosen level of asset risk or is there is bargaining between the bank's claimholders. For example, if the corporation's asset value decreases further to 62, risk shifting can take place under each of the cases of different control rights, but its degree increases with stockholders' relative bargaining power. Equilibrium risk is highest when stockholders have full control, 59.7%, accordingly DIPD receives its highest value of 8.5%. The lowest asset risk is observed when subdebtholders have full control, 26.2% (similarly DIPD is the lowest at 26.2%). If claimholders have joint control and side payments are not possible equilibrium risk is between 26.2% and 59.7% depending on the relative bargaining power. In the case that claimholders have joint control and side payments are possible, or if the bank had no subdebt and is funded by stock and deposits, the equilibrium level of risk is 45% and the cost of DIPS is 15.9%.

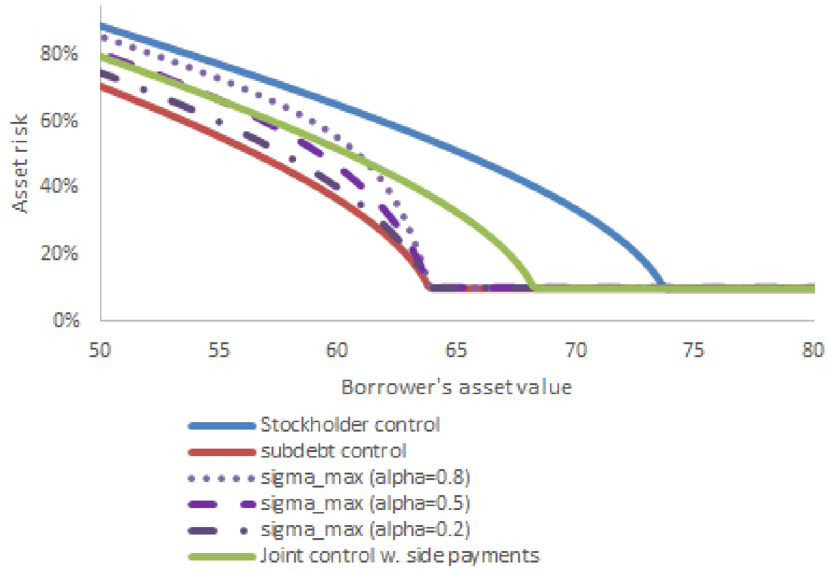
A similar picture emerges from Panel (a) of Figure 7, which presents the equilibrium level of borrower's asset risk. The equilibrium level of risk is at its highest level when the bank is controlled by the stockholders and lowest when controlled by the subdebtholders. Equilibrium risk is between these to levels when the bank is jointly controlled. Risk shifting occurs only for asset values for which subdebtholders would increase risk if they controlled the bank. The introduction of side payments may increase or decrease equilibrium risk, depending on the stockholders relative bargaining power. Panel (b) of Figure 7 presents a mirror image of this results: the cost of deposit insurance per dollar of insured deposits. As can be seen, higher equilibrium risk translates into higher cost of deposit insurance.

The effect of replacing stock with subdebt, depends on the relative power of the subdebtholders and the stockholders. If subdebtholders do not have the ability to affect the level of asset risk, the issuance of subdebt will increase the level of equilibrium risk due to the increased leverage. If subdebtholders have some bargaining power, the introduction of subdebt will decrease the equilibrium risk for asset values in the range between $V^{**}_{B_{Sub}}$ and $V^{**}_{S_B+B_{sub}}$, since with subdebt there will be no risk-shifting in this range at all. However the effect of the issuance of subdebt on equilibrium risk for asset values below $V^{**}_{B_{Sub}}$ depends on the relative power of the stockholders, which is captured in our model by the parameter $\alpha$. When stockholders' power is low enough, the introduction of subdebt will decrease asset risk in equilibrium for all asset values. In contrast, for relatively high stockholers' bargaining power, risk decreases following the issuance of subdebt only for asset values between a threshold $\tilde{V}$ and $V^{**}_{B_{Sub}}$ (defined in Equation 11), but will increase following the issuance of subdebt for asset values below $\tilde{V}$. The threshold $\tilde{V}$ itself is decreasing with the stockholders' bargaining power $\alpha$.

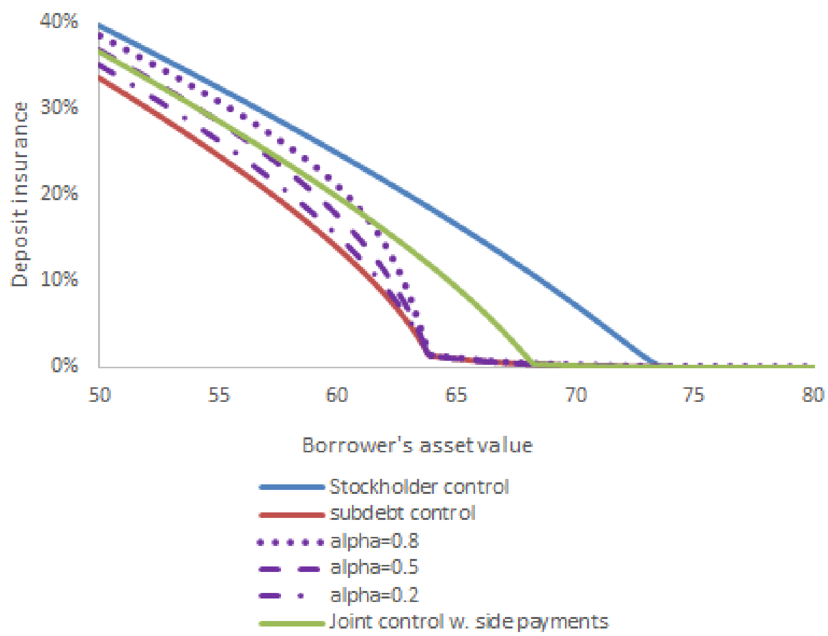
In our example, using the base case parameters, when $\alpha$ is smaller than 0.45, the introduction of subdebt, to replace stock, always leads to a decrease in the level of risk in equilibrium relative to a bank with no subdebt. In contrast, when $\alpha$ is above 0.45, equilibrium risk with subdebt will be higher than in a bank with no subdebt for low enough asset values. For example, when $\alpha = 0.8$, the introduction of subdebt would lead to lower equilibrium risk for asset values above $\tilde{V} = 61.1$, but to an increase in equilibrium risk for asset values below $\tilde{V} = 61.1$.

Figure 8 presents the effect of the amount of subdebt on the equilibrium level of asset risk. For any level of the subdetholders bargaining power, as the size of stock capital which is replaced with subdebt increases, and consequently stock capital decreases, the equilibrium level of risk increases. This is shown for the case where stockholders control the bank in Panel (a) and for the case where subdebtholds control the bank in Panel (b).

The equilibrium level of risk is not affected by a banks leverage ratio if side payments are possible and consequently issuing of subdebt has no effect on the equilibrium level of risk.

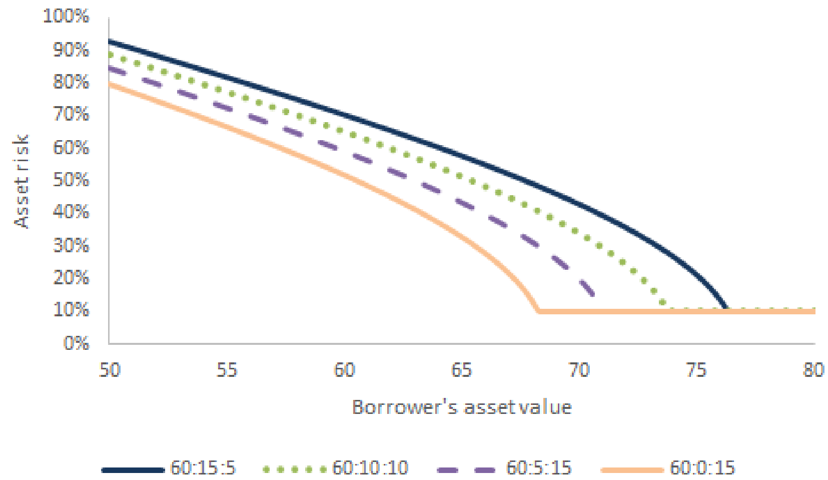



**(a)** Asset risk

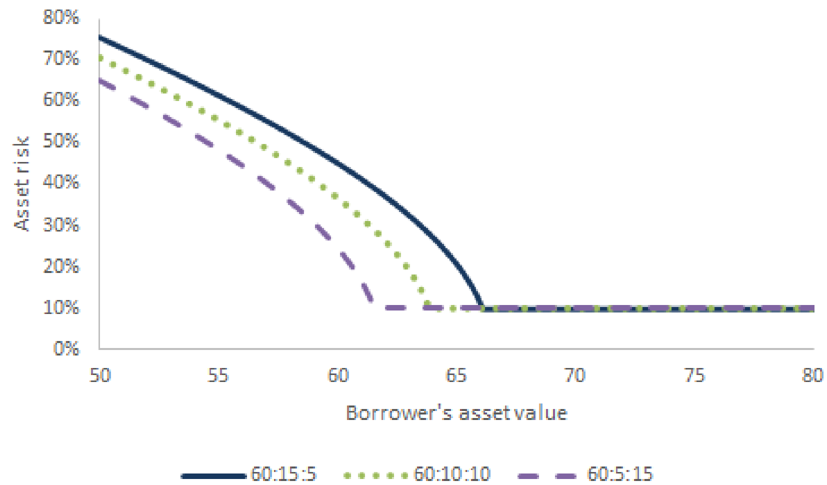



**(b)** Value of deposit insurance per dollar of insured deposits

**Figure 7: The equilibrium level of the bank's borrower asset risk and the value of insured deposits per dollar of insured deposits.** The figure depicts the equilibrium level of asset risk for all the cases discussed in Section 4. The face value of the borrower's debt is $F_C = 80$. The face value of the bank's deposits is $F_{Dep} = 60$ and of the subordinated debt is $F_{sub} = 10$. The initial level of risk is $\sigma_0 = 10\%$. In addition, the time to maturity is one year and the risk free rate is $r = 1\%$.




**(a)** Asset risk chosen by stockholders




**(b)** Asset risk chosen by subdebtholders

**Figure 8: The equilibrium level of the bank's borrower asset risk for different amounts of subdebt.** The top panel of the figure depicts the equilibrium level of asset risk chosen by the stockholders while the bottom panel depicts the equilibrium level of asset risk chosen by the subdebtholders. The different lines correspond to different face values of subdebt, $F_{sub}$, ranging from 15 to zero, i.e. a bank funded only by stock and deposits. The face value of the borrower's debt is $F_C = 80$ and the face value of the bank's deposits is $F_{Dep} = 60$. The initial level of risk is $\sigma_0 = 10\%$. In addition, the time to maturity is one year and the risk free rate is $r = 1\%$.

# 6 Conclusion

The accepted belief among policymakers and regulators before the 2007-2009 financial crisis, was that uninsured subordinated creditors can effectively reduce financial institutions' excessive risk taking. However, after the 2007-2009 financial crisis the effectiveness of subdebt as a monitoring tool was questioned. On the one hand, many financial institutions with subdebt as part of their capital structure were bailed out using taxpayers' money. On the other hand, the empirical literature found that subdebt can reduce bank risk taking. Motivated by these mixed conceptions and the revision to banking regulation (Basel III) that followed the crisis, we enhance the understanding of subdebt's effect on bank risk taking by analyzing the interaction between the bank's different claimholders. The analysis of equilibrium risk levels with different subdebtholders' bargaining power allows us to reconcile between different theories that explain the ineffectiveness of subdebt as disciplinary tool and also to connect between these theories and the empirical research on subdebt.

We find that when the stockholder fully controls the banks decision regarding the level of its borrowers asset risk, risk shifting takes place when the value of the bank's borrowers assets is below the discounted geometric average of the face value of debt of the borrower and of the total debt of the bank. Thus, risk shifting occurs only if the bank's borrower is in financial distress and in such case the equilibrium level of asset risk has an interior solution and is not unlimited as predicted by Gorton and Santomero [1990]. This case is in line with the claim that subdebtholders cannot affect bank risk taking as they are uninformed about the banks level of asset risk. In the other extreme case, when the subdebtholders control the banks level of risk, risk shifting occurs only when the borrowers asset value is blow the discounted geometric mean of the face value of the bank total debt and of its deposits. Thus, risk taking is limited to less states of the borrower's asset value and when it does occur, it is to lower levels of asset risk than when the stockholders control the level of risk. This case is aligned with the assumption that subdebtholders are informed about the banks level of asset risk, but risk shifting can still occur when they are better off with such a shift.

We further analyze the more realistic case, where the stockholders and subdebtholders jointly control the bank. In this case the equilibrium level of asset risk is between the levels in the case where the subdebtholders control asset risk and the case where the stockholders do. Still, the asset values for which risk shifting takes place are the same as when the subdebtholders determine the level of asset risk. We show that risk shifting decreases as the bargaining power of the subdebtholders increases. In addition, we show that replacing bank stock with subdebt leads to less asset values in which risk shifting can take place, but when the barraging power of the stockholders is relatively high, replacing stock with subdebt can lead to a higher level of risk in equilibrium. When side payments between the claim holders are possible, the range of states for which risk shifting occurs increases to the same level as when the stockholders control asset risk, but, the equilibrium level of asset risk can be below the one which is achieved in a process with bargaining and no side payments. We also show, that risk shifting behavior in this case is identical to what would be observed if subdebt was replaced by additional stock.

Following the 2007-2009 financial crisis, the issue of bank transparency was debated. The international regulatory framework (Basel III) calls to increase bank transparency by conducting stress tests involving an unprecedented amount of disclosure. On the one hand, transparency can prevent excessive risk shifting by banks. On the other hand, it is often argued that transparency has significant disadvantages in banking given banks' role in liquidity provision and risk sharing. We contribute to this debate by showing counterintuitively, that as the level of asset risk which is determined at regulatory audit event increases the equilibrium level of asset risk increases as well. As a result, a more restrictive corrective measure imposed by the regulator can motivate claim holders to agree on a higher level of risk in the bargaining process. Thus, the efficiency of the subdebt as a disciplinary tool declines with the enforcement of traditional regulatory tools such as on-site supervision.

## Appendix A: Notation

| **The bank's borrower (the corporation)** | |
|---|---|
| $S_C$ | Value of stock of the bank's borrower. |
| $F_C$ | Face value of debt of the bank's borrower. |
| $B_C$ | Value of the borrower's debt. |
| $T$ | Time in which all debt matures (the time till next audit). |
| $V_C$ | Value of the bank's borrower assets. |
| $\mu$ | Instantaneous expected return on the borrower's assets. |
| $\sigma$ | Instantaneous volatility of the borrower's assets - asset risk. |
| $dW$ | Standard Wiener process. |
| **The bank** | |
| $V_B$ | Value of bank's assets. |
| $S_B$ | Value of bank's stock. |
| $F_{Dep}$ | Face value of deposits. |
| $B_{Dep}$ | Value of deposits. |
| $F_{Sub}$ | Face value of subdebt. |
| $B_{Sub}$ | Market value of subordinated debt. |
| $DI$ | Value of deposit insurance. |
| $V^*_{S_B}$ | Threshold below which the bank's stockholders prefer risk higher than zero. |
| $V^{**}_{S_B}$ | Threshold below which the bank's stockholders prefer risk higher than the initial risk. |
| $\sigma^{max}_{S_B}$ | Bank stockholders' preferred asset risk when $V_C < V^*_{S_B}$. |
| $V^*_{B_{Sub}}$ | Threshold below which the subdebtholders prefer risk higher than zero. |
| $V^{**}_{B_{Sub}}$ | Threshold below which the subdebtholders prefer risk higher than the initial risk. |
| $\sigma^{max}_{B_{Sub}}$ | Subdebtholder's preferred asset risk when $V^*_{B_{Sub}} < V_{C,t}$. |
| $\alpha$ | Bargaining power of the bank stockholders relative to subdebtholders. |
| $\sigma^{max}_{\alpha}$ | Preferred asset risk when stockholders and subdebtholders have joint control. |
| $X_\alpha$ | Side payment. |
| $V^*_{S_B+B_{sub}}$ | Threshold below which the bank's claim holders prefer risk higher than zero. |
| $V^{**}_{S_B+B_{sub}}$ | Threshold below which the bank's claim holders prefer risk higher than the initial risk. |
| $\sigma^{max}_{S_B+B_{sub}}$ | Preferred risk with side payments. |

## Appendix B: Proofs

The payoff to both the bank's stockholders (Eq. 4) and subdebtholders (Eq. 3), as well as the sum of their payoffs (Eq. 13), are all equivalent to a portfolio of two call options on the value of the borrower's assets where each payoff is defined by options with different strike price. All these payoffs can be represented generally using the strike-prices $F_1$ and $F_2$ as:

$$P_t = Call_t(V_{C,t}, F_1, \sigma, T-t, r) - Call_t(V_{C,t}, F_2, \sigma, T-t, r).$$

To find the level of the borrower's asset risk that maximizes the value of any of these payoffs we calculate the derivative of the value of equity with respect to asset risk. Since it is well known that:

$$\frac{\partial call_t}{\partial \sigma} = \frac{\sqrt{T}}{\sqrt{2\pi}} \cdot V^C \cdot e^{-\frac{1}{2}\cdot(d_1(F_i))^2},$$

where $d_1 = \frac{1}{\sigma\sqrt{T-t}} \cdot \left[\ln\left(\frac{V_{C,t}}{F_i}\right) + \left(r + \frac{1}{2}\sigma^2\right)\cdot(T-t)\right]$, we get:

$$\frac{\partial P_t}{\partial \sigma} = \frac{\sqrt{T}}{\sqrt{2\pi}} \cdot V^C \cdot e^{-\frac{1}{2}\cdot(d_1(F_1))^2} - \frac{\sqrt{T}}{\sqrt{2\pi}} \cdot V^C \cdot e^{-\frac{1}{2}\cdot(d_1(F_2))^2}.$$

After rearranging it can be shown that the derivative is equal:

$$\frac{\partial P_t}{\partial \sigma} = \frac{\sqrt{T-t}}{\sqrt{2\pi}} \cdot V_{C,t} \cdot e^{-\frac{1}{2\cdot T\cdot\sigma^2}} \left[e^a - e^b\right]$$

where $a$ and $b$ are defined as:

$$a = -2\cdot\ln V_{C,t}\cdot\ln F_1 + (\ln(F_1))^2 - 2\cdot\ln(F_1)\cdot\left(r + \frac{\sigma^2}{2}\right)\cdot(T-t)$$

$$b = -2\cdot\ln V_{C,t}\cdot\ln F_2 + (\ln(F_2))^2 - 2\cdot\ln(F_2)\cdot\left(r + \frac{\sigma^2}{2}\right)\cdot(T-t).$$

The payoff is maximized with respect to asset risk in cases where the first derivative equals zero. This happens when either $V_{C,t} = 0$ or $a = b$. Since the first option is of no interest economically we focus on the second option. We find that $a = b$ when:

$$V_{C,t} = e^{-\left(r+\frac{1}{2}\sigma^2\right)\cdot(T-t)} \cdot \sqrt{F_1 \cdot F_2} \quad \text{(B.1)}$$

Based on Equation B.1, we define $V^{**}$ as the borrower's asset value for which the claim's value is maximized given a level of asset risk $\sigma$. This threshold is identical to the one found in Black and Cox [1976] and Gorton and Santomero [1990] for subdebtholders. We also note that the derivative changes its sign from positive to negative above the threshold, $V^{**}$, meaning that the bank's claimholder would like to increase risk below that level and to decrease it above that level of assets.

Alternatively, fix the level of assets, we find that $a = b$ when:

$$\sigma_{max} = \sqrt{\frac{1}{T-t} \ln\left(\frac{F_1 \cdot F_2}{(V_{C,t})^2}\right) - 2r}. \tag{B.2}$$

However, for both Equations B.1 and B.2 to hold - that is, for an internal solution to exist - the corporation's asset value must be below $V^*$ defined as: $V^* \equiv e^{-r\cdot(T-t)}\sqrt{F_1 \cdot F_2}$.

## References


Franklin Allen and Douglas Gale. Financial contagion. *Journal of political economy*, 108(1): 1–33, 2000.

Franklin Allen, Elena Carletti, Itay Goldstein, and Agnese Leonello. Moral hazard and government guarantees in the banking industry. *Journal of Financial Regulation*, 1(1): 30–50, 2015.

Deniz Anginer, Asli Demirguc-Kunt, and Min Zhu. How does deposit insurance affect bank risk? evidence from the recent crisis. *Journal of Banking & Finance*, 48:312–321, 2014.

Robert B Avery, Terrence M Belton, and Michael A Goldberg. Market discipline in regulating bank risk: New evidence from the capital markets. *Journal of Money, Credit and Banking*, 20(4):597–610, 1988.

Bhanu Balasubramnian and Ken B Cyree. Market discipline of banks: Why are yield spreads on bank-issued subordinated notes and debentures not sensitive to bank risks? *Journal of Banking & Finance*, 35(1):21–35, 2011.

Basel Committee on Banking Supervision. Framework, a new capital adequacy. *Bank for International Settlements, June*, 1999.

Basel Committee on Banking Supervision. Basel III: A global regulatory framework for more resilient banks and banking systems. *Basel Committee on Banking Supervision, Basel*, 2010.

Mohamed Belkhir. Do subordinated debt holders discipline bank risk-taking? evidence from risk management decisions. *Journal of Financial Stability*, 9(4):705–719, 2013.

Allen N Berger and Sally M Davies. The information content of bank examinations. *Journal of Financial Services Research*, 14(2):117–144, 1998.

Allen N Berger, Sally M Davies, and Mark J Flannery. Comparing market and supervisory assessments of bank performance: Who knows what when? *Journal of Money, Credit and Banking*, 32(3 Part II):27–29, 2000.

Fischer Black and John C Cox. Valuing corporate securities: Some effects of bond indenture provisions. *Journal of Finance*, 31(2):351–367, 1976.

Fischer Black and Myron Scholes. The pricing of options and corporate liabilities. *Journal of Political Economy*, 81(3):637–654, 1973.

Charles W Calomiris. Building an incentive-compatible safety net. *Journal of Banking and Finance*, 23(10):1499–1519, 1999.

Charles W Calomiris and Richard J Herring. How to design a contingent convertible debt requirement that helps solve our too-big-to-fail problem. *Journal of Applied Corporate Finance*, 25(2):39–62, 2013.

Gerard Caprio and Ross Levine. *Financial Sector Governance: The roles of the Public and Private sectors*, chapter Corporate governance in finance: Concepts and international observations, pages 17–50. Brookings Institution Press, Washington DC, 2002.

Yuk-Shee Chan, Stuart I Greenbaum, and Anjan V Thakor. Is fairly priced deposit insurance possible? *The Journal of Finance*, 47(1):227–245, 1992.

Yehning Chen and Iftekhar Hasan. Subordinated debt, market discipline, and bank risk. *Journal of Money, Credit and Banking*, 43(6):1043–1072, 2011.

Rebel A Cole and Jeffery W Gunther. Predicting bank failures: A comparison of on-and off-site monitoring systems. *Journal of Financial Services Research*, 13(2):103–117, 1998.

Michel Crouhy and Dan Galai. A contingent claim analysis of a regulated depository institution. *Journal of Banking & Finance*, 15(1):73–90, 1991.

Drew Dahl, Gerald A Hanweck, and John O'Keefe. The influence of auditors and examiners on accounting discretion in the banking industry. In *Academy of Financial Services Conference*, 1995.

Piotr Danisewicz, Danny McGowan, Enrico Onali, and Klaus Schaeck. Debt priority structure, market discipline, and bank conduct. *Review of Financial Studies*, 2017.

Jean-Paul Decamps, Jean-Charles Rochet, and Benoit Roger. The three pillars of Basel II: optimizing the mix. *Journal of Financial Intermediation*, 13(2):132–155, 2004.

Jean Dermine. Basel III leverage ratio requirement and the probability of bank runs. *Journal of Banking & Finance*, 53:266–277, 2015.

Jean Dermine and Fatma Lajeri. Credit risk and the deposit insurance premium: a note. *Journal of Economics and Business*, 53:497–508, 2001.

Mathias Dewatripont and Jean Tirole. Efficient governance structure: implications for banking regulation. *Capital markets and financial intermediation*, pages 12–35, 1993.

Robert DeYoung, Mark J Flannery, William W Lang, Sorin M Sorescu, et al. Could publication of bank camel ratings improve market discipline? In *Federal Reserve Bank of Chicago Proceedings*, number 600, 1998.

Robert DeYoung, Mark J Flannery, William W Lang, and Sorin M Sorescu. The information content of bank exam ratings and subordinated debt prices. *Journal of Money, Credit and Banking*, 33(4):900–925, 2001.

Douglas D Evanoff and Larry D Wall. Subordinated debt and bank capital reform. *FRB Chicago Working Paper No. 2000-07*, 2000.

Douglas D Evanoff and Larry D Wall. Sub-debt yield spreads as bank risk measures. *Journal of Financial Services Research*, 20(2-3):121–145, 2001.

Mark J Flannery. The faces of market discipline. *Journal of Financial Services Research*, 20 (2):107–119, 2001.

Mark J Flannery and Joel F Houston. The value of a government monitor for us banking firms. *Journal of Money, Credit and Banking*, pages 14–34, 1999.

Mark J Flannery and Sorin M Sorescu. Evidence of bank market discipline in subordinated debenture yields: 1983–1991. *The Journal of Finance*, 51(4):1347–1377, 1996.

Mark J Flannery, Simon H Kwan, and Mahendrarajah Nimalendran. The 2007–2009 financial crisis and bank opaqueness. *Journal of Financial Intermediation*, 22(1):55–84, 2013.

Xavier Freixas and Jean-Charles Rochet. Fair pricing of deposit insurance. is it possible? yes. is it desirable? no. *Research in Economics*, 52(3):217–232, 1998.

Frederick T Furlong, Michael C Keeley, et al. Subordinated debt as bank capital. *FRBSF Economic Letter*, 1987.

Prasanna Gai and Sujit Kapadia. Contagion in financial networks. In *Proceedings of the Royal Society of London A: Mathematical, Physical and Engineering Sciences*, volume 466, pages 2401–2423. The Royal Society, 2010.

Dan Galai and Ronald W Masulis. The option pricing model and the risk factor of stock. *Journal of Financial Economics*, 3(1):53–81, 1976.

Michael Gofman. Efficiency and stability of a financial architecture with too-interconnected-to-fail institutions. *Journal of Financial Economics*, 124(1):113–146, 2017.

Itay Goldstein and Yaron Leitner. Stress tests and information disclosure. *FRB of Philadelphia Working Paper*, 2015.

Itay Goldstein and Haresh Sapra. Should banks' stress test results be disclosed? an analysis of the costs and benefits. *Foundations and Trends in Finance*, 8(1):1–54, 2014.

William Gornall and Ilya A Strebulaev. Financing as a supply chain: The capital structure of banks and borrowers. *Working paper*, 2013.

Gary Gorton and Anthony M Santomero. Market discipline and bank subordinated debt: Note. *Journal of money, credit and Banking*, 22(1):119–128, 1990.

Diana Hancock and Myron L Kwast. Using subordinated debt to monitor bank holding companies: Is it feasible? *Journal of Financial Services Research*, 20(2-3):147–187, 2001.

Jing-Zhi Huang and Ming Huang. How much of the corporate-treasury yield spread is due to credit risk? *Review of Asset Pricing Studies*, 2(2):153–202, 2012.

M.C. Jensen and W.H Meckling. Theory of the firm: managerial behavior, agency costs and ownership structure. *Journal of Financial Economics*, 3:305–360, 1976.

Kose John, Hamid Mehran, and Yiming Qian. Outside monitoring and CEO compensation in the banking industry. *Journal of Corporate Finance*, 16(4):383–399, 2010.

Ehud Kalai. Nonsymmetric nash solutions and replications of 2-person bargaining. *International Journal of Game Theory*, 6(3):129–133, 1977.

CNV Krishnan, Peter H Ritchken, and James B Thomson. Monitoring and controlling bank risk: Does risky debt help? *The Journal of Finance*, 60(1):343–378, 2005.

Yaron Leitner. Financial networks: Contagion, commitment, and private sector bailouts. *The Journal of Finance*, 60(6):2925–2953, 2005.

Alan J Marcus and Israel Shaked. The valuation of FDIC deposit insurance using option-pricing estimates. *Journal of Money, Credit and Banking*, 16(4):446–460, 1984.

Robert C Merton. On the pricing of corporate debt: The risk structure of interest rates. *Journal of Finance*, 29(2):449–470, 1974.

Robert C Merton. An analytic derivation of the cost of deposit insurance and loan guarantees an application of modern option pricing theory. *Journal of Banking & Finance*, 1:3–11, 1977.

Donald P Morgan. Rating banks: Risk and uncertainty in an opaque industry. *FRB of New York Staff Report*, (105), 2000.

Donald P Morgan and Kevin J Stiroh. Market discipline of banks: The asset test. *Journal of Financial Services Research*, 20(2-3):195–208, 2001.

Stefan Nagel and Amiyatosh Purnanandam. Bank risk dynamics and distance to default. *Working paper*, 2015.

John F Nash. The bargaining problem. *Econometrica: Journal of the Econometric Society*, pages 155–162, 1950.

Tu Nguyen. The disciplinary effect of subordinated debt on bank risk taking. *Journal of empirical finance*, 23:117–141, 2013.

Martin J Osborne and Ariel Rubinstein. *Bargaining and markets*. Academic press, 1990.

Sharon Peleg Lazar and Alon Raviv. Bank risk dynamics where assets are risky debt claims. *European Financial Management*, 23(1):3–31, 2017.

Ehud I Ronn and Avinash K Verma. Pricing risk-adjusted deposit insurance: An option-based model. *Journal of Finance*, 41(4):871–896, 1986.

Andrea Sironi. Testing for market discipline in the european banking industry: evidence from subordinated debt issues. *Journal of Money, Credit, and Banking*, 35(3):443–472, 2003.

Rahul Vashishtha, Qi Chen, Itay Goldstein, and Zeqiong Huang. Bank transparency and deposit flows. *Workig paper*, 2018.

Lawrence A Weiss. Bankruptcy resolution: Direct costs and violation of priority of claims. *Journal of Financial Economics*, 27(2):285–314, 1990.