

In Defense of Market Efficiency: Evidence from Two Million Strategies

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Abstract

We develop a data mining approach to generate many trading strategy: some of which have already been studied and published, some have likely been studied but not published, and some that have never (and likely never will) be studied. Using the large cross-section of trading strategies we construct precise multiple testing hypothesis adjusted t-statistics that account for the cross-correlation in the data used to generate such strategies (i.e., accounting variables and returns data). After taking such adjustments into account, most strategies that have already been studied would not be statistically significant (even in-sample), and the ones that would be significant, and that have yet to be studied, could be dismissed based on economic considerations.

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1 Introduction

A large body of literature studies the profitability of trading strategies based on signals obtained from publicly available information. The list of strategies that display “anomalous” returns when compared to what is predicted by popular asset pricing models has grown since the first papers published on the topic (eg., Blume and Husic (1973) and Basu (1977)). Researchers are currently tracking a large number of strategies: for example, Harvey, Liu, and Zhu (2015, henceforth HLZ) document the existence of 316 factors; Green, Hand, and Zhang (2013) also study over 300 strategies.

In his presidential address, Harvey (2017) questions whether the performance of various strategies is real or apparent due to a number of possible problems with the way in which these strategies are discovered. For example, the manner in which trading strategies are evaluated does not align with the actual research process: many strategies are investigated, but only the ones that are significant have a viable path to publication. Lo and MacKinlay (1990) argue that data snooping plays a big role in finding significant strategies. HLZ (2015) propose that a multiple testing approach be used when evaluating the performance of trading strategies. Taking a different route, some studies evaluate the out-of-sample performance of strategies to gauge their persistence. McLean and Pontiff (2015) show that the performance of the different trading strategies declines after a research paper that claims the discovery is published in one of the main finance journals. Linnainmaa and Roberts (2016) consider the performance of the strategies in the period before and after the one that is studied in the paper that claims discovery and find that the out-of-sample performance is substantially weaker, pointing to data mining.

We propose a different approach. Instead of studying the out-sample performance of strategies, we infer their statistical properties by augmenting the asset space on which multiple hypothesis testing is performed. We compute the performance of a large number of trading strategies that encompasses the majority of ways in which public information from prices and balance sheets is currently used to construct trading signals. In particular, we consider the list of all accounting variables on Compustat and basic market variables on CRSP. We then construct strategies based on these variables and growth rates of these variables. In addition, we consider all possible permutations of three variables by computing ratios of the difference of first two variables to the third. In total, we construct over two million different strategies.

Our approach to generating trading signals has one important benefit: it provides a complete description of the profitability of the trading strategies that have been studied and published, as well as of those that have been studied but not published (likely because they

do not surpass traditionally accepted statistical hurdles), and of those that have yet to be studied (likely because their economic foundation is not immediately justifiable or simply because researchers have not thought about them). Precisely because we account for such a large number of possibilities, we can properly identify statistical hurdles that take into account cross-correlation among strategies and multiple hypothesis testing, without having to rely exclusively on strategies that have been published, as in HLZ (2015). Therefore, while we conduct a large scale data-mining exercise, our only purpose in doing so is to provide a reasonable set of strategies that allows the proper application of multiple hypothesis testing. Since we are not necessarily interested in promoting any particular strategy, the reader should think of the exercise not as a fishing expedition to find new strategy but as a data mining exercise as defined by Leamer (1978) (i.e., a thorough use of the data to properly evaluate an hypothesis). Moreover, since we do not have any a priori idea of which strategy will appear to be formidable, one should also not think of the exercise as to being a result of data snooping (i.e., which is what we are ultimately trying help control by applying multiple hypothesis testing).

We evaluate the performance of the two million strategies in two different ways. First, we compute their alphas using the Fama and French (2015) five factor model augmented with Carhart (1997) momentum factor. Since it is well-known that factor models are incomplete, as a second measure, we calculate Fama and MacBeth (1973, henceforth FM) coefficients for these signals following Brennan, Chordia, and Subrahmanyam (1998) approach.

In examining the different trading strategies we uncover several interesting facts. Despite applying stringent rules that affect the composition of our universe of stocks and signals (e.g., we eliminate stocks that are in the bottom quintile of the NYSE size distribution and that have prices below three dollars), we find some exceptionally large average returns, t -statistics and Sharpe ratios in the tails of the distribution. For example, the most profitable strategy has an average absolute return of 1.07% per month (with a t -statistic of 4.40) and an annualized Sharpe ratio of 0.8. Considering all the filters that we apply to the process, this would constitute an incredible investment opportunity. Most of the strategies in the tails are new and appear unrelated to existing anomalies (we control for the well-known anomalies in the factor models and regressions).

It is not too surprising that among a sample of over two million strategies, we uncover *some* strategies in the tails that appear exceptional. Interestingly, we find evidence of *many* strategies with extraordinary performance. For example, about 30% of t -statistics are significant at the five percent confidence level for both alphas and FM coefficients. The very large number of rejections, under the null of no outperformance, suggests that the classical significance levels do not adequately control for the fact that we are testing a large number

of trading strategies.

In order to better understand the cross-sectional distribution of these t -statistics, we follow Kosowski, Timmermann, Wermers, and White (2006), Fama and French (2010), and Yan and Zheng (2016) and a bootstrap method to obtain an empirical distribution under the null of no outperformance. The bootstrap allows us to preserve the cross-correlations as well as the autocorrelations in the return structure. We find that the bootstrapped distribution of the t -statistics under the null is close to normal, partly as a result of the fact that we have a large number of strategies. The average (across bootstraps) value of the 97.5th percentile of t -statistic under the null is close to 1.96. These facts imply that the cross-sectional distribution of actual t -statistics is fat-tailed in the sense that, again, we find that over 30% of strategies have t -statistics that exceed the empirical bootstrapped percentiles at five percent levels. These results are similar to those in Yan and Zheng (2016) who report a significant amount of market inefficiency using the bootstrap approach. Therefore, if one judges the performance of actual t -statistics based on statistical thresholds suggested by the bootstrap, one will uncover a lot of outperformance (because the bootstrap thresholds are roughly the same as the classical thresholds). While the bootstrap approach is useful for learning about the properties of the cross-sectional distribution of t -statistics, it is not designed to inform us about the relative proportion of the true rejections of the null versus the false positives.

In this paper, therefore, we rely on formal multiple hypothesis testing (MHT). MHT is designed to handle the problem that conventional statistical levels are not relevant for testing many hypotheses. For instance, while a significance level of 5% implies that Type I error (probability of false rejection) is 5% in testing one strategy, the rate of Type I error in *jointly* testing ten (independent) strategies is $1 - 0.95^{10} = 40\%$. The literature has proposed a variety of ways for controlling the Type I error in testing multiple hypotheses. We focus on the three most common approaches: family-wise error rate (FWER), false discovery ratio (FDR), and false discovery proportion (FDP). FWER controls for the probability of making even one false rejection, FDP controls for probability of a user-specified proportion of false rejections in a given sample, while FDR controls expected (across different samples) proportion of false rejections. Besides the conceptual distinction in what they are trying to control, these methods also differ in their underlying assumptions. For our purposes, the most important of these assumptions (that some of these methods impose) is that of independent, uncorrelated strategies. Since this assumption is unlikely to be satisfied in our setting, we put more weight on MHT methods that do not rely on the independence assumption but, rather, account for the cross-correlations that are present in the data. Relaxing the independent assumption ought to deliver more precise statistical critical values. We defer a more detailed discussion

of relative merits of these methods to later in the paper but focus on results from FDP methods for the rest of this introductory section. We note here that FDP methods deliver statistical cutoffs that *rely* on the cross-correlations present in the data by using a bootstrap method similar to the one used in Harvey and Liu (2016).

Imposing a tolerance of 5% of false discoveries (false discovery proportion) and a significance level of 5%, we find that critical value for factor-model alpha t -statistic (t_α) is 3.81 while that for FM coefficient t -statistic (t_λ) is 3.13. While these critical values are, obviously, quite a bit higher than the conventional levels, they are not far from the recent proposal by HLZ (2015) to use a critical value of three. Our higher threshold is due to our sample of two million strategies vis-à-vis slightly over 300 strategies in HLZ. At these thresholds, 2.65% of strategies have significant alphas and 10.70% have significant FM coefficients. The larger critical values for FM coefficient t -statistics than those for alphas are due to the fact that the cross-strategy distribution of the former has longer tails (i.e., the standard deviation of the distribution of t_λ is equal to 1.93, while the standard deviation of t_α is 1.82). Translated into numbers of strategies, we find that 55,874 strategies have t_α greater than 3.81 and 225,800 strategies have t_λ greater than 3.14. Thus, purely from a statistical point of view, even after imposing high cutoffs dictated by the big sample of strategies that are being evaluated, we still find a large number of outperforming strategies.

The main thrust of our paper is on the economic significance of these strategies. It is possible that some of these good strategies are just lucky. Recall, though, that our MHT procedures are designed to guard against luck in the discovery process and to prevent (many) false discoveries. Notably, statistical hurdles could be made even harder to pass if one were able to formally include in MHT some adjustment for data-snooping à la Lo and MacKinlay (1990). Nevertheless, despite using the state-of-the-art statistics, some false discoveries may still slip through the net. Equally importantly, we would like to consider strategies that display not only good statistical properties but are also economically meaningful and relevant.

Towards the end of gauging economic significance, we impose several additional hurdles on strategies that survive statistical thresholds. First, we would like to guarantee consistency between results obtained by studying portfolio returns and those derived from FM regressions. The reasons for this are three-fold and follow the advice of Fama and French (2010). First, as we mentioned earlier, portfolio alphas suffer from the joint hypothesis problem while FM regressions do not. Second, we only look at the alphas of the long-short portfolio effectively considering the efficacy of the strategy in only 20% of the sample while FM regressions consider the entire sample. Third, FM regressions impose linearity while portfolio returns are a non-parametric way of looking at the data. There are, thus, advantages and disadvantages to both portfolio sorts and regressions. We would like a trading signal to not only generate

a high long-short portfolio alpha but also to explain the broader cross-section of returns in a regression setting. Therefore, we reject strategies that have statistically significant t_α but insignificant t_λ or vice-versa. Imposing this filter drastically reduces the number of strategies to only 670, which is only 0.03% of the original number of total strategies.

Our final task is to take a step back and look at the economic magnitudes of these remaining strategies. Recall that these strategies already have a high t_α . This means that these strategies already have high risk-adjusted abnormal returns and imposing a filter on the magnitude of the alpha (which will be ad-hoc in any case) is not going to be useful. Instead, we opt for another metric that is often used in performance evaluation, that of Sharpe ratio. We impose the restriction that, to be considered a viable candidate, any strategy must have a Sharpe ratio at least as big as that of the market. The additional qualification finally decreases the sample to 16 strategies (out of 2.1 million) that are both statistically and economically significant.

A closer inspection of the signals (i.e., the variables that are used to construct the signals) that generate the 16 surviving strategies leaves us with some hesitation due to ostensible lack of any obvious possible rationalization. For example, one of the strategy that survives is the produced by sorting the ratios of the difference between Total Other Liabilities and the value of Property Sales to the Number of Common Shares. Thus, we conclude that, despite almost half a century after Fama (1970), the standard of market efficiency remains as strong as ever. Our exercise underscores another important problem related to the evaluation of trading strategies: statistical significance does not guarantee an economic plausible explanation.

Our paper joins the list of the growing literature that studies the proliferation of discoveries of abnormally profitable trading strategies and/or pricing factors and its relation to data-snooping biases in finance. See Leamer (1978), Lo and MacKinlay (1990), and MacKinlay (1995) for early work emphasizing statistical biases in hypothesis testing. Towards the turn of the century, more formal statistical approaches were developed and applied to the problem of evaluating multiple strategies (see, for example, Sullivan, Timmermann, and White (1999) and White (2000)). The question of whether the profitability of published strategies survives the test of time is studied in Schwert (2003), McLean and Pontiff (2015), and Linnainmaa and Roberts (2016). The multiple hypothesis testing has been applied to financial settings in Romano and Wolf (2005), Barras, Scaillet, and Wermers (2010), HLZ (2015), and emphasized in the presidential address of Harvey (2017). Our paper is also closely related to Yan and Zheng (2016). Both paper share the goal of evaluating a broader universe of strategies than just the published ones. Beyond inevitable differences in sample construction etc., our conclusions about market efficiency differ markedly from theirs for two main reasons. One is our use of formal statistical approaches to MHT rather than heuristic

based bootstrapped approach. Second is our insistence on economic efficiency.

2 Data and trading strategies

Monthly returns and prices are gathered from CRSP. Annual accounting data come from the merged CRSP/COMPUSTAT files. We collect all items included in the balance sheet, the income statement, the cash-flow statement and other miscellaneous items for the years 1972 and 2015. We chose 1972 as the beginning of our sample as it corresponds to the first year of trading at Nasdaq that dramatically increased the number of stocks in the CRSP dataset. All our results are robust to beginning the sample in 1963, which is the first date on which the COMPUSTAT data are not affected by backfilling bias. Following convention, we set a six-month lag between the end of the fiscal year and the availability of accounting information.

We impose several filters on the data. First, we include only common stocks with CRSP share codes of 10 or 11. Second, we require that data for each variable be available for at least 300 firms each month for at least 30 years during the sample period. Third, in FM (1973) regressions described later, we require that data be available for all independent variables (including the variable of interest) for at least 300 firms each month for at least 30 years during the sample period. Fourth, at portfolio formation at the end of June of each year (exact procedure described later), we require stocks to have a price higher than three dollars and market capitalization to be higher than the bottom twentieth percent of the NYSE capitalization distribution. The last filter ensures that we eliminate micro-cap stocks alleviating concerns about transaction costs (Novy-Marx and Velikov (2015)).

There are 156 variables that clear our filters and can be used as a trading signal. The list of these variables is provided in Appendix Table A1. We refer to these variables as *Levels*. We also construct *Growth rates* from one year to the next for these variables. Since it is common in the literature to construct ratios of different variables we also compute all possible combinations of ratios of two levels, denoted *Ratios of two*, and ratios of any two growth rates, denoted *Ratios of growth rates*. Finally, we also compute all possible combinations that can be expressed as a ratio between the difference of two variables to a third variable (i.e., $(x_1 - x_2)/x_3$). We refer to this last group as *Ratios of three*. We obtain a total of 2,090,365 possible signals.

2.1 Hedge portfolios

We follow standard procedures in forming portfolios. We sort firms into value-weighted deciles on June 30 of each year and rebalance these portfolios annually. The first portfolio formation is June 1973 and the last formation date is June 2015. We require a minimum of 30 stocks in each decile (300 stocks in total) in a month to consider that month as having a valid return. The signal is considered to have generated a valid portfolio if there are at least 360 months of valid returns. We consider long-short portfolios only. Thus, we compute a hedge portfolio return that is long in decile ten and short in decile one. Since we do not know ex-ante which of the two extreme portfolios has the largest average return, our hedge portfolios can have either positive or negative average returns. Obviously, it is always possible to obtain a positive average return for a hedge portfolio that has a negative average return by taking the opposite positions. For expositional convenience, we decide not to force average returns to be positive.

2.2 Strategy evaluation

Following common practice in the empirical asset pricing literature we evaluate trading strategies in two ways: by estimating abnormal performance of the hedge portfolios using a factor model, and by evaluating the ability of the signal in explaining the cross-section of firms' abnormal returns.

2.2.1 Factor models

Our benchmark evaluation factor model is composed of the five factors in Fama and French (2015) plus the momentum factor. The five factors include the return of market value weighted portfolio in excess of the risk-free rate, the large minus small portfolio, the high minus low book to market portfolio, the robust minus weak profitability factor, the conservative minus aggressive investment factor. For each trading strategy originated by one of the signals, we run a time-series regression of the corresponding hedge portfolio returns on the six factors and obtain the α as well as its heteroskedasticity-adjusted t -statistic (t_α).

2.2.2 Fama-MacBeth regressions

It is well-known that factor models provide an incomplete characterization of risk-adjusted returns as portfolio alphas suffer from the joint hypothesis problem. Moreover, the alphas of the long-short portfolio effectively consider the efficacy of the strategy in only 20% of the sample. Therefore, we also evaluate a signal's ability to predict returns in the cross-section

of stocks using FM (1973) regressions. In particular, we evaluate the ability of the signal to explain stock returns by estimating the following cross-sectional regression each month:

$$R_{it} - \widehat{\beta}_i F_t = \lambda_{0t} + \lambda_{1t} X_{it-1} + \lambda_{2t} Z_{it-1} + e_{it}, \quad (1)$$

where X is the variable that represents the signal and Z 's are control variables. We use the most commonly used control variables, namely size (i.e., the natural logarithm of the firm's market capitalization), natural logarithm of the book-to-market ratio, past one-month and 11-month return (skipping the most recent month), asset growth, and profitability ratio. Book-to-market is calculated following Fama and French (1992) while asset growth and profitability are calculated following Fama and French (2015).

We risk-adjust the returns on the left-hand-side of equation (1) following Brennan, Chordia, and Subrahmanyam (1998). We use the same six-factor model used to calculate hedge portfolio alphas, and calculate full-sample betas $\widehat{\beta}$ for each stock. We require at least 60 months of valid returns to estimate the time-series regression. After obtaining the betas, we subtract from each month's stock return the factor model return (i.e., $\widehat{\beta}_i F_t$) for that month. In estimating the λ_t coefficients of the cross-sectional regressions, we require a minimum of 300 stocks in a month. Finally, we require a minimum of 360 valid monthly cross-sectional estimates during the sample period to calculate a valid λ_1 coefficient for a signal. Thus, we calculate the FM coefficient λ_1 as well its heteroskedasticity-adjusted t -statistic (t_λ). Given that we require a valid beta for each stock and data on additional control variables, the data requirements for regressions are slightly more stringent than those for portfolio formation.

3 Strategy performance

In this section we discuss the statistical properties of the trading strategy returns. We analyze raw returns and Sharpe ratios in Section 3.1, and abnormal returns and regression coefficients in Section 3.2.

3.1 Raw returns and Sharpe ratios

Table 1 reports summary statistics of raw returns on the hedge portfolios. We report cross-sectional means, medians, standard deviation, minimum, and maximum across portfolios. These statistics are reported for the sample of all portfolios as well as the sub-sample of portfolios formed by the different trading signals (i.e., ratio of two, ratio of three, etc.). We report monthly average returns in Panel A, t -statistics for returns in Panel B, and monthly

Sharpe ratios in Panel C. Each panel also reports the number and percentage of portfolios that cross specific thresholds.

Looking at Panel A, we see that the cross-sectional mean and median average return of the portfolios are close to zero. The cross-sectional standard deviation of returns at 0.18% coupled with the fact that we have over two million portfolios implies that there are many portfolios with very large absolute returns. For example, there are 17,192 portfolios with absolute average monthly return greater than 0.5%. Panel B shows that a large number of portfolios also have average returns that exceed conventional statistical significance levels. 22,237 spread portfolios have average return t -statistics larger than 2.57 (in absolute value); albeit this represents only about 1% of the total number of portfolios. The economic importance of these portfolios is also very impressive: many portfolios have monthly Sharpe ratios larger than the historical market Sharpe ratio (approximately 0.116), with only one with a Sharpe ratio above 0.232. These facts, while not perhaps surprising, are, nevertheless, interesting because they are obtained despite the stringent rules that affect the composition of our universe of stocks and signals (e.g., we eliminate stocks that are in the bottom quintile of the NYSE size distribution and that have prices below three dollars).

As is to be expected, the dispersion in the performance of strategies is largest in the subset of strategies *Ratios of three*. The most profitable and statistically significant returns come from this group. The largest absolute average return is 1.07 per cent per month, and the largest absolute t -statistic is 5.41.

In order to examine the tails of the distribution, in Tables A2, A3, and A4 we list the top 50 strategies by average returns, return t -statistic, and Sharpe ratio, respectively. Most of the strategies in the tails are new and appear unrelated to existing anomalies (as it should be, since we control for the well-known anomalies in the factor models and regressions). For example, the most profitable strategy in terms of raw returns is the ratio of the difference between Capital surplus-share premium reserve (CAPS) and Cash and cash equivalent increase/decrease (CHECH) to advertising expense (XAD). This strategy has an average return of -1.07 per cent per month with a t -statistic of -4.40 .

3.2 Abnormal returns and Fama-MacBeth regression coefficients

We next compute abnormal returns for our strategies using the Fama and French (2015) five-factor model augmented with the momentum factor. We report summary statistics in Table 2.

The distribution of alphas in Panel A of Table 2 reveals even more exceptional performance of strategies than that in raw returns of Panel A of Table 1. There are 222,566 monthly

alphas larger than 0.5% (in absolute value). Panel B shows that the cross-sectional distribution of alphas' t -statistics has mean and median close to zero but a standard deviation of 1.82 resulting in a large number of t -statistics in the tails. For example, about 30% of the absolute t -statistics are significant at the five percent confidence level and a staggering 17% are significant at the one percent confidence level. As is the case for average returns, most of the extreme alphas come from the subset of *Ratios of three* strategies.

Panel C of Table 2 reports descriptive statistics on Fama-MacBeth (1973) coefficients. Once again, we find that almost 30% of the absolute t -statistics are larger than 1.96 and almost 19% are larger than 2.57.

Figure 1 provides a visual representation of the distribution of these t -statistics.

It is not too surprising that among a sample of over two million strategies, we uncover *some* strategies in the tails that appear exceptional. However, the fact that we find *many* exceptional strategies with very large t -statistics presents prima facie evidence against direct application of classical conventional significance levels, thus motivating our use of the multiple hypothesis testing, which we present in Section 5.

As a preliminary investigative step to formal multiple hypothesis testing we present, in the next section, a description of the empirical distribution obtained by bootstrapping the data under the null hypothesis (i.e., of zero alpha and of zero FM coefficient). The exercise serves not only to present the small-sample properties of the data, but also introduces the bootstrap method that we use when applying MHT.

4 Bootstrap

Kosowski, Timmermann, Wermers, and White (2006) and Fama and French (2010) propose a bootstrap technique to assess skill in mutual fund returns. The idea is to identify the mutual funds that outperform the benchmarks in the cross-section of mutual fund returns. The approach relies on bootstrapping the cross-section of fund returns through time, therefore, preserves the cross-sectional dependence structure in fund returns and ultimately their alpha estimates. More recently, Yan and Zheng (2016) use this approach to analyze multiple trading strategies in an exercise similar to ours.

We follow Fama and French (2010) and construct bootstrap distributions of the alphas and their t -statistics under the null hypothesis that the alphas are zero. We subtract the six-factor alpha from the time-series of portfolio returns. We then bootstrap 1,000 times the returns and compute six-factor alphas and their t -statistics for each of the bootstrap samples for each hedge portfolio. Each bootstrap run is a random sample (with replacement) of the alpha-adjusted returns and the factors over 522 months of the sample period 1972 to 2015. To

preserve the cross-sectional correlation we apply the same bootstrap draws to each portfolio and factor returns. To preserve possible autocorrelation in the return structure, we construct the stationary bootstrap of Politis and Romano (1994) by drawing random blocks with an average length of six months. Due to the computational constraints imposed by the large scale of our exercise we limit the exercise to 1,000 bootstrap samples, as opposed to the 10,000 runs implemented by Fama and French (2010).

For each bootstrap run we obtain the spread portfolio alphas and their t -statistics under the null of zero alpha. Following Fama and French (2010) we then compare the percentiles of the t -statistics from the actual data sample to the corresponding percentiles in the bootstrap samples (i.e., the collection of x -th percentile from each bootstrap run). We focus on t -statistics rather than on the coefficients themselves because t -statistics control for the precision of coefficients and are advocated by, for example, Romano, Shaikh, and Wolf (2008).

Table 3 documents selected percentiles of the t -statistics from the actual distribution (Data) and the average (across bootstraps) t -statistic for that percentile (Boot). Following Yan and Zheng (2016), we report percentage (from the entire set of trading strategies) of actual t -statistics that are bigger than the average bootstrapped t -statistic (% Data). Finally, following Fama and French (2010), we also report the fraction of iterations where the bootstrapped percentile was bigger than the actual percentile (% Boot).

Consider the 99th percentile. The actual alpha t -statistic (t_α) from the data is 4.03 while the average (across iterations) bootstrap t_α under the null is 2.35. There are 10.35% actual t_α 's that are bigger than the cutoff of 2.35. At the same time in the 99th percentile bootstrap distribution (i.e., the collection of 99th percentiles from each bootstrap run) we do not find any t_α larger than 4.03. Similar observations apply to other percentiles implying that, relative to bootstrap distribution under the null of zero alpha, the extreme of the distributions of t_α in the data are atypical.

We conduct a similar experiment for Fama-MacBeth coefficients. In particular, for each signal variable we start by subtracting the average from the time-series of λ_{1t} coefficients from equation (1), thus obtaining a time-series of adjusted coefficients under the null of no explanatory power. We then bootstrap 1,000 times the time-series of pseudo coefficients and calculate the means and t -statistics for each bootstrap iterations. Finally, for each percentile of interest we collect the the corresponding quantity from each bootstrap cross-sectional distribution of Fama-MacBeth coefficients. We then compare the t_λ based on the data to the corresponding bootstrap quantities in the same way as we do for the t_α . We report the comparisons in the right panel of Table 3. We find very similar patterns than those observed for alphas. Let's consider for example the 95th percentile of the actual t_λ 's, which is equal to 3.29. The distribution of the corresponding bootstrap percentiles has an average of 1.95.

No element of such distribution is larger than 3.29 (last column in the table), while 12.77% of the t_λ is the data is larger than its mean (i.e., 1.95). Therefore the very large t_λ observed in the data appear atypical when compared to their bootstrap distributions.

While the type of analysis that we report in Table 3 is informative of the general properties of the empirical distribution of t -statistics, it presents some important limitations when used as a basis to conduct formal inference. In fact, the cross-section of alphas does provide some information about luck versus skill (i.e., true versus false null hypotheses), but it does not inform us about the relative proportion of true versus false rejections of the null. As illustrated by Barras, Scaillet, and Wermers (2010), this is particularly true of the tails of the distribution. To take an example, assuming normally distributed test statistics, if one observes that 10% of t -statistics are above the threshold 2.57 for significance level of 1% in a two tailed test, then the econometrician can infer that there are some strategies that do beat the benchmark. However, she will not be able to infer how many of these strategies represents a true discovery (i.e., for which the null should be rejected) without knowing the proportion of strategies that have truly no alpha but were lucky in generating abnormal performance in the sample (i.e., false positives). In other words, comparing the data to the bootstrap is a useful first diagnostic but one needs a formal multiple hypothesis testing approach to the problem of assessing the proportion of outperforming strategies.

5 Multiple hypotheses testing

Classical single hypothesis testing uses a significance level α to control Type I error (discovery of false positives). In multiple hypothesis testing (MHT), using α to test each individual hypothesis does not control the overall probability of false positives.¹ For instance, if test statistics are normally distributed and we set the significance level at 5%, then the rate of Type I error (i.e., the probability of making at least one false discovery) is $1 - 0.95^{10} = 40\%$ in testing ten hypotheses and over 99% in testing 100 hypotheses. There are three broad approaches in the statistics literature to deal with this problem: family-wise error rate (FWER), false discovery rate (FDR), and false discovery proportion (FDP). In this section, we describe these approaches and provide details on their implementation.

We are interested in testing the performance of trading strategies by analyzing the abnormal returns generated by M signals. The test statistic is the t -statistic on either the factor-model alphas of these strategies (equivalently the p -value of these alphas) or on the Fama and MacBeth (1972) coefficient estimate of the variable used as the signal to con-

¹The use of symbol α to denote both the significance level as well as the abnormal returns from a factor model is standard. We hope that this does not cause any confusion and the usage is clear from the context.

struct the trading strategy. The null hypothesis corresponding to each strategy is labeled as H_m . For ease of notation, we will relabel the strategies and order them from the best (highest t -statistic) to the worst (lowest t -statistic). In other words, it is assumed that $t_1 \geq t_2 \geq \dots \geq t_M$, or equivalently the p -values $p_1 \leq p_2 \leq \dots \leq p_M$. Some of the methods used in this section use a bootstrap procedure which is the same as that described in the previous section.

5.1 FWER

The strictest idea in MHT is to try to avoid any false rejections. This translates to controlling the FWER, which is defined as the probability of rejecting even one of the true null hypotheses:

$$\text{FWER} = \text{Prob}\{\text{Reject even one true null hypothesis}\}.$$

Thus, FWER measures the probability of even one false discovery, i.e., rejecting even one true null hypothesis (type I error). A testing method is said to control the FWER at a significance level α if $\text{FWER} \leq \alpha$. There are many approaches to controlling FWER.

5.1.1 Bonferroni method

The Bonferroni method, at level α , rejects H_m if $p_m \leq \alpha/M$. The Bonferroni method is a single-step procedure because all p -values are compared to a single critical value. This critical p -value is equal to α/M . For a very large number of strategies, this leads to an extremely small (large) critical p -value (t -statistic). While widely used for its simplicity, the biggest disadvantage of the Bonferroni method is that it is very conservative leading to a loss of power. One of the main reasons for the lack of power is that the Bonferroni method implicitly treats all test statistics as independent and, consequently, ignores the cross-correlations that are bound to be present in most financial applications.

5.1.2 Holm method

This is a stepwise method based on Holm (1979) and works as follows. The null hypothesis H_m is rejected at level α if $p_i \leq \alpha/(M - i + 1)$ for $i = 1, \dots, m$. In comparison with the Bonferroni method, the criterion for the smallest p -value is equally strict at α/M but it becomes less and less strict for larger p -values. Thus, the Holm method will typically reject more hypotheses and is more powerful than the Bonferroni method. However, because it also does not take into account the dependence structure of the individual p -values, the Holm method is also very conservative.

5.1.3 Bootstrap reality check

Bootstrap reality check (BRC) is based on White (2000). The idea is to estimate the sampling distribution of the largest test statistic taking into account the dependence structure of the individual test statistics, thereby asymptotically controlling FWER.

The implementation of the method proceeds as follows. Bootstrap the data using procedure described later in this section. For each bootstrapped iteration b , calculate the highest (absolute) t -statistic across all strategies and call it $t_{\max}^{(b)}$, where the superscript b is used to clarify that these t -statistics come from the bootstrap. The critical value is computed as the $(1 - \alpha)$ empirical percentile of B bootstrap iterations values $t_{\max}^{(1)}, t_{\max}^{(2)}, \dots, t_{\max}^{(B)}$.

Statistically speaking BRC can be viewed as a method that improves upon Bonferroni by using the bootstrap to get a less conservative critical value. From an economic point of view, RBC addresses the question of whether the strategy that appears the best in the observed data really beats the benchmark. However, it does not attempt to identify as many outperforming strategies as possible.

5.1.4 StepM method

This method, based on Romano and Wolf (2005) addresses the problem of detecting as many out-performing strategies as possible. The stepwise StepM method is an improvement over the single-step BRC method in very much the same way as the stepwise Holm method improves upon the single-step Bonferroni method. The implementation of this procedure proceeds as follows:

1. Consider the set of all M strategies. For each cross-sectional bootstrap iteration, compute the maximum t -statistic, thus obtaining the set $t_{\max}^{(1)}, t_{\max}^{(2)}, \dots, t_{\max}^{(B)}$. Then compute the critical value c_1 as the $(1 - \alpha)$ empirical percentile of the set of maximal t -statistics, as in BRC method. Apply now the c_1 threshold to the set of original t -statistics and determine the number of strategies for which the null can be rejected. Say that there are M_1 strategies, for which $t_m \geq c_1$. We have now $M - M_1$ strategies remaining with t -statistics ordered as $t_{M_1+1}, t_{M_1+2}, \dots, t_M$.
2. Consider the set of remaining $M - M_1$ strategies. For each bootstrapped iteration b , calculate the highest (absolute) t -statistic across all remaining strategies. To avoid cluttering up the notation, we will use the same symbols as before and call the maximal t -statistics of the b bootstrap iteration across the $M - M_1$ remaining strategies as $t_{\max}^{(b)}$. The critical value c_2 is computed as the $(1 - \alpha)$ empirical percentile of B bootstrap iterations values $t_{\max}^{(1)}, t_{\max}^{(2)}, \dots, t_{\max}^{(B)}$. Say that there are M_2 strategies, for which $t_m \geq$

c_2 , and are, therefore, rejected in this step. After this step, $M - M_1 - M_2$ strategies remain with t -statistics ordered as $t_{M_2+1}, t_{M_2+2}, \dots, t_M$.

3. Repeat the procedure until there are no further strategies that are rejected. The StepM critical value for the entire procedure is equal to the critical value of the last step and the number of strategies that are rejected is equal to the sum of the number of strategies that are rejected in each step.

Like the Holm method, the StepM method is a stepdown method that starts by examining the most significant strategies. The main advantage of the method is that, because it relies on bootstrap, it is valid under arbitrary correlation structure of the test statistics. As mentioned before, this method will detect many more out-performing strategies than the Bonferroni method or the BRC approach.

It is easy to see that the BRC approach amounts to only step one of the above procedure, namely computing only the critical value c_1 . By continuing the method after the first step, more false hypothesis can be rejected. Moreover, since typically $c_1 > c_2 > \dots$, the critical value in StepM method is less conservative than that in BRC approach. Nevertheless, the StepM procedure still asymptotically controls FWER at significance level α .

However, the StepM method still suffers from low power, as noted by Bajgrowicz and Scaillet (2012), as it is very susceptible to determining a threshold that rejects lucky strategies even if there remain an important number of outperforming strategies in the population.

5.2 k -FWER

By relaxing the strict FWER criterion, one can reject more false hypotheses. For instance, k -FWER is defined as the probability of rejecting at least k of the true null hypotheses:

$$k\text{-FWER} = \text{Prob}\{\text{Reject at least } k \text{ of the true null hypothesis}\}.$$

A testing method is said to control for k -FWER at a significance level α if $k\text{-FWER} \leq \alpha$. Testing methods such as Bonferroni and Holm, discussed earlier, can be generalized for k -FWER testing. Please refer to Romano, Shaikh, and Wolf (2008) for further details. Here we discuss only the extension of the StepM method which is known as the k -StepM method.

5.2.1 k -StepM method

The implementation of this procedure proceeds as follows:

1. Consider the set of all M strategies. For each bootstrapped iteration b , calculate the k -highest (absolute) t -statistic across all strategies and call it $t_{k\text{-max}}^{(b)}$, where the

superscript b is used to clarify that these t -statistics come from the bootstrap. Compute the critical value c_1 as the $(1 - \alpha)$ empirical percentile of B bootstrap iterations values $t_{k\text{-max}}^{(1)}, t_{k\text{-max}}^{(2)}, \dots, t_{k\text{-max}}^{(B)}$. Say that there are M_1 strategies, for which $t_m \geq c_1$, and are, therefore, rejected in this step. After this step, $M - M_1$ strategies remain with t -statistics ordered as $t_{M_1+1}, t_{M_1+2}, \dots, t_M$. Apart from the use of k -max instead of max, this step is identical to the first step of StepM procedure.

2. Consider the set of remaining $M - M_1$ strategies. Call this set **Remain**. Also consider a number $k - 1$ of strategies from the set of already rejected strategies. Call this set **Reject**. Now consider the union of these two sets, **Consider** = **Remain** \cup **Reject**. For each bootstrapped iteration b , calculate the k -highest (absolute) t -statistic across all strategies in the set **Consider** and call it $t_{k\text{-max}}^{(b)}$. Compute the $(1 - \alpha)$ empirical percentile of B bootstrap iterations values $t_{k\text{-max}}^{(1)}, t_{k\text{-max}}^{(2)}, \dots, t_{k\text{-max}}^{(B)}$. This empirical percentile will depend on which $k - 1$ strategies were included in the set **Reject**. Given that there are $\binom{M_1}{k-1}$ possible ways of choosing $k - 1$ strategies from a set of M_1 strategies, the critical value c_2 is computed as the maximum across all these permutations. Say that there are M_2 strategies, for which $t_m \geq c_2$, and are, therefore, rejected in this step. After this step, $M - M_1 - M_2$ strategies remain with t -statistics ordered as $t_{M_2+1}, t_{M_2+2}, \dots, t_M$.
3. Repeat the procedure until there are no further strategies that are rejected. The critical value of the procedure is equal to the critical value of the last step and the number of strategies that are rejected is equal to the sum of the number of strategies that are rejected in each step.

The key innovation in the k -StepM procedure is in the inclusion of (some of the) rejected strategies while calculating subsequent critical values (c_2 and thereafter). The intuition is as follows. Remember that ideally we want to calculate the empirical critical value from the set of strategies that are true under the null hypothesis. This set is unknown in practice. However, we can use the results of the first step to arrive at this set. The set **Remain** of remaining strategies that have not (yet) been rejected is an obvious candidate for strategies that are true under the null. As the k -StepM, it stands to reason that the first step of the procedure is not able to control k -FWER. In other words, less than k true null hypotheses have been rejected in the first step. Let's say that number is in fact $k - 1$. Obviously, we do not know with precision which $k - 1$ true nulls have been rejected among the many strategies rejected in the first step. Therefore, to be cautious, Romano, Shaikh, and Wolf (2008) suggest looking at all possible combinations of $k - 1$ rejected hypotheses from the set **Reject**.

5.3 False Discovery Ratio (FDR)

In many applications, we are willing to tolerate a larger number of false rejections if there are a large number of total rejections. In other words, rather than controlling for the “number” of false rejections, one can control for the “proportion” of false rejections or the False Discovery Proportion (FDP). FDR measures and controls the expected FDP among all discoveries. More formally, a multiple testing method is said to control FDR at level γ if $\text{FDR} \equiv \text{E}(\text{FDP}) \leq \gamma$. The level γ is a user-defined parameter which should not be confused with a significance level α . Since FDR is already an expectation, controlling for FDR does not need additional specification of probabilistic significance level. Nevertheless, the literature often uses γ and α interchangeably. It is to be noted though that choosing FDR γ to be the same as the significance level α in FWER would imply that FDR method is more lenient than the FWER methods as FDR tolerates a larger number of false rejections. Harvey, Liu, and Zhu (2016) explore γ of both 5% and 1%.

One of the earliest methods to controlling FDR is by Benjamini and Hochberg (1995) and proceeds in a stepwise fashion as follows. Assuming as before that the individual p -values are ordered from the smallest to largest, and defining:

$$j^* = \max \left\{ j : p_j \leq \frac{j \times \gamma}{M} \right\},$$

one rejects all hypotheses H_1, H_2, \dots, H_{j^*} (i.e., j^* is the index of the largest p -value among all hypotheses that are rejected). This is a step-up method that starts with examining the least significant hypothesis and moves up to more significant test statistics.

Benjamini and Hochberg show that their method controls FDR if the p -values are mutually independent. Benjamini and Yekutieli (2001) show that a more general control of FDR under a more arbitrary dependence structure of p -values can be achieved by replacing the definition of j^* with:

$$j^* = \max \left\{ j : p_j \leq \frac{j \times \gamma}{M \times C_M} \right\},$$

where the constant $C_M = \sum_{i=1}^M 1/i \approx \log(M) + 0.5$. However, the Benjamini and Yekutieli method is less powerful than that of Benjamini and Hochberg. Moreover, even under the conditions of Benjamini and Yekutieli, these methods (henceforth referred to as BHY methods) are still conservative.

Storey (2002) suggests an improvement to power by replacing M , the total number of

strategies, with an estimate M_0 of the number of true null hypotheses. This is given by:

$$M_0 = \frac{\#\{p_i > \lambda\}}{1 - \lambda},$$

where $\lambda \in (0, 1)$ is a user-specified parameter. Bajgrowicz and Scaillet (2012) find that setting $\lambda = 0.6$ works reasonably well. Using this M_0 , the critical index j^* is defined as:

$$j^* = \max \left\{ j : p_j \leq \frac{j \times \gamma}{M_0} \right\}.$$

Unfortunately, the Storey method (henceforth BHYS) comes at the cost of assuming stronger dependence conditions on the individual p -values than the BHY procedures.

Finally, it is important to note that FDR, being the mean of FDP, only controls the central tendency of the sampling distribution of FDP. In a given application, the realized FDP could still be far away from the level γ .

5.4 False Discovery Proportion (FDP)

One caveat with FDR is that it is designed to control the expected value of the FDP. Its application is, therefore, better suited for cases where a researcher can analyze a large number of data sets thus allowing one to make confidence statements about the realized average FDP across the various data sets. Since our application of multiple hypothesis testing is based on a single dataset, it is more appropriate to use a method that directly controls the FDP.²

A multiple testing method is said to control FDP at level α if $\text{Prob}(\text{FDP} > \gamma) \leq \alpha$. Lehman and Romano (2005) and Romano and Shaikh (2006) develop extensions of the Holm method for FDP control. Here we discuss only the extension of the StepM procedure developed by Romano and Wolf (2007).

5.4.1 FDP-StepM method

The StepM procedure for control of FDP is as follows:

1. Let $j = 1$ and $k_1 = 1$.
2. Apply the k_j -StepM method and denote by M_j the number of hypotheses rejected.
3. If $M_j < k_j/\gamma - 1$, then stop. Else let $j = j + 1$, $k_j = k_{j-1} + 1$, and return to step 2.

²We thank Michael Wolf for explaining this important difference to us.

The FDP-StepM method is, thus, a sequence of k -StepM procedures. The intuition of applying an increasing series of k 's is as follows. Consider controlling FDP at rate $\gamma = 0.1$. We start by applying the 1-StepM method. Denote by M_1 the number of strategies rejected at this stage. Since the basic StepM procedures control for FWER, we can be confident that no false rejections have occurred so far, which in turn also implies that FDP has also been controlled. Consider now the issue of rejecting the strategy H_{M_1+1} , the next most significant strategy (recall that StepM is a stepdown procedure).

Rejection of H_{M_1+1} , if the null of this strategy is true, renders the FDP equal to $1/(M_1+1)$. Since we are willing to tolerate 10% of false rejections, we would be willing to tolerate rejecting this strategy if $1/(M_1 + 1) < 0.1$ which is true if $M_1 > 9$. Thus if $M_1 < 9$ the procedure would stop at the first step. Alternatively, if $M_1 > 9$, the procedure would continue with the 2-StepM method, which by design should not reject more than one true hypothesis.

Besides the fact that the FDP-StepM method allows the researcher to directly control FDP, one other big advantage of this method is that it accounts for generalized dependence structure in the data and, therefore, in the individual p -values.

6 Statistical and economic hurdles

6.1 Adjusted confidence levels

As we detailed in the previous section, all MHT methods essentially consists of adjustments to the threshold p -value or t -statistic associated with a desired level of significance. In this section we calculate the adjusted statistical significance levels for the FWER, FDR, and FDP methods and report the results in Table 4. In particular, we tabulate the t -statistic thresholds corresponding to one and five per cent statistical significance for FWER methods in Panel A. Theory does not tell us what FDR proportion γ to tolerate. For FDR we report critical values corresponding to the BHY and BHYS methods controlling γ at one and five percent in Panel B (recall that there is no significance level associated with FDR). For FDP, we report critical values corresponding to the FDP-StepM method controlling γ at one and five percent with significance levels of one and five percent in Panel C.

The FWER critical value at five and one percent significance are extremely high at 5.58 and 5.86, respectively and are virtually identical for either alpha and FM coefficient t -statistics. There is also no difference in the critical values calculated from the Bonferroni and the Holm method. One reason for these extremely high critical values is the large number of strategies that we analyze and the fact that FWER methods are known to be overly

conservative (as they account for the probability of making *one* Type I error). In terms of the strategies, at a 5% significance level, the FWER methods find only 487 strategies with significant alphas but around about 9,100 significant FM coefficients. However, these strategies are less than 0.5% of the total number of strategies considered implying that the FWER methods fail to find a lot of evidence of outperformance.

FDR methods, by tolerating a proportion of Type I errors (as opposed to just one), are less conservative. Using the BHY method and using false discovery proportion of 5%, the critical values are 4.83 and 4.29 for alpha and FM coefficient t -statistics, respectively. The number of rejections of the null hypothesis for alpha is 4,257 (0.2% of total number of strategies) and for the FM coefficient is 56,254 (2.69% of the total number of strategies).³ As the BHYS method is less conservative, it allows for lower critical values and a larger number of significant strategies. Considering again the five per cent proportion, we obtain critical values of 3.18 and 3.08 for alpha and FM t -statistics, with a considerably larger number of trading strategies being found significant.

One important aspect of FWER and FDR methods is that they do not account (or account in a limited way) for cross-correlation in the statistics used to evaluate the null hypothesis. Such cross-correlation arises from two sources. On the one hand, different trading strategies rely on firm level data that are economically related through the balance sheet, the income statement, or the market assessment of such data. Therefore the trading signals are not independent. On the other hand, even if the signals were truly independent, they are still applied to a common set of stock returns that co-move in time because of aggregate forces. Thus, it is important to use methods that not only do not rely on restrictive assumptions about cross-correlations but are able to take into account the actual cross-correlations present in the data to deliver more precise critical values. For these reasons (and for reasons discussed earlier regarding appropriateness to our setting), we dedicate more attention to a method that controls FDP.

Panel C of Table 4 shows that, for a significance level of five per cent and false discovery proportion of five per cent, the critical values for alpha and FM coefficient t -statistic are 3.79 and 3.12, respectively. HLZ (2015) suggest a critical t -statistic of three for their sample of 316 strategies. Given our sample of two million strategies, it is not surprising that when applying multiple hypothesis testing the confidence level about any strategy's performance is lower relative to the case where only 316 strategies are observed.

At these critical values, we find 57,753 strategies (2.76% of total) that can be rejected for

³The fact that a lower threshold for FM coefficient t -statistics (relative to alpha t -statistics) leads to a higher number of rejections is due to the fact that the cross-sectional distribution of FM coefficient t -statistics has much longer tails than the distribution of alpha t -statistics. One simple way to verify this is to compare the standard deviation of the cross-sectional distributions of t -statistics from Panel B and C of Table 2.

the null of zero alpha and 225,677 strategies (10.80% of total) that can be rejected for the null of zero FM coefficient. Therefore, given the lower critical values relative to FWER methods, the FDP-StepM method finds many more strategies that outperform. At the same time, the number of strategies that survive these statistical hurdles seems large in an absolute sense. At first glance, this would suggest that we have managed to uncover many more trading signals that have been published in the literature and that have the potential to generate profitable trading strategies.

However, before reaching that conclusion, we would like to subject these remaining strategies to the test of economic significance.

6.2 Economic hurdles

The main goal of our paper is to gauge the economic significance of a large set of strategies. It is possible that some of the strategies that pass the statistical thresholds are just lucky. Although our MHT procedures are designed to guard against luck in the discovery process, some false discoveries may still slip through the net. In fact, both the FDR and the FDP methods are designed to tolerate a certain fraction of false discoveries. We would, therefore, like to consider strategies that display not only good statistical properties but are also economically meaningful and relevant.

Therefore, we impose here additional hurdles that impose consistency and economic restrictions on the strategies that survive statistical thresholds. First, we would like to guarantee consistency between results obtained by studying portfolio returns and those derived from Fama–MacBeth regressions. As discussed in Section 2.2.2, there are advantages and disadvantages to both portfolio sorts and regressions. We would like a trading signal to not only generate a high long-short portfolio alpha but also to explain the broader cross-section of returns in a regression setting. Therefore, we reject strategies that have statistically significant t_α but insignificant t_λ or vice-versa. Imposing this filter drastically reduces the number of strategies (we report exact numbers slightly later in this section).

Second, we take a step back and look at the economic magnitudes of these remaining strategies. Recall that our statistical hurdles are based on t -statistics as these test statistics have been shown to be better than just magnitude of alphas. Since, there is a close relation between the magnitude of alpha and its t -statistic, the strategies that survive our statistical hurdles are also invariably strategies that have large alphas. For example, strategies for which both alpha and FM t -statistics clear the FDP-StepM critical values at five per cent significance and proportion have an average alpha of 0.72% per month (in absolute value). The magnitude of alpha is however hard to interpret as it is not obvious whether

a sophisticated investor would be willing to accept the time-series variation associated with a large alpha. For this reason, we opt for another metric that is often used in performance evaluation, that of Sharpe ratio.

MacKinlay (1995) argues that risk based explanations for the rejections of the null hypothesis result in Sharpe ratios that are bounded while non-risk explanations would result in unbounded Sharpe ratios. We relate the strategy's Sharpe ratio to that of the market (SRM). Since risk based rejections of the null could in principle be captured by the proper risk models we want to focus on the non-risk based rejections of the null. We use various cutoffs from half to twice the market's Sharpe ratio (i.e., the Sharpe ratio of the value-weighted market portfolio). For the entire sample and on a monthly basis the SRM is 0.116, corresponding to an annualized figure of approximately 0.4.

We also impose the restriction suggested by Linnainmaa and Roberts (2016) that a certain amount of persistence in profitability should be expected across in- and out-of-sample estimates. As our data is entirely in-sample, we impose the condition that the Sharpe ratios of the strategies should exceed the cutoffs in the entire sample, but also in two halves of the sample. For the first half of the sample, from June of 1973 to May of 1994, the market has a SR of 0.091, while for the second part of the sample, June 1994 to May 2015, SRM is 0.143.

In particular, Table 5 reports the number of strategies that satisfy the consistency and economic hurdles. We stratified them into four groups: between 0 and half of SRM; between half of SRM and SRM; between SRM and twice SRM; and larger than twice SRM. For each group, we report the number of strategies that are in the respective group for the full sample period, first half sample, second half sample, and in full sample period as well as in both half subsample periods.

We start by discussing Panel A which presents the strategies that survive the FDR-BHY rejections in Table 4. Thus, for false discovery proportion of 5%, we are looking for the intersection of 4,257 strategies from the alpha t -statistic rejection and 56,254 strategies from the rejection of the FM coefficient t -statistic. This leaves us with a total of 136 strategies (less than 0.01% of the total number of considered strategies) that we subject to the economic hurdles. Of the 136 strategies, only five have Sharpe ratios greater than that of the market over the entire sample period. There is no strategy that has Sharpe ratio greater than that of the market in either the entire sample period and in the two halves. There is also no strategy that has Sharpe ratio greater than twice that of the market in the full sample period.

Analyzing the corresponding figures, tabulated in Panel B, obtained by applying the less stringent FDR-BHYS critical values (also at 5% proportion) we observe a larger number of surviving strategies. However, the space of strategies that outperform shrinks considerably when the performance is compared to the market. Only 345 strategies have Sharpe ratios

larger than the market in the entire period, and none crosses the threshold of two times the market Sharpe ratio. Moreover, only 52 strategies are consistent across the entire sample period, when also considering the two half samples.

Panel C tabulates results obtained by applying critical values derived from FDP-StepM. Since the critical values for FDP method are lower than those based on BHY and higher than the ones from BHYS, we find slightly higher number of strategies that cross the statistical and economic thresholds than those reported in Panel A and lower than those reported in Panel B. At a significance level of 5% and a false discovery proportion of 5%, the intersection of 57,753 strategies for which the zero-alpha null hypothesis is rejected and the 225,677 strategies for which the FM coefficient t -statistic is found to be significant is a set that contains 806 elements (0.04% of the total number of considered strategies). Of these 806 strategies, only 17 have Sharpe ratios greater than that of the market over the entire sample period and only four have persistent performance. There is no strategy that has Sharpe ratio greater than twice that of the market in the full sample period.

In summary, on the one hand, in the most optimistic scenario where we consider the least stringent approach (i.e., BHYS), and therefore neglect to account for cross-correlation in the data, we find at most 345 economically significant strategies (52 if we impose some persistence in economic performance). In the least optimistic scenario, we find 5. On the other hand, if we properly account for the statistical properties of the data-generating process, we are left with a handful of exceptional investment opportunities. If we adopt an all-together conservative approach and control FDR or FDP at $\gamma = 1\%$ (i.e., we accept one per cent of lucky discovery among all discoveries on average or in our sample), we are bound to reject the totality of strategies in two of the three cases reported in Table 4.

Even if very few strategies survive, it is worth considering that those could in fact be a super or subset of those documented in the literature. Finding some intersection between our exceptional strategies and those that are believed to be (i.e., because they appear in the top finance journals) would lend some credibility to the idea that market inefficiencies do exist on a relatively large scale. We list the 17 strategies in Table A8.

7 Additional tests

We present here some robustness checks. First, we expand the sample of stocks by including all stocks, thus removing the restriction that stocks, at portfolio formation, must be above the 20th percentile of NYSE market capitalization and have price bigger than \$3. We aim to check whether the inclusion of micro-cap stocks yields stronger evidence of market inefficiency. As shown by Fama and French (2010) and Hou, Xue, and Zhang (2017), anomalies

are in fact more prevalent in the stocks that we exclude from our main analyses. Second, we use different factor models as benchmark for assessing abnormal performance in both time-series and cross-sectional regressions. We choose three additional factor models: (i) Fama and French (1993) three-factor model (FF3), (ii) Barillas and Shanken (2015) five-factor model (BS), and (iii) Hou, Xue, and Zhang (2015) four-factor model augmented with the momentum factor (HXZ).⁴

We present results in two tables. Table 6 shows summary statistics of the cross-sectional distribution of alpha and FM t -statistics. Table 7 presents the critical values and the number of strategies that pass the statistical and economic thresholds. To reduce clutter, we present critical values from the FDP-StepM method only using false discovery proportion of 5% and statistical significance level of 5%.

Focusing first on the sample of all stocks, we find that distributions of t -statistics (both alpha and FM) have longer tails than those reported in Table 2 for large stocks. For instance, the minimum (maximum) of alpha t -statistic using the sample of all stocks is -7.44 (8.55) while it is only -6.75 (7.35) in the sample of non-microcap stocks. Nevertheless, we find a slightly lower proportion of t_α larger than conventional critical levels, and a slightly lower proportion of t_λ . The net effect is that the number of strategies that have t -statistics larger than the critical values derived by applying the FDP-StepM method (i.e., 3.98 and 3.04, respectively) is essentially equal to that for the universe of strategies based on non microcap stocks (i.e., 729 versus 809).

This is not the result of lower critical values (e.g., t_α and t_λ are equal to 3.79 and 3.12 respectively, using the main sample of large stocks). Rather it is due to the fact that the tails of the cross-sectional distribution are longer, and the fact that the factor models and cross-sectional regressions fail in at a similar rate at attributing the returns to risk and/or characteristics of the stocks that end up in the spread portfolios. That is particularly evident when we consider that there are in fact 440 strategies with a Sharpe ratio bigger than that of the market in the sample that includes small stocks. However, while 440 is quite larger than the number of accepted discoveries for large stocks (i.e. 17), it still represents an insignificantly small fraction of the two million strategies we consider.

Remarkably, even in the list of 440 strategies we fail to find many of the strategies that are analyzed by Harvey, Liu, and Zhu (2015) and Hou, Xue, and Zhang (2017), for example.

As mentioned above, our second set of robustness checks is related to the choice of factor models. Looking at Table 6, we find that the FF model generates the lowest fraction of

⁴As described in Section 2.2.2 the left hand side of the cross-sectional regression is the risk adjusted realized return. Therefore we recompute all Fama-MacBeth coefficients and their relative t -statistics using the different factor models. The alpha t -statistics from the Fama and French three factor model is therefore paired with the FM t -statistics obtained using the same factor model to risk-adjust stock returns.

t -statistics (both t_α and t_λ) higher than 1.96. The BS model has the widest distribution of t_α with a cross-sectional standard deviation of 2.42 resulting in 45.83% strategies that cross the conventional cutoff of 1.96. On the other hand, the cross-sectional distribution of t_λ from BS and HXZ models is similar to that of FF6 model.

The results in Table 7 broadly follow the descriptive statistics presented earlier. For example, FF alphas showed lower outperformance using conventional levels and the same picture emerges in MHT; there are less than 50 strategies overall that survive the statistical hurdles for both t_α and t_λ . The BS model has the highest number of strategies that cross the statistical hurdles at around four thousand but only 80 of these strategies have Sharpe ratios higher than that of the market. The HXZ model also produces a lower number of strategies for which both alpha and FM t -statistics are over the respective critical values, but singles out the same number of strategies with SR higher than the market's (i.e., 17 total).

8 Conclusions

We develop a data mining approach to generate over two million trading strategies. We compute statistical thresholds that are robust to multiple hypothesis testing. Applying the statistical thresholds in conjunction to economic considerations leads us to conclude that very few strategies appear abnormally profitable. Our findings are in line with recent studies that examine the out-of-sample profitability of many trading strategies that have been published in finance journals.

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Figure 1: Empirical distributions of spread portfolios characteristics

The figure displays histograms for various characteristics of the distribution of trading strategies returns. We show average returns, average return t -statistics, Sharpe ratios, Fama-French six factor alphas and relative t -statistics, Fama-MacBeth coefficients t -statistics. The sample is composed by all companies that have stock returns between 1970 and 2015. Trading strategies are constructed using combinations of one, two and three variables constructed as described in Section 2.1. Variables are extracted from Compustat and CRSP. There are a total of 2,110,823 trading strategies. All data is at the monthly frequency and span the period between July of 1972 and June of 2015.

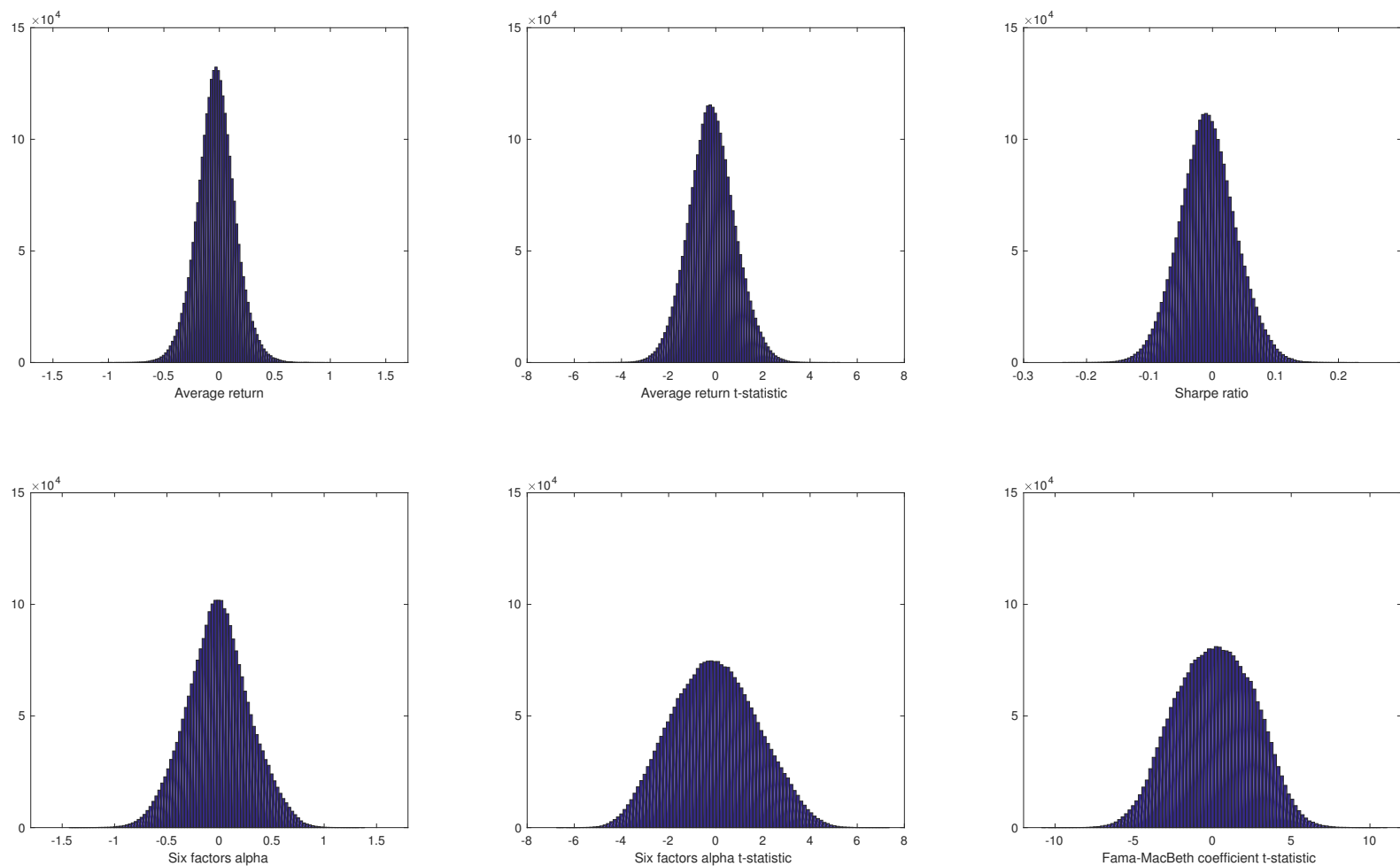


Table 1: Descriptive statistics of portfolio raw returns on trading strategies

We construct trading strategies as described in the text. This table reports cross-sectional mean, median, standard deviation, minimum, and maximum of monthly average return (Panel A), t -statistic (Panel B) and monthly Sharpe ratio (Panel C). All returns are reported in percentages. We also report the number and percentage of strategies that cross specific thresholds in each panel. The sample period is 1972 to 2015.

Panel A: Average return										
	N	Mean	Median	Std	Min	Max	ret > 0.5%		ret > 1.0%	
							#	%	#	%
All	2,090,365	-0.03	-0.03	0.18	-1.07	0.99	17,192	0.82	5	0.00
Levels	156	-0.02	-0.04	0.15	-0.34	0.62	3	1.92	0	0.00
Growth rates	126	-0.17	-0.16	0.21	-0.68	0.48	7	5.56	0	0.00
Ratios of two	10,668	-0.02	-0.02	0.17	-0.78	0.78	110	1.03	0	0.00
Ratios of growth rates	7,762	-0.02	-0.02	0.14	-0.70	0.55	7	0.09	0	0.00
Ratios of three	2,071,653	-0.03	-0.03	0.18	-1.07	0.99	17,065	0.82	5	0.00
Panel B: Average return t -statistic										
	N	Mean	Median	Std	Min	Max	t_μ > 1.96		t_μ > 2.57	
							#	%	#	%
All	2,090,365	-0.17	-0.19	0.98	-5.41	5.01	105,756	5.06	22,237	1.06
Levels	156	-0.26	-0.26	0.86	-2.69	2.46	8	5.13	3	1.92
Growth rates	126	-1.11	-1.20	1.39	-4.14	3.58	41	32.54	22	17.46
Ratios of two	10,668	-0.11	-0.13	0.99	-4.31	3.54	515	4.83	122	1.14
Ratios of growth rates	7,762	-0.15	-0.14	1.07	-4.00	3.96	543	7.00	150	1.93
Ratios of three	2,071,653	-0.17	-0.19	0.98	-5.41	5.01	104,649	5.05	21,940	1.06
Panel C: Sharpe ratio										
	N	Mean	Median	Std	Min	Max	SR > 0.116		SR > 0.232	
							#	%	#	%
All	2,090,365	-0.01	-0.01	0.04	-0.24	0.23	24,211	1.16	1	0.00
Levels	156	-0.01	-0.01	0.04	-0.12	0.11	2	1.28	0	0.00
Growth rates	126	-0.05	-0.05	0.06	-0.18	0.19	21	16.67	0	0.00
Ratios of two	10,668	-0.01	-0.01	0.05	-0.19	0.16	149	1.40	0	0.00
Ratios of growth rates	7,762	-0.01	-0.01	0.05	-0.18	0.18	153	1.97	0	0.00
Ratios of three	2,071,653	-0.01	-0.01	0.04	-0.24	0.23	23,886	1.15	1	0.00

Table 2: Descriptive statistics of portfolio abnormal returns and regression coefficients of trading strategies

We construct trading strategies as described in the text. This table reports cross-sectional mean, median, standard deviation, minimum, and maximum of monthly alpha (Panel A), t -statistic of alpha (Panel B) and t -statistic of Fama and MacBeth (1973) coefficient (Panel C). Abnormal returns are computed relative to the Fama and French (2015) five-factor model augmented with a momentum factor. All alphas are reported in percentages. We also report the number and percentage of strategies that cross specific thresholds in each panel. The sample period is 1972 to 2015.

	Panel A: Alpha									
	N	Mean	Median	Std	Min	Max	$ \alpha > 0.5\%$		$ \alpha > 1.0\%$	
							#	%	#	%
All	2,090,365	-0.01	-0.01	0.30	-1.43	1.39	222,566	10.65	1,574	0.08
Levels	156	-0.04	-0.65	0.79	0.06	3.13	7	4.49	0	0.00
Growth rates	126	0.01	-0.62	0.59	-0.14	3.44	5	3.97	0	0.00
Ratios of two	10,668	-0.09	-1.20	1.03	0.08	3.15	1,104	10.35	11	0.10
Ratios of growth rates	7,762	0.02	-0.97	0.65	-0.10	4.11	23	0.30	0	0.00
Ratios of three	2,071,653	-0.01	-1.43	1.39	0.02	2.96	221,427	10.69	1,563	0.08
	Panel B: Alpha t -statistic									
	N	Mean	Median	Std	Min	Max	$ t_\alpha > 1.96$		$ t_\alpha > 2.57$	
							#	%	#	%
All	2,090,365	-0.05	-0.09	1.82	-6.75	7.36	638,825	30.56	353,914	16.93
Levels	156	-0.35	-0.09	1.76	-4.38	3.69	51	32.69	22	14.10
Growth rates	126	0.15	0.02	1.40	-3.12	3.77	23	18.25	10	7.94
Ratios of two	10,668	-0.54	-0.58	1.73	-4.96	5.71	3,222	30.20	1,799	16.86
Ratios of growth rates	7,762	0.16	0.15	1.03	-3.86	4.00	477	6.15	128	1.65
Ratios of three	2,071,653	-0.05	-0.09	1.82	-6.75	7.36	635,052	30.65	351,955	16.99
	Panel C: Fama-MacBeth coefficient t -statistic									
	N	Mean	Median	Std	Min	Max	$ t_\lambda > 1.96$		$ t_\lambda > 2.57$	
							#	%	#	%
All	2,090,365	0.11	0.12	1.93	-11.01	11.39	643,236	30.77	384,390	18.39
Levels	156	-0.25	-0.54	1.77	-4.79	4.07	42	26.92	23	14.74
Growth rates	126	-0.81	-1.17	1.83	-4.38	3.91	49	38.89	22	17.46
Ratios of two	10,668	0.28	0.33	1.97	-8.34	7.26	3,420	32.06	2,085	19.54
Ratios of growth rates	7,762	0.05	0.03	1.34	-4.60	5.04	1,136	14.64	423	5.45
Ratios of three	2,071,653	0.11	0.12	1.93	-11.01	11.39	638,589	30.83	381,837	18.43

Table 3: Bootstrapped distributions of t -statistics

The table reports results of the bootstrap exercise described in Section 4. We run 1,000 bootstraps preserving cross-correlation between strategy returns and factors (please see the text for further details). For each percentile (i.e., each row in the table), we report the percentile of the actual t -statistics (Data) and the average (across bootstraps) t -statistic for that percentile (Boot). We also report percentage of actual t -statistics that are bigger than the average bootstrapped t -statistic (% Data) and the fraction of iterations where the bootstrapped percentile was bigger than the actual percentile (% Boot).

Percentile	Alpha t -statistic, t_α				Fama-MacBeth t -statistic, t_λ			
	Data	Boot	% Data	% Boot	Data	Boot	% Data	% Boot
0.5	-4.15	-2.57	91.54	100.00	-4.93	-2.54	91.43	100.00
1.0	-3.87	-2.32	88.97	100.00	-4.41	-2.30	89.42	100.00
2.5	-3.41	-1.97	84.46	100.00	-3.72	-1.95	85.81	100.00
5.0	-2.97	-1.66	79.88	100.00	-3.10	-1.65	82.06	100.00
10.0	-2.42	-1.30	73.78	100.00	-2.36	-1.29	76.90	100.00
20.0	-1.66	-0.85	65.56	100.00	-1.50	-0.86	69.45	100.00
30.0	-1.08	-0.53	59.08	99.90	-0.89	-0.54	63.38	100.00
40.0	-0.57	-0.25	53.36	98.00	-0.37	-0.26	57.84	98.50
50.0	-0.09	0.01	48.02	76.00	0.12	-0.00	52.47	0.00
60.0	0.41	0.27	42.75	16.10	0.60	0.26	47.06	0.00
70.0	0.94	0.55	37.29	1.80	1.11	0.53	41.35	0.00
80.0	1.55	0.87	31.19	0.10	1.72	0.86	34.86	0.00
90.0	2.38	1.31	23.63	0.00	2.58	1.29	26.78	0.00
95.0	3.02	1.68	18.21	0.00	3.29	1.64	21.08	0.00
97.5	3.51	1.99	14.22	0.00	3.89	1.95	16.88	0.00
99.0	4.03	2.35	10.35	0.00	4.61	2.29	12.77	0.00
99.5	4.36	2.59	8.16	0.00	5.10	2.53	10.42	0.00

Table 4: Multiple hypothesis testing critical values

The table shows alpha and Fama-MacBeth statistical thresholds adjusted for multiple hypothesis testing, as well number of strategies rejected and relative percentage to the number of strategies considered. We report FWER (Bonferroni and Holm) adjusted thresholds in Panel A; FDR (BHY) in Panel B, and FDP (FDP-StepM) in Panel C. The numbers are reported for significance levels of 1% and 5% in Panels A and C (there is no significance level associated with FDR). We use 1% and 5% as two proportions of false discoveries for Panels B and C. The total number of strategies is 2,090,365. The sample period is 1972 to 2015.

Panel A: FWER - Bonferroni and Holm						
Significance = 5%			Significance = 1%			
<i>t</i> -stat	#	%	<i>t</i> -stat	#	%	
Alpha <i>t</i> -statistic						
Bonferroni	5.58	487	0.02	5.86	178	0.01
Holm	5.58	487	0.02	5.86	178	0.01
Fama-MacBeth <i>t</i> -statistic						
Bonferroni	5.58	9,188	0.44	5.85	6,002	0.29
Holm	5.58	9,205	0.44	5.85	6,003	0.29
Panel B: FDR - BHY and BHYS						
BHY			BHYS			
<i>t</i> -stat	#	%	<i>t</i> -stat	#	%	
Alpha <i>t</i> -statistic						
Proportion = 1%	4.03	34,731	1.66	2.31	465,965	22.29
Proportion = 5%	4.83	4,257	0.20	3.18	163,879	7.84
Fama-MacBeth <i>t</i> -statistic						
Proportion = 1%	3.75	112,205	5.37	2.30	488,218	23.36
Proportion = 5%	4.29	56,254	2.69	3.08	236,634	11.32
Panel C: FDP - StepM						
Significance = 5%			Significance = 1%			
<i>t</i> -stat	#	%	<i>t</i> -stat	#	%	
Alpha <i>t</i> -statistic						
Proportion = 1%	4.94	3,369	0.16	6.21	71	0.00
Proportion = 5%	3.79	57,753	2.76	4.49	11,554	0.55
Fama-MacBeth <i>t</i> -statistic						
Proportion = 1%	4.04	78,275	3.74	4.64	35,287	1.69
Proportion = 5%	3.12	225,677	10.80	3.70	119,489	5.72

Table 5: Strategies that survive the statistical and economic hurdles

This table reports the number of trading strategies that survive the statistical thresholds from Table 4. For example, for FDR-BHY methods with false discovery proportion of 5%, the strategy should have alpha t -statistic greater than 4.04 and, at the same time, FM coefficient t -statistic greater than 3.75. These strategies are further classified for various levels of economic significance which are determined by comparing the level of the absolute value of the strategy's Sharpe ratio to various targets determined by the market Sharpe ratio (SRM) in the corresponding period. The market Sharpe ratio for the entire sample is 0.113, 0.091 for the first half, and 0.143 for the second half of the sample. We report the number of strategies stratified into four groups: between 0 and half of SRM; between half of SRM and SRM; between SRM and twice SRM; and larger than twice SRM. For each group, we report the number of strategies that are in the respective group for the full sample period, first half sample, second half sample, and in full sample period as well as in both half subsample periods. The critical values applied to half sample period are not the ones reported in Table 4 but are recalculated for that respective sample period. The total number of strategies is 2,090,365. The sample period is 1972 to 2015.

Panel A: FDR - BHY								
	0 to SRM/2	SRM/2 to SRM	SRM to 2×SRM	More than 2×SRM	0 to SRM/2	SRM/2 to SRM	SRM to 2×SRM	More than 2×SRM
	Proportion = 5%				Proportion = 1%			
Full sample	80	51	5	0	2	2	0	0
First-half sample	83	32	18	3	2	0	2	0
Second-half sample	53	75	8	0	2	2	0	0
Full sample and both half subsamples	37	6	0	0	2	0	0	0

Panel B: FDR - BHYS								
	Proportion = 5%				Proportion = 1%			
Full sample	13,014	3,476	345	0	1,707	449	43	0
First-half sample	8,546	5,303	2,781	205	1,207	669	307	16
Second-half sample	11,001	5,230	603	1	1,287	842	70	0
Full sample and both half subsamples	5,758	697	52	0	730	90	10	0

Panel C: FDP - StepM								
	Significance = 5%, Proportion = 5%				Significance = 5%, Proportion = 1%			
Full sample	591	198	17	0	2	2	0	0
First-half sample	471	221	109	5	2	0	2	0
Second-half sample	445	326	35	0	2	2	0	0
Full sample and both half subsamples	264	31	4	0	2	0	0	0

Table 6: Robustness checks: Descriptive statistics

This table reports cross-sectional mean, median, standard deviation, minimum, and maximum of alpha and Fama-MacBeth t -statistic for (i) sample including all stocks) and (ii) alternative factor models. We use the three-factor model of Fama and French (1993) (FF), six-factor model of Barillas and Shanken (2015) (BS), and the four-factor model of Hou, Xue and Zhang (2015) augmented with the momentum factor (HXZ). We report alpha t -statistics in Panel A and Fama-MacBeth t -statistics in Panel B. We also report the number and percentage of strategies that cross specific thresholds in each panel.

Panel A: Alpha t -statistic										
	N	Mean	Median	Std	Min	Max	$ t_\alpha > 1.96$		$ t_\alpha > 2.57$	
							#	%	#	%
All stocks	2,090,365	-0.18	-0.18	1.63	-7.44	8.56	491,550	23.52	250,258	11.97
FF3	2,090,365	-0.41	-0.44	1.47	-6.09	6.85	428,502	20.50	188,911	9.04
BS	2,090,365	-0.09	-0.12	2.42	-7.94	7.73	958,043	45.83	673,077	32.20
HXZ	2,090,365	-0.15	-0.14	1.73	-6.33	6.51	583,995	27.94	302,648	14.48

Panel B: Fama-MacBeth t -statistic										
	N	Mean	Median	Std	Min	Max	$ t_\lambda > 1.96$		$ t_\lambda > 2.57$	
							#	%	#	%
All stocks	2,090,365	0.13	0.14	2.06	-12.91	12.49	684,108	32.73	424,315	20.30
FF3	2,090,365	-0.03	-0.03	1.33	-7.17	7.30	280,523	13.42	122,560	5.86
BS	2,090,365	0.11	0.11	1.99	-10.99	11.35	655,539	31.36	398,040	19.04
HXZ	2,090,365	0.11	0.12	1.99	-11.30	11.14	662,381	31.69	404,239	19.34

Table 7: Robustness checks: Statistical and economic thresholds

The table reports statistical thresholds for alpha and Fama-MacBeth t -statistics based on the FDP-StepM method (with significance equal to 5% and false discovery proportion of 5%) as well as the number of strategies that qualify based on economic hurdles based on the strategy Sharpe ratios relative to the market Sharpe ratio (in the same manner as in Table 5). Quantities are reported for the four robustness check cases presented in Table 6 (i.e., Fama and French (2015) five-factor model together with the momentum factor for all stocks (FF6), Fama and French (1993) three factor model (FF3), Barillas and Shanken (2015) six-factor model (BS), and Hou, Xue and Zhang (2015) factor model together with the momentum factor (HXZ)).

	t_α	t_λ		0 to SRM/2	SRM/2 to SRM	SRM to 2×SRM	More than 2×SRM
All stocks	3.98	3.04	Full sample	109	180	433	7
			Full sample and halves	50	47	80	0
FF3	4.28	3.95	Full sample	0	9	37	1
			Full sample and halves	0	4	13	0
BS	2.76	3.08	Full sample	3,955	917	80	0
			Full sample and halves	1,833	167	14	0
HXZ	3.85	3.08	Full sample	426	131	17	0
			Full sample and halves	209	25	1	0

Table A1: Basic variables used to construct trading strategies

#	Short	Long	#	Short	Long
1	aco	Current Assets - Other - Total	61	idit	Interest and Related Income - Total
2	acox	Current Assets - Other - Sundry	62	intan	Intangible Assets - Total
3	act	Current Assets - Total	63	intc	Interest Capitalized
4	am	Amortization of Intangibles	64	invfg	Inventories - Finished Goods
5	ao	Assets - Other	65	invrm	Inventories - Raw Materials
6	aox	Assets - Other - Sundry	66	invt	Inventories - Total
7	aqs	Acquisitions - Sales Contribution	67	invwip	Inventories - Work In Process
8	at	Assets - Total	68	itcb	Investment Tax Credit (Balance Sheet)
9	bkvlps	Book Value Per Share	69	itci	Investment Tax Credit (Income Account)
10	caps	Capital Surplus-Share Premium Reserve	70	ivaeq	Investment and Advances - Equity
11	capx	Capital Expenditures	71	ivao	Investment and Advances - Other
12	capxv	Capital Expend Property, Plant and Equipment Schd V	72	ivst	Short-Term Investments - Total
13	ceq	Common-Ordinary Equity - Total	73	lco	Current Liabilities - Other - Total
14	ceql	Common Equity - Liquidation Value	74	lcox	Current Liabilities - Other - Sundry
15	ceqt	Common Equity - Tangible	75	lct	Current Liabilities - Total
16	ch	Cash	76	lifr	LIFO Reserve
17	che	Cash and Short-Term Investments	77	lifrp	LIFO Reserve - Prior
18	chech	Cash and Cash Equivalents - Increase-(Decrease)	78	lse	Liabilities and Stockholders Equity - Total
19	cogs	Cost of Goods Sold	79	lt	Liabilities - Total
20	csbfd	Common Shares Used to Calc Earnings Per Share - Fully Diluted	80	mib	Noncontrolling Interest (Balance Sheet)
21	csho	Common Shares Outstanding	81	mibt	Noncontrolling Interests - Total - Balance Sheet
22	cshpri	Common Shares Used to Calculate Earnings Per Share - Basic	82	mii	Noncontrolling Interest (Income Account)
23	cshr	Common-Ordinary Shareholders	83	mrc1	Rental Commitments - Minimum - 1st Year
24	ctk	Common-Ordinary Stock (Capital)	84	mrc2	Rental Commitments - Minimum - 2nd Year
25	ctkcv	Common Stock-Carrying Value	85	mrc3	Rental Commitments - Minimum - 3rd Year
26	ctske	Common Stock Equivalents - Dollar Savings	86	mrc4	Rental Commitments - Minimum - 4th Year
27	dc	Deferred Charges	87	mrc5	Rental Commitments - Minimum - 5th Year
28	dcpstk	Convertible Debt and Preferred Stock	88	mrct	Rental Commitments - Minimum - 5 Year Total
29	dcvsr	Debt - Senior Convertible	89	msa	Marketable Securities Adjustment
30	dcvsub	Debt - Subordinated Convertible	90	ni	Net Income (Loss)
31	dcvt	Debt - Convertible	91	niadj	Net Income Adjusted for Common-Ordinary Stock (Capital) Equivalents
32	dd5	Debt - Due in 5th Year	92	nopi	Nonoperating Income (Expense)
33	dlto	Other Long-term Debt	93	nopio	Nonoperating Income (Expense) - Other
34	dltp	Long-Term Debt - Tied to Prime	94	np	Notes Payable - Short-Term Borrowings
35	dltt	Long-Term Debt - Total	95	ob	Order Backlog
36	dm	Debt - Mortgages Other Secured	96	oiadp	Operating Income After Depreciation
37	dn	Debt - Notes	97	oibdp	Operating Income Before Depreciation
38	dp	Depreciation and Amortization	98	pi	Pretax Income
39	dpact	Depreciation, Depletion and Amortization (Accumulated)	99	ppeg	Property, Plant and Equipment - Total (Gross)
40	dpc	Depreciation and Amortization (Cash Flow)	100	ppent	Property, Plant and Equipment - Total (Net)
41	dpvieb	Depreciation (Accumulated) - Ending Balance (Schedule VI)	101	ppeveb	Property, Plant, and Equipment - Ending Balance (Schedule V)
42	ds	Debt-Subordinated	102	prstkc	Purchase of Common and Preferred Stock
43	dv	Cash Dividends (Cash Flow)	103	pstk	Preferred-Preference Stock (Capital) - Total
44	dvc	Dividends Common-Ordinary	104	pstkc	Preferred Stock - Convertible
45	dvp	Dividends - Preferred-Preference	105	pstkl	Preferred Stock - Liquidating Value
46	dvpa	Preferred Dividends in Arrears	106	pstkn	Preferred-Preference Stock - Nonredeemable
47	dvt	Dividends - Total	107	pstkr	Preferred-Preference Stock - Redeemable
48	ebit	Earnings Before Interest and Taxes	108	pstkrv	Preferred Stock - Redemption Value
49	ebitda	Earnings Before Interest	109	re	Retained Earnings
50	emp	Employees	110	rea	Retained Earnings - Restatement
51	esub	Equity in Earnings - Unconsolidated Subsidiaries	111	reajo	Retained Earnings - Other Adjustments
52	esubc	Equity in Net Loss - Earnings	112	recco	Receivables - Current - Other
53	fca	Foreign Exchange Income (Loss)	113	recd	Receivables - Estimated Doubtful
54	fopo	Funds from Operations - Other	114	recta	Retained Earnings - Cumulative Translation Adjustment
55	gp	Gross Profit (Loss)	115	rectr	Receivables - Trade
56	ib	Income Before Extraordinary Items	116	reuna	Retained Earnings - Unadjusted
57	ibadj	Income Before Extraordinary Items - Adjusted for Common Stock Equivalents	117	revt	Revenue - Total
58	ibc	Income Before Extraordinary Items (Cash Flow)	118	sale	Sales-Turnover (Net)
59	ibcom	Income Before Extraordinary Items - Available for Common	119	seq	Stockholders Equity - Parent
60	icapt	Invested Capital - Total	120	spi	Special Items

#	Short	Long	#	Short	Long
121	sppe	Sale of Property	139	xad	Advertising Expense
122	sstk	Sale of Common and Preferred Stock	140	xido	Extraordinary Items and Discontinued Operations
123	tstkc	Treasury Stock - Common	141	xidoc	Extraordinary Items and Discontinued Operations (Cash Flow)
124	tstkn	Treasury Stock - Number of Common Shares	142	xint	Interest and Related Expense - Total
125	tstkp	Treasury Stock - Preferred	143	xlr	Staff Expense - Total
126	txc	Income Taxes - Current	144	xopr	Operating Expenses - Total
127	txdb	Deferred Taxes (Balance Sheet)	145	xpp	Prepaid Expenses
128	txdc	Deferred Taxes (Cash Flow)	146	xpr	Pension and Retirement Expense
129	txdi	Income Taxes - Deferred	147	xrd	Research and Development Expense
130	txditc	Deferred Taxes and Investment Tax Credit	148	xrdp	Research Development - Prior
131	txfed	Income Taxes - Federal	149	xrent	Rental Expense
132	txfo	Income Taxes - Foreign	150	xsga	Selling, General and Administrative Expense
133	txp	Income Taxes Payable	151	ret1	1m Past Return
134	txs	Income Taxes - State	152	ret3	3m Past Return
135	txt	Income Taxes - Total	153	ret6	6m Past Return
136	txw	Excise Taxes	154	ret9	9m Past Return
137	wcap	Working Capital (Balance Sheet)	155	ret12	1y Past Return
138	xacc	Accrued Expenses	156	vol	1y Return Volatility

Table A2: Top 50 strategies by average returns

		Mean	t_μ	SR	α	t_α	t_λ
(caps - chech) / xad	(Capital Surplus-Share Premium Reserve - Cash and Cash Equivalents - Increase-(Decrease)) / Advertising Expense	-1.07	-4.40	-0.23	-0.55	-2.27	-1.21
(aqs - xrdp) / dvt	(Acquisitions - Sales Contribution - Research Development - Prior) / Dividends - Total	-1.06	-4.04	-0.21	-1.03	-3.72	3.70
(csho - cshpri) / mrc1	(Common Shares Outstanding - Common Shares Used to Calculate Earnings Per Share - Basic) / Rental Commitments - Minimum - 1st Year	-1.05	-4.38	-0.20	-0.37	-1.79	1.18
(csho - cshpri) / mrc4	(Common Shares Outstanding - Common Shares Used to Calculate Earnings Per Share - Basic) / Rental Commitments - Minimum - 4th Year	-1.05	-4.36	-0.20	-0.34	-1.57	0.49
(csho - cshpri) / mrc2	(Common Shares Outstanding - Common Shares Used to Calculate Earnings Per Share - Basic) / Rental Commitments - Minimum - 2nd Year	-1.00	-4.04	-0.18	-0.29	-1.39	1.89
(dvpa - ret6) / xad	(Preferred Dividends in Arrears - 6m Past Return) / Advertising Expense	-1.00	-3.82	-0.18	-0.60	-2.00	-0.23
(bkvlps - sstk) / xad	(Book Value Per Share - Sale of Common and Preferred Stock) / Advertising Expense	0.99	3.98	0.18	0.24	0.91	1.98
(intan - xrdp) / dvc	(Intangible Assets - Total - Research Development - Prior) / Dividends Common-Ordinary	-0.99	-4.07	-0.20	-0.96	-3.84	-1.69
(csho - cshpri) / mrc3	(Common Shares Outstanding - Common Shares Used to Calculate Earnings Per Share - Basic) / Rental Commitments - Minimum - 3rd Year	-0.98	-3.92	-0.18	-0.26	-1.26	0.48
(aqs - xrd) / dvc	(Acquisitions - Sales Contribution - Research and Development Expense) / Dividends Common-Ordinary	-0.95	-3.84	-0.18	-0.82	-3.09	1.52
(idit - mrc1) / invfg	(Interest and Related Income - Total - Rental Commitments - Minimum - 1st Year) / Inventories - Finished Goods	-0.95	-3.16	-0.16	-0.29	-1.07	-1.03
(dvpa - ret6) / xsga	(Preferred Dividends in Arrears - 6m Past Return) / Selling, General and Administrative Expense	-0.94	-3.90	-0.18	-0.55	-2.41	0.22
(lct - xpp) / dvolume	(Current Liabilities - Total - Prepaid Expenses) / Dollar Traded Volume	0.94	3.65	0.16	0.25	1.40	1.99
(caps - xsga) / xrdp	(Capital Surplus-Share Premium Reserve - Selling, General and Administrative Expense) / Research Development - Prior	-0.94	-3.91	-0.19	-0.75	-2.95	-0.35
(csho - cshpri) / mrc2	(Common Shares Outstanding - Common Shares Used to Calculate Earnings Per Share - Basic) / Rental Commitments - Minimum - 5 Year Total	-0.93	-4.52	-0.21	-0.42	-2.14	0.91
(txw - xrd) / xad	(Excise Taxes - Research and Development Expense) / Advertising Expense	0.93	2.52	0.13	-0.18	-0.56	-2.07
(esubc - reuna) / xrdp	(Equity in Net Loss - Earnings - Retained Earnings - Unadjusted) / Research Development - Prior	-0.93	-3.28	-0.17	-0.43	-1.95	-2.35
(ret6 - vol) / xad	(6m Past Return - 1y Return Volatility) / Advertising Expense	0.93	4.18	0.19	0.78	3.36	2.84
(csho - mrc2) / cshpri	(Common Shares Outstanding - Rental Commitments - Minimum - 2nd Year) / Common Shares Used to Calculate Earnings Per Share - Basic	-0.93	-3.86	-0.18	-0.21	-1.17	-1.67
(txdi - xrdp) / dv	(Income Taxes - Deferred - Research Development - Prior) / Cash Dividends (Cash Flow)	-0.93	-3.30	-0.16	-1.33	-5.41	1.42
(ret6 - vol) / act	(6m Past Return - 1y Return Volatility) / Current Assets - Total	0.93	4.15	0.18	0.55	2.32	0.13
(csho - ret6) / cshpri	(Common Shares Outstanding - 6m Past Return) / Common Shares Used to Calculate Earnings Per Share - Basic	-0.92	-5.41	-0.24	-0.30	-1.73	1.96
(aqs - ret6) / txs	(Acquisitions - Sales Contribution - 6m Past Return) / Income Taxes - State	-0.92	-4.69	-0.22	-0.40	-2.03	1.45
(dvpa - ret6) / cshr	(Preferred Dividends in Arrears - 6m Past Return) / Common-Ordinary Shareholders	-0.92	-4.07	-0.19	-0.45	-2.12	-1.16
(csho - mrc3) / cshpri	(Common Shares Outstanding - Rental Commitments - Minimum - 3rd Year) / Common Shares Used to Calculate Earnings Per Share - Basic	-0.92	-3.63	-0.17	-0.18	-0.97	-1.38
(ret6 - vol) / rectr	(6m Past Return - 1y Return Volatility) / Receivables - Trade	0.92	4.12	0.19	0.65	2.84	2.05
(xpp - dvolume) / lct	(Prepaid Expenses - Dollar Traded Volume) / Current Liabilities - Total	0.91	3.48	0.15	0.21	1.17	2.61
(chech - xsga) / lse	(Cash and Cash Equivalents - Increase-(Decrease) - Selling, General and Administrative Expense) / Liabilities and Stockholders Equity - Total	-0.91	-4.34	-0.22	-0.42	-2.17	-1.35
(chech - xsga) / at	(Cash and Cash Equivalents - Increase-(Decrease) - Selling, General and Administrative Expense) / Assets - Total	-0.91	-4.34	-0.22	-0.42	-2.17	-1.35
(aqs - xrdp) / cstk	(Acquisitions - Sales Contribution - Research Development - Prior) / Common-Ordinary Stock (Capital)	-0.91	-3.33	-0.17	-1.13	-3.86	-0.86
(ret6 - vol) / cshr	(6m Past Return - 1y Return Volatility) / Common-Ordinary Shareholders	0.91	4.49	0.21	0.52	2.79	1.59
(csho - mrc1) / cshpri	(Common Shares Outstanding - Rental Commitments - Minimum - 1st Year) / Common Shares Used to Calculate Earnings Per Share - Basic	-0.91	-3.94	-0.18	-0.18	-1.03	-1.79
(nopi - xrdp) / dv	(Nonoperating Income (Expense) - Research Development - Prior) / Cash Dividends (Cash Flow)	-0.91	-3.73	-0.18	-1.14	-4.68	2.77
(sstk - ret9) / xad	(Sale of Common and Preferred Stock - 9m Past Return) / Advertising Expense	-0.91	-3.77	-0.17	-0.19	-0.77	-2.10
(txdc - xrdp) / dv	(Deferred Taxes (Cash Flow) - Research Development - Prior) / Cash Dividends (Cash Flow)	-0.91	-3.11	-0.15	-1.17	-4.43	1.84
(lct - recd) / dvolume	(Current Liabilities - Total - Receivables - Estimated Doubtful) / Dollar Traded Volume	0.90	3.14	0.14	0.13	0.61	2.05
(aqs - ret3) / dvc	(Acquisitions - Sales Contribution - 3m Past Return) / Dividends Common-Ordinary	-0.90	-4.99	-0.23	-0.46	-2.69	1.83
(revt - tstkc) / csho	(Revenue - Total - Treasury Stock - Common) / Common Shares Outstanding	0.90	3.35	0.17	0.15	0.85	-1.19
(sale - tstkc) / csho	(Sales-Turnover (Net) - Treasury Stock - Common) / Common Shares Outstanding	0.90	3.35	0.17	0.15	0.85	-1.19
(dvpa - ret6) / xopr	(Preferred Dividends in Arrears - 6m Past Return) / Operating Expenses - Total	-0.90	-3.84	-0.18	-0.48	-2.08	0.33
(dvpa - ret3) / cstk	(Preferred Dividends in Arrears - 3m Past Return) / Common-Ordinary Stock (Capital)	-0.90	-4.17	-0.19	-0.79	-3.51	2.92
(ret6 - vol) / xsga	(6m Past Return - 1y Return Volatility) / Selling, General and Administrative Expense	0.90	4.17	0.18	0.54	2.47	-0.29
(intc - ret6) / xad	(Interest Capitalized - 6m Past Return) / Advertising Expense	-0.90	-3.19	-0.15	-0.83	-3.17	3.03
(invfg - reuna) / xrdp	(Inventories - Finished Goods - Retained Earnings - Unadjusted) / Research Development - Prior	-0.90	-3.19	-0.16	-0.35	-1.64	-1.77
(dvpa - xrd) / xad	(Preferred Dividends in Arrears - Research and Development Expense) / Advertising Expense	0.90	2.88	0.14	-0.06	-0.21	0.61
(aqs - xrd) / dv	(Acquisitions - Sales Contribution - Research and Development Expense) / Cash Dividends (Cash Flow)	-0.90	-3.74	-0.18	-0.86	-3.42	2.74
(icapt - xrdp) / xsga	(Invested Capital - Total - Research Development - Prior) / Selling, General and Administrative Expense	-0.90	-3.94	-0.19	-0.88	-3.92	2.34
(ret6 - vol) / mrc2	(6m Past Return - 1y Return Volatility) / Rental Commitments - Minimum - 2nd Year	0.89	4.28	0.20	0.59	2.87	0.76
(idit - xrent) / invfg	(Interest and Related Income - Total - Rental Expense) / Inventories - Finished Goods	-0.89	-3.29	-0.15	-0.09	-0.37	-3.14
(sstk - xrdp) / dv	(Sale of Common and Preferred Stock - Research Development - Prior) / Cash Dividends (Cash Flow)	-0.89	-3.77	-0.18	-0.71	-2.99	2.92

Table A3: Top 50 strategies by average returns t -statistic

		Mean	t_μ	SR	α	t_α	t_λ
(csho - ret6) / cshpri	(Common Shares Outstanding - 6m Past Return) / Common Shares Used to Calculate Earnings Per Share - Basic	-0.92	-5.41	-0.24	-0.30	-1.73	1.96
(csho - ret3) / cshpri	(Common Shares Outstanding - 3m Past Return) / Common Shares Used to Calculate Earnings Per Share - Basic	-0.89	-5.03	-0.22	-0.23	-1.38	3.07
(dd4 - dltis) / ppeg	(Debt - Due in 4th Year - Long-Term Debt - Issuance) / Property, Plant and Equipment - Total (Gross)	0.59	5.01	0.23	0.52	4.73	1.45
(aqs - ret3) / dvc	(Acquisitions - Sales Contribution - 3m Past Return) / Dividends Common-Ordinary	-0.90	-4.99	-0.23	-0.46	-2.69	1.83
(csho - ret9) / cshpri	(Common Shares Outstanding - 9m Past Return) / Common Shares Used to Calculate Earnings Per Share - Basic	-0.83	-4.97	-0.22	-0.37	-2.21	1.90
(aqs - ret3) / dv	(Acquisitions - Sales Contribution - 3m Past Return) / Cash Dividends (Cash Flow)	-0.83	-4.82	-0.22	-0.40	-2.54	1.55
(aqs - lct) / size	(Acquisitions - Sales Contribution - Current Liabilities - Total) / Market Capitalization	-0.84	-4.75	-0.22	-0.46	-2.89	-2.24
(aqs - ret6) / txs	(Acquisitions - Sales Contribution - 6m Past Return) / Income Taxes - State	-0.92	-4.69	-0.22	-0.40	-2.03	1.45
(recco - ret6) / dv	(Receivables - Current - Other - 6m Past Return) / Cash Dividends (Cash Flow)	-0.81	-4.68	-0.21	-0.56	-3.33	-1.56
(msa - ret3) / dv	(Marketable Securities Adjustment - 3m Past Return) / Cash Dividends (Cash Flow)	-0.79	-4.68	-0.22	-0.37	-2.12	-1.43
(ch - sstk) / txt	(Cash - Sale of Common and Preferred Stock) / Income Taxes - Total	0.65	4.67	0.21	0.32	2.05	-0.49
(cshpri - do) / csho	(Common Shares Used to Calculate Earnings Per Share - Basic - Discontinued Operations) / Common Shares Outstanding	0.57	4.66	0.21	0.21	1.76	2.82
(np - ret3) / dv	(Notes Payable - Short-Term Borrowings - 3m Past Return) / Cash Dividends (Cash Flow)	-0.73	-4.65	-0.20	-0.46	-3.03	-2.01
(dd4 - dltis) / ch	(Debt - Due in 4th Year - Long-Term Debt - Issuance) / Cash	0.51	4.64	0.21	0.38	3.29	-1.71
(np - ret3) / dvt	(Notes Payable - Short-Term Borrowings - 3m Past Return) / Dividends - Total	-0.72	-4.64	-0.20	-0.42	-2.85	-0.34
(rea - ret3) / dv	(Retained Earnings - Restatement - 3m Past Return) / Cash Dividends (Cash Flow)	-0.76	-4.62	-0.20	-0.37	-2.35	0.45
(ap - cshpri) / xrd	(Accounts Payable - Trade - Common Shares Used to Calculate Earnings Per Share - Basic) / Research and Development Expense	0.73	4.57	0.20	0.60	3.72	2.73
(aqc - ret3) / dvt	(Acquisitions - 3m Past Return) / Dividends - Total	-0.72	-4.57	-0.20	-0.53	-3.21	0.78
(dvc - ret3) / dv	(Dividends Common-Ordinary - 3m Past Return) / Cash Dividends (Cash Flow)	-0.76	-4.57	-0.20	-0.35	-2.15	-2.01
(ap - csho) / xrd	(Accounts Payable - Trade - Common Shares Outstanding) / Research and Development Expense	0.75	4.56	0.20	0.60	3.61	2.47
(dvc - ret3) / dvt	(Dividends Common-Ordinary - 3m Past Return) / Dividends - Total	-0.75	-4.56	-0.20	-0.39	-2.44	0.31
(intc - ret3) / dvt	(Interest Capitalized - 3m Past Return) / Dividends - Total	-0.79	-4.55	-0.21	-0.48	-2.78	-1.68
(recco - ret3) / dv	(Receivables - Current - Other - 3m Past Return) / Cash Dividends (Cash Flow)	-0.76	-4.55	-0.20	-0.63	-3.75	-1.22
(msa - ret3) / dvc	(Marketable Securities Adjustment - 3m Past Return) / Dividends Common-Ordinary	-0.76	-4.55	-0.21	-0.24	-1.43	0.34
(dc - ret3) / dv	(Deferred Charges - 3m Past Return) / Cash Dividends (Cash Flow)	-0.81	-4.54	-0.21	-0.55	-3.06	-2.09
(dvpa - ret3) / dvt	(Preferred Dividends in Arrears - 3m Past Return) / Dividends - Total	-0.78	-4.53	-0.21	-0.44	-2.65	-1.92
(aqs - ret3) / dvt	(Acquisitions - Sales Contribution - 3m Past Return) / Dividends - Total	-0.77	-4.52	-0.21	-0.37	-2.32	0.76
(csho - cshpri) / mrc	(Common Shares Outstanding - Common Shares Used to Calculate Earnings Per Share - Basic) / Rental Commitments - Minimum - 5 Year Total	-0.93	-4.52	-0.21	-0.42	-2.14	0.91
(aqs - ret3) / txs	(Acquisitions - Sales Contribution - 3m Past Return) / Income Taxes - State	-0.87	-4.51	-0.21	-0.49	-2.44	1.56
(dd4 - dltis) / ap	(Debt - Due in 4th Year - Long-Term Debt - Issuance) / Accounts Payable - Trade	0.49	4.51	0.21	0.35	3.36	0.66
(csho - txr) / cshpri	(Common Shares Outstanding - Income Tax Refund) / Common Shares Used to Calculate Earnings Per Share - Basic	-0.73	-4.51	-0.20	-0.21	-1.36	0.19
(dd3 - dltis) / ppeg	(Debt - Due in 3rd Year - Long-Term Debt - Issuance) / Property, Plant and Equipment - Total (Gross)	0.52	4.50	0.21	0.45	3.85	2.07
(ret6 - vol) / cshr	(6m Past Return - 1y Return Volatility) / Common-Ordinary Shareholders	0.91	4.49	0.21	0.52	2.79	1.59
(dd4 - dltis) / ppevb	(Debt - Due in 4th Year - Long-Term Debt - Issuance) / Property, Plant, and Equipment - Ending Balance (Schedule V)	0.54	4.49	0.21	0.48	4.14	0.35
(csho - ret1) / cshpri	(Common Shares Outstanding - 1m Past Return) / Common Shares Used to Calculate Earnings Per Share - Basic	-0.76	-4.49	-0.20	-0.16	-1.07	3.86
(dd4 - dltis) / gp	(Debt - Due in 4th Year - Long-Term Debt - Issuance) / Gross Profit (Loss)	0.54	4.48	0.20	0.48	3.96	-3.63
(lco - sstk) / cshr	(Current Liabilities - Other - Total - Sale of Common and Preferred Stock) / Common-Ordinary Shareholders	0.63	4.48	0.21	0.41	3.00	-2.72
(csho - cstke) / cshpri	(Common Shares Outstanding - Common Stock Equivalents - Dollar Savings) / Common Shares Used to Calculate Earnings Per Share - Basic	-0.77	-4.47	-0.20	-0.20	-1.31	2.93
(ret6 - vol) / revt	(6m Past Return - 1y Return Volatility) / Revenue - Total	0.89	4.45	0.20	0.41	2.03	0.58
(ret6 - vol) / sale	(6m Past Return - 1y Return Volatility) / Sales-Turnover (Net)	0.89	4.45	0.20	0.41	2.03	0.58
(dd5 - dltis) / ch	(Debt - Due in 5th Year - Long-Term Debt - Issuance) / Cash	0.46	4.44	0.20	0.40	3.60	-1.14
(csho - idit) / cshpri	(Common Shares Outstanding - Interest and Related Income - Total) / Common Shares Used to Calculate Earnings Per Share - Basic	-0.83	-4.44	-0.20	-0.67	-3.63	1.55
(ret6 - vol) / dp	(6m Past Return - 1y Return Volatility) / Depreciation and Amortization	0.85	4.43	0.20	0.82	4.31	2.46
(ch - ds) / dvolume	(Cash - Debt-Subordinated) / Dollar Traded Volume	0.55	4.42	0.19	0.50	4.01	-0.15
(cstk - epsfx) / dvt	(Common-Ordinary Stock (Capital) - Earnings Per Share (Diluted) - Excluding Extraordinary Items) / Dividends - Total	-0.61	-4.41	-0.19	-0.31	-2.35	2.00
(caps - chech) / xad	(Capital Surplus-Share Premium Reserve - Cash and Cash Equivalents - Increase-(Decrease)) / Advertising Expense	-1.07	-4.40	-0.23	-0.55	-2.27	-1.21
(epsfx - sstk) / txp	(Earnings Per Share (Diluted) - Excluding Extraordinary Items - Sale of Common and Preferred Stock) / Income Taxes Payable	0.50	4.39	0.19	0.26	2.11	-2.15
(aqc - ret6) / dv	(Acquisitions - 6m Past Return) / Cash Dividends (Cash Flow)	-0.70	-4.39	-0.19	-0.39	-2.36	-0.35
(tstk - ret3) / dv	(Treasury Stock - Total (All Capital) - 3m Past Return) / Cash Dividends (Cash Flow)	-0.77	-4.39	-0.19	-0.62	-3.83	1.05
(csho - cshpri) / mrc1	(Common Shares Outstanding - Common Shares Used to Calculate Earnings Per Share - Basic) / Rental Commitments - Minimum - 1st Year	-1.05	-4.38	-0.20	-0.37	-1.79	1.18

Table A4: Top 50 strategies by Sharpe ratio

		Mean	t_μ	SR	α	t_α	t_λ
(csho - ret6) / cshpri	(Common Shares Outstanding - 6m Past Return) / Common Shares Used to Calculate Earnings Per Share - Basic	-0.92	-5.41	-0.24	-0.30	-1.73	1.96
(dd4 - dltis) / ppeg	(Debt - Due in 4th Year - Long-Term Debt - Issuance) / Property, Plant and Equipment - Total (Gross)	0.59	5.01	0.23	0.52	4.73	1.45
(caps - chech) / xad	(Capital Surplus-Share Premium Reserve - Cash and Cash Equivalents - Increase-(Decrease)) / Advertising Expense	-1.07	-4.40	-0.23	-0.55	-2.27	-1.21
(aqs - ret3) / dvc	(Acquisitions - Sales Contribution - 3m Past Return) / Dividends Common-Ordinary	-0.90	-4.99	-0.23	-0.46	-2.69	1.83
(chech - xsga) / lse	(Cash and Cash Equivalents - Increase-(Decrease) - Selling, General and Administrative Expense) / Liabilities and Stockholders Equity - Total	-0.91	-4.34	-0.22	-0.42	-2.17	-1.35
(chech - xsga) / at	(Cash and Cash Equivalents - Increase-(Decrease) - Selling, General and Administrative Expense) / Assets - Total	-0.91	-4.34	-0.22	-0.42	-2.17	-1.35
(csho - ret3) / cshpri	(Common Shares Outstanding - 3m Past Return) / Common Shares Used to Calculate Earnings Per Share - Basic	-0.89	-5.03	-0.22	-0.23	-1.38	3.07
(aqs - ret3) / dv	(Acquisitions - Sales Contribution - 3m Past Return) / Cash Dividends (Cash Flow)	-0.83	-4.82	-0.22	-0.40	-2.54	1.55
(aqs - lct) / size	(Acquisitions - Sales Contribution - Current Liabilities - Total) / Market Capitalization	-0.84	-4.75	-0.22	-0.46	-2.89	-2.24
(dvpa - recta) / ivao	(Preferred Dividends in Arrears - Retained Earnings - Cumulative Translation Adjustment) / Investment and Advances - Other	-0.69	-4.30	-0.22	-0.51	-3.10	1.31
(csho - ret9) / cshpri	(Common Shares Outstanding - 9m Past Return) / Common Shares Used to Calculate Earnings Per Share - Basic	-0.83	-4.97	-0.22	-0.37	-2.21	1.90
(aqs - ret6) / txs	(Acquisitions - Sales Contribution - 6m Past Return) / Income Taxes - State	-0.92	-4.69	-0.22	-0.40	-2.03	1.45
(msa - ret3) / dv	(Marketable Securities Adjustment - 3m Past Return) / Cash Dividends (Cash Flow)	-0.79	-4.68	-0.22	-0.37	-2.12	-1.43
(dcvrs - recta) / ivch	(Debt - Senior Convertible - Retained Earnings - Cumulative Translation Adjustment) / Increase in Investments	-0.71	-4.16	-0.21	-0.33	-1.99	-0.07
(dd4 - dltis) / ch	(Debt - Due in 4th Year - Long-Term Debt - Issuance) / Cash	0.51	4.64	0.21	0.38	3.29	-1.71
(intc - ret3) / dvt	(Interest Capitalized - 3m Past Return) / Dividends - Total	-0.79	-4.55	-0.21	-0.48	-2.78	-1.68
(msa - ret3) / dvc	(Marketable Securities Adjustment - 3m Past Return) / Dividends Common-Ordinary	-0.76	-4.55	-0.21	-0.24	-1.43	0.34
(dc - ret3) / dv	(Deferred Charges - 3m Past Return) / Cash Dividends (Cash Flow)	-0.81	-4.54	-0.21	-0.55	-3.06	-2.09
(dvpa - ret3) / dvt	(Preferred Dividends in Arrears - 3m Past Return) / Dividends - Total	-0.78	-4.53	-0.21	-0.44	-2.65	-1.92
(dcvrs - recta) / ivao	(Debt - Senior Convertible - Retained Earnings - Cumulative Translation Adjustment) / Investment and Advances - Other	-0.67	-4.10	-0.21	-0.45	-2.71	1.57
(aqs - ret3) / txs	(Acquisitions - Sales Contribution - 3m Past Return) / Income Taxes - State	-0.87	-4.51	-0.21	-0.49	-2.44	1.56
(ret6 - vol) / cshr	(6m Past Return - 1y Return Volatility) / Common-Ordinary Shareholders	0.91	4.49	0.21	0.52	2.79	1.59
(lco - sstk) / cshr	(Current Liabilities - Other - Total - Sale of Common and Preferred Stock) / Common-Ordinary Shareholders	0.63	4.48	0.21	0.41	3.00	-2.72
(aqs - ret3) / dvt	(Acquisitions - Sales Contribution - 3m Past Return) / Dividends - Total	-0.77	-4.52	-0.21	-0.37	-2.32	0.76
(chech - mrc2) / txfed	(Cash and Cash Equivalents - Increase-(Decrease) - Rental Commitments - Minimum - 2nd Year) / Income Taxes - Federal	-0.78	-3.98	-0.21	-0.50	-2.51	-3.28
(csho - cshpri) / mrct	(Common Shares Outstanding - Common Shares Used to Calculate Earnings Per Share - Basic) / Rental Commitments - Minimum - 5 Year Total	-0.93	-4.52	-0.21	-0.42	-2.14	0.91
(recco - ret6) / dv	(Receivables - Current - Other - 6m Past Return) / Cash Dividends (Cash Flow)	-0.81	-4.68	-0.21	-0.56	-3.33	-1.56
(aqs - xrdp) / dvt	(Acquisitions - Sales Contribution - Research Development - Prior) / Dividends - Total	-1.06	-4.04	-0.21	-1.03	-3.72	3.70
(dd4 - dltis) / ap	(Debt - Due in 4th Year - Long-Term Debt - Issuance) / Accounts Payable - Trade	0.49	4.51	0.21	0.35	3.36	0.66
(ch - sstk) / txt	(Cash - Sale of Common and Preferred Stock) / Income Taxes - Total	0.65	4.67	0.21	0.32	2.05	-0.49
(dd3 - dltis) / ppeg	(Debt - Due in 3rd Year - Long-Term Debt - Issuance) / Property, Plant and Equipment - Total (Gross)	0.52	4.50	0.21	0.45	3.85	2.07
(chech - mrct) / txfed	(Cash and Cash Equivalents - Increase-(Decrease) - Rental Commitments - Minimum - 5 Year Total) / Income Taxes - Federal	-0.71	-3.96	-0.21	-0.47	-2.48	-2.29
(cshpri - do) / csho	(Common Shares Used to Calculate Earnings Per Share - Basic - Discontinued Operations) / Common Shares Outstanding	0.57	4.66	0.21	0.21	1.76	2.82
(dd4 - dltis) / ppevb	(Debt - Due in 4th Year - Long-Term Debt - Issuance) / Property, Plant, and Equipment - Ending Balance (Schedule V)	0.54	4.49	0.21	0.48	4.14	0.35
(dm - xsga) / ebitda	(Debt - Mortgages Other Secured - Selling, General and Administrative Expense) / Earnings Before Interest	-0.67	-4.08	-0.20	-0.61	-3.01	3.14
(dm - xsga) / oibdp	(Debt - Mortgages Other Secured - Selling, General and Administrative Expense) / Operating Income Before Depreciation	-0.67	-4.08	-0.20	-0.61	-3.01	3.14
(ivch - recta) / txfed	(Increase in Investments - Retained Earnings - Cumulative Translation Adjustment) / Income Taxes - Federal	-0.68	-4.01	-0.20	-0.31	-2.11	1.13
(np - ret3) / dv	(Notes Payable - Short-Term Borrowings - 3m Past Return) / Cash Dividends (Cash Flow)	-0.73	-4.65	-0.20	-0.46	-3.03	-2.01
(dd4 - dltis) / gp	(Debt - Due in 4th Year - Long-Term Debt - Issuance) / Gross Profit (Loss)	0.54	4.48	0.20	0.48	3.96	-3.63
(np - ret3) / dvt	(Notes Payable - Short-Term Borrowings - 3m Past Return) / Dividends - Total	-0.72	-4.64	-0.20	-0.42	-2.85	-0.34
(chech - xsga) / ebitda	(Cash and Cash Equivalents - Increase-(Decrease) - Selling, General and Administrative Expense) / Earnings Before Interest	-0.71	-4.00	-0.20	-0.26	-1.45	1.36
(chech - xsga) / oibdp	(Cash and Cash Equivalents - Increase-(Decrease) - Selling, General and Administrative Expense) / Operating Income Before Depreciation	-0.71	-4.00	-0.20	-0.26	-1.45	1.36
(dcvt - recta) / ivch	(Debt - Convertible - Retained Earnings - Cumulative Translation Adjustment) / Increase in Investments	-0.69	-4.00	-0.20	-0.41	-2.61	0.09
(chech - xrent) / txfed	(Cash and Cash Equivalents - Increase-(Decrease) - Rental Expense) / Income Taxes - Federal	-0.74	-3.99	-0.20	-0.39	-1.99	-2.25
(rea - ret3) / dv	(Retained Earnings - Restatement - 3m Past Return) / Cash Dividends (Cash Flow)	-0.76	-4.62	-0.20	-0.37	-2.35	0.45
(dd5 - dltis) / ch	(Debt - Due in 5th Year - Long-Term Debt - Issuance) / Cash	0.46	4.44	0.20	0.40	3.60	-1.14
(dvpa - ret3) / dvc	(Preferred Dividends in Arrears - 3m Past Return) / Dividends Common-Ordinary	-0.76	-4.38	-0.20	-0.42	-2.51	1.11
(ret6 - vol) / xrdp	(6m Past Return - 1y Return Volatility) / Research Development - Prior	0.85	4.14	0.20	0.96	4.81	0.64
(chech - sstk) / dpvieb	(Cash and Cash Equivalents - Increase-(Decrease) - Sale of Common and Preferred Stock) / Depreciation (Accumulated) - Ending Balance (Schedule VI)	0.69	3.96	0.20	0.31	1.84	-1.96
(epspx - nopi) / rectr	(Earnings Per Share (Basic) - Excluding Extraordinary Items - Nonoperating Income (Expense)) / Receivables - Trade	0.60	4.36	0.20	0.23	1.74	1.18

Table A5: Top 50 strategies by Alpha

		Mean	t_μ	SR	α	t_α	t_λ
(itcb - itci) / invwip	(Investment Tax Credit (Balance Sheet) - Investment Tax Credit (Income Account)) / Inventories - Work In Process	-0.79	-2.24	-0.11	-1.43	-4.30	2.32
(sppe - xrdp) / dvt	(Sale of Property - Research Development - Prior) / Dividends - Total	-0.76	-2.49	-0.12	-1.42	-5.47	0.63
(rectr - xrd) / lt	(Receivables - Trade - Research and Development Expense) / Liabilities - Total	-0.65	-2.32	-0.11	-1.41	-5.41	3.69
(dltr - xrdp) / dpact	(Long-Term Debt - Reduction - Research Development - Prior) / Depreciation, Depletion and Amortization (Accumulated)	-0.57	-1.89	-0.09	-1.39	-5.35	1.83
(xrdp - vol) / invt	(Research Development - Prior - 1y Return Volatility) / Inventories - Total	0.18	0.52	0.03	1.39	4.79	1.75
(recd - xrdp) / xint	(Receivables - Estimated Doubtful - Research Development - Prior) / Interest and Related Expense - Total	-0.51	-1.59	-0.08	-1.37	-4.55	-1.65
(aqi - xrdp) / xint	(Acquisitions - Income Contribution - Research Development - Prior) / Interest and Related Expense - Total	-0.25	-0.68	-0.03	-1.35	-4.46	-3.02
(recta - xrd) / dpvieb	(Retained Earnings - Cumulative Translation Adjustment - Research and Development Expense) / Depreciation (Accumulated) - Ending Balance (Schedule VI)	-0.26	-0.64	-0.03	-1.35	-4.07	2.15
(aqi - xrdp) / cstk	(Acquisitions - Income Contribution - Research Development - Prior) / Common-Ordinary Stock (Capital)	-0.39	-1.20	-0.06	-1.34	-4.95	0.32
(aqs - xrdp) / rectr	(Acquisitions - Sales Contribution - Research Development - Prior) / Receivables - Trade	-0.61	-1.86	-0.09	-1.34	-4.79	-1.15
(xint - xrdp) / dvc	(Interest and Related Expense - Total - Research Development - Prior) / Dividends Common-Ordinary	-0.80	-3.08	-0.15	-1.34	-5.52	3.48
(dltr - xrdp) / dpvieb	(Long-Term Debt - Reduction - Research Development - Prior) / Depreciation (Accumulated) - Ending Balance (Schedule VI)	-0.45	-1.55	-0.08	-1.34	-5.20	0.62
(cstkev - xrdp) / dv	(Common Stock-Carrying Value - Research Development - Prior) / Cash Dividends (Cash Flow)	-0.71	-2.49	-0.13	-1.33	-5.62	1.80
(txdi - xrdp) / dv	(Income Taxes - Deferred - Research Development - Prior) / Cash Dividends (Cash Flow)	-0.93	-3.30	-0.16	-1.33	-5.41	1.42
(ds - xrdp) / dd1	(Debt-Subordinated - Research Development - Prior) / Long-Term Debt Due in One Year	-0.60	-2.26	-0.11	-1.32	-5.64	-0.15
(xrdp - xsga) / xint	(Research Development - Prior - Selling, General and Administrative Expense) / Interest and Related Expense - Total	-0.64	-2.17	-0.11	-1.32	-5.42	-2.92
(aqs - xrd) / xint	(Acquisitions - Sales Contribution - Research and Development Expense) / Interest and Related Expense - Total	-0.61	-1.99	-0.09	-1.32	-4.81	-1.92
(recta - xrdp) / dpvieb	(Retained Earnings - Cumulative Translation Adjustment - Research Development - Prior) / Depreciation (Accumulated) - Ending Balance (Schedule VI)	-0.40	-1.06	-0.05	-1.32	-4.14	1.56
(dvc - itci) / dltp	(Dividends Common-Ordinary - Investment Tax Credit (Income Account)) / Long-Term Debt - Tied to Prime	-0.81	-2.61	-0.12	-1.32	-5.10	-0.68
(aqs - xrdp) / rect	(Acquisitions - Sales Contribution - Research Development - Prior) / Receivables - Total	-0.60	-1.84	-0.09	-1.31	-4.75	-1.10
(tstkn - xrdp) / cstk	(Treasury Stock - Number of Common Shares - Research Development - Prior) / Common-Ordinary Stock (Capital)	-0.43	-1.49	-0.07	-1.31	-5.44	0.33
(dclo - dpvieb) / xrdp	(Debt - Capitalized Lease Obligations - Depreciation (Accumulated) - Ending Balance (Schedule VI)) / Research Development - Prior	0.42	1.11	0.05	1.31	4.32	-4.28
(ds - xrdp) / cstk	(Debt-Subordinated - Research Development - Prior) / Common-Ordinary Stock (Capital)	-0.52	-1.91	-0.09	-1.30	-5.79	1.60
(tstkc - xrd) / intan	(Treasury Stock - Common - Research and Development Expense) / Intangible Assets - Total	-0.75	-2.70	-0.14	-1.30	-4.93	3.28
(tstkc - xrdp) / intan	(Treasury Stock - Common - Research Development - Prior) / Intangible Assets - Total	-0.75	-2.69	-0.14	-1.30	-4.86	2.96
(ds - xrd) / dpvieb	(Debt-Subordinated - Research and Development Expense) / Depreciation (Accumulated) - Ending Balance (Schedule VI)	-0.10	-0.31	-0.01	-1.30	-5.64	3.65
(txp - xrdp) / dvc	(Income Taxes Payable - Research Development - Prior) / Dividends Common-Ordinary	-0.84	-3.25	-0.16	-1.30	-5.54	2.19
(dltp - xacc) / dpvieb	(Long-Term Debt - Tied to Prime - Accrued Expenses) / Depreciation (Accumulated) - Ending Balance (Schedule VI)	-0.49	-1.95	-0.09	-1.29	-5.83	2.05
(invt - xrd) / xrdp	(Inventories - Total - Research and Development Expense) / Research Development - Prior	-0.14	-0.41	-0.02	-1.29	-5.37	4.07
(mii - xad) / cstk	(Noncontrolling Interest (Income Account) - Advertising Expense) / Common-Ordinary Stock (Capital)	-0.54	-2.00	-0.09	-1.29	-5.14	1.40
(cstkev - xrdp) / dvt	(Common Stock-Carrying Value - Research Development - Prior) / Dividends - Total	-0.64	-2.26	-0.12	-1.29	-5.45	1.13
(rectr - xrdp) / idit	(Receivables - Trade - Research Development - Prior) / Interest and Related Income - Total	-0.26	-0.68	-0.03	-1.29	-4.44	-0.84
(ivaq - xrdp) / cstk	(Investment and Advances - Equity - Research Development - Prior) / Common-Ordinary Stock (Capital)	-0.63	-2.07	-0.10	-1.28	-4.95	0.82
(ebitda - xrdp) / idit	(Earnings Before Interest - Research Development - Prior) / Interest and Related Income - Total	-0.20	-0.48	-0.02	-1.28	-4.25	-0.42
(oibdp - xrdp) / idit	(Operating Income Before Depreciation - Research Development - Prior) / Interest and Related Income - Total	-0.20	-0.48	-0.02	-1.28	-4.25	-0.42
(act - fca) / cstk	(Current Assets - Total - Foreign Exchange Income (Loss)) / Common-Ordinary Stock (Capital)	0.49	1.62	0.08	1.28	4.76	-2.03
(invwip - xrdp) / rectr	(Inventories - Work In Process - Research Development - Prior) / Receivables - Trade	-0.54	-1.47	-0.07	-1.27	-4.15	-1.74
(xint - xrdp) / dv	(Interest and Related Expense - Total - Research Development - Prior) / Cash Dividends (Cash Flow)	-0.76	-2.87	-0.14	-1.27	-5.42	3.51
(am - tstk) / intan	(Amortization of Intangibles - Treasury Stock - Total (All Capital)) / Intangible Assets - Total	0.48	1.58	0.08	1.27	4.97	-0.63
(recta - xrdp) / dv	(Retained Earnings - Cumulative Translation Adjustment - Research Development - Prior) / Cash Dividends (Cash Flow)	-0.73	-2.39	-0.12	-1.27	-5.39	3.15
(am - tstkc) / intan	(Amortization of Intangibles - Treasury Stock - Common) / Intangible Assets - Total	0.47	1.56	0.08	1.27	4.96	-0.59
(epspx - xrdp) / dv	(Earnings Per Share (Basic) - Excluding Extraordinary Items - Research Development - Prior) / Cash Dividends (Cash Flow)	-0.60	-2.14	-0.10	-1.27	-5.31	2.64
(aqs - xrdp) / dd2	(Acquisitions - Sales Contribution - Research Development - Prior) / Debt - Due in 2nd Year	-0.77	-2.93	-0.15	-1.27	-4.99	1.32
(txdi - xrdp) / lct	(Income Taxes - Deferred - Research Development - Prior) / Current Liabilities - Total	-0.69	-1.98	-0.10	-1.27	-5.13	2.21
(cstkev - xrd) / dv	(Common Stock-Carrying Value - Research and Development Expense) / Cash Dividends (Cash Flow)	-0.63	-2.15	-0.11	-1.27	-5.29	2.88
(xrdp - ret6) / invt	(Research Development - Prior - 6m Past Return) / Inventories - Total	0.12	0.36	0.02	1.27	4.60	1.04
(txdi - xrdp) / dvc	(Income Taxes - Deferred - Research Development - Prior) / Dividends Common-Ordinary	-0.82	-2.93	-0.14	-1.26	-4.91	1.02
(txdi - xrdp) / ppeveb	(Income Taxes - Deferred - Research Development - Prior) / Property, Plant, and Equipment - Ending Balance (Schedule V)	-0.57	-1.75	-0.09	-1.26	-4.69	0.91
(txdi - xrdp) / dpvieb	(Income Taxes - Deferred - Research Development - Prior) / Depreciation (Accumulated) - Ending Balance (Schedule VI)	-0.48	-1.43	-0.07	-1.26	-4.45	1.19
(rectr - xrdp) / revt	(Receivables - Trade - Research Development - Prior) / Revenue - Total	-0.74	-2.46	-0.12	-1.26	-4.68	-0.26

Table A6: Top 50 strategies by alpha t -statistic

		Mean	t_μ	SR	α	t_α	t_λ
(ch - ds) / lifr	(Cash - Debt-Subordinated) / LIFO Reserve	0.37	1.89	0.09	0.95	7.36	0.33
(ch - dd2) / lse	(Cash - Debt - Due in 2nd Year) / Liabilities and Stockholders Equity - Total	0.37	1.43	0.07	1.26	7.24	-4.01
(ch - dd2) / at	(Cash - Debt - Due in 2nd Year) / Assets - Total	0.37	1.43	0.07	1.26	7.24	-4.01
(ch - xpp) / lse	(Cash - Prepaid Expenses) / Liabilities and Stockholders Equity - Total	0.55	2.65	0.12	1.14	7.21	-2.36
(ch - xpp) / at	(Cash - Prepaid Expenses) / Assets - Total	0.55	2.65	0.12	1.14	7.21	-2.36
(ch - dd2) / icapt	(Cash - Debt - Due in 2nd Year) / Invested Capital - Total	0.43	1.79	0.08	1.21	7.05	-4.07
(ch - dd1) / lse	(Cash - Long-Term Debt Due in One Year) / Liabilities and Stockholders Equity - Total	0.33	1.37	0.06	1.18	6.94	-3.51
(ch - dd1) / at	(Cash - Long-Term Debt Due in One Year) / Assets - Total	0.33	1.37	0.06	1.18	6.94	-3.51
(ch - xpp) / lt	(Cash - Prepaid Expenses) / Liabilities - Total	0.48	2.09	0.09	1.18	6.94	-0.77
(che - dlc) / cstk	(Cash and Short-Term Investments - Debt in Current Liabilities - Total) / Common-Ordinary Stock (Capital)	0.35	1.86	0.08	1.04	6.90	1.42
(dlc - xacc) / ppegt	(Debt in Current Liabilities - Total - Accrued Expenses) / Property, Plant and Equipment - Total (Gross)	-0.36	-1.87	-0.08	-0.94	-6.75	2.16
(ch - dd1) / cstk	(Cash - Long-Term Debt Due in One Year) / Common-Ordinary Stock (Capital)	0.29	1.47	0.06	1.00	6.75	-1.56
(ch - dd1) / icapt	(Cash - Long-Term Debt Due in One Year) / Invested Capital - Total	0.26	1.25	0.05	0.96	6.75	-3.07
(ch - ds) / recco	(Cash - Debt-Subordinated) / Receivables - Current - Other	0.32	1.85	0.08	0.83	6.69	-0.71
(ch - dcvt) / icapt	(Cash - Debt - Convertible) / Invested Capital - Total	0.51	3.36	0.15	0.92	6.68	-3.23
(ch - dcvt) / lse	(Cash - Debt - Convertible) / Liabilities and Stockholders Equity - Total	0.47	2.58	0.11	1.10	6.68	-3.21
(ch - dcvt) / at	(Cash - Debt - Convertible) / Assets - Total	0.47	2.58	0.11	1.10	6.68	-3.21
(ch - dd3) / cstk	(Cash - Debt - Due in 3rd Year) / Common-Ordinary Stock (Capital)	0.47	2.32	0.11	1.07	6.66	-1.17
(ch - dd3) / lse	(Cash - Debt - Due in 3rd Year) / Liabilities and Stockholders Equity - Total	0.39	1.53	0.07	1.16	6.65	-4.03
(ch - dd3) / at	(Cash - Debt - Due in 3rd Year) / Assets - Total	0.39	1.53	0.07	1.16	6.65	-4.03
(ch - sppe) / lse	(Cash - Sale of Property) / Liabilities and Stockholders Equity - Total	0.34	1.35	0.06	1.13	6.63	-4.34
(ch - sppe) / at	(Cash - Sale of Property) / Assets - Total	0.34	1.35	0.06	1.13	6.63	-4.34
(ch - xpr) / dltr	(Cash - Pension and Retirement Expense) / Long-Term Debt - Reduction	0.30	1.56	0.07	0.88	6.58	0.27
(ch - ds) / icapt	(Cash - Debt-Subordinated) / Invested Capital - Total	0.32	1.65	0.07	1.00	6.56	-2.49
(lcox - sppe) / tstkn	(Current Liabilities - Other - Sundry - Sale of Property) / Treasury Stock - Number of Common Shares	0.29	1.58	0.07	0.86	6.53	-0.23
(ch - dcvsb) / lse	(Cash - Debt - Subordinated Convertible) / Liabilities and Stockholders Equity - Total	0.37	1.93	0.09	1.05	6.52	-3.63
(ch - dcvsb) / at	(Cash - Debt - Subordinated Convertible) / Assets - Total	0.37	1.93	0.09	1.05	6.52	-3.63
(dd2 - xsga) / emp	(Debt - Due in 2nd Year - Selling, General and Administrative Expense) / Employees	-0.35	-1.89	-0.09	-0.99	-6.51	3.38
(ch - rectr) / revt	(Cash - Receivables - Trade) / Revenue - Total	0.38	1.43	0.07	1.19	6.51	0.38
(ch - rectr) / sale	(Cash - Receivables - Trade) / Sales-Turnover (Net)	0.38	1.43	0.07	1.19	6.51	0.38
(ch - dd3) / icapt	(Cash - Debt - Due in 3rd Year) / Invested Capital - Total	0.41	1.70	0.08	1.12	6.51	-4.14
(ch - dltr) / icapt	(Cash - Long-Term Debt - Reduction) / Invested Capital - Total	0.32	1.57	0.07	1.11	6.50	0.12
(np - xacc) / ppegt	(Notes Payable - Short-Term Borrowings - Accrued Expenses) / Property, Plant and Equipment - Total (Gross)	-0.44	-2.21	-0.10	-0.96	-6.47	3.49
(ch - ds) / cshpri	(Cash - Debt-Subordinated) / Common Shares Used to Calculate Earnings Per Share - Basic	0.44	3.62	0.16	0.75	6.46	-0.17
(ch - rectr) / lct	(Cash - Receivables - Trade) / Current Liabilities - Total	0.39	1.64	0.08	1.13	6.45	-2.89
(ch - txp) / dltr	(Cash - Income Taxes Payable) / Long-Term Debt - Reduction	0.35	1.67	0.07	0.91	6.45	-0.63
(ch - dd2) / cstk	(Cash - Debt - Due in 2nd Year) / Common-Ordinary Stock (Capital)	0.39	1.93	0.09	1.05	6.43	-1.03
(che - np) / cstk	(Cash and Short-Term Investments - Notes Payable - Short-Term Borrowings) / Common-Ordinary Stock (Capital)	0.37	1.95	0.09	0.96	6.43	1.04
(ch - xpr) / lt	(Cash - Pension and Retirement Expense) / Liabilities - Total	0.33	1.26	0.06	1.15	6.39	-2.16
(ch - rectr) / lct	(Cash - Receivables - Total) / Current Liabilities - Total	0.36	1.61	0.07	1.10	6.38	-1.16
(ch - ds) / lse	(Cash - Debt-Subordinated) / Liabilities and Stockholders Equity - Total	0.24	1.07	0.05	1.06	6.36	-3.30
(ch - ds) / at	(Cash - Debt-Subordinated) / Assets - Total	0.24	1.07	0.05	1.06	6.36	-3.30
(esubc - txdi) / dpvieb	(Equity in Net Loss - Earnings - Income Taxes - Deferred) / Depreciation (Accumulated) - Ending Balance (Schedule VI)	0.64	3.45	0.15	1.08	6.36	3.94
(ch - sppe) / icapt	(Cash - Sale of Property) / Invested Capital - Total	0.34	1.42	0.06	1.10	6.33	-3.88
(ch - emp) / lse	(Cash - Employees) / Liabilities and Stockholders Equity - Total	0.29	1.22	0.05	1.13	6.32	-3.65
(ch - emp) / at	(Cash - Employees) / Assets - Total	0.29	1.22	0.05	1.13	6.32	-3.65
(acox - np) / ppegt	(Current Assets - Other - Sundry - Notes Payable - Short-Term Borrowings) / Property, Plant and Equipment - Total (Net)	0.26	1.44	0.06	0.83	6.31	1.08
(dn - xacc) / ppegt	(Debt - Notes - Accrued Expenses) / Property, Plant and Equipment - Total (Gross)	-0.41	-1.96	-0.09	-1.10	-6.31	1.38
(ch - ds) / tstkc	(Cash - Debt-Subordinated) / Treasury Stock - Common	0.36	1.62	0.08	0.91	6.31	-0.40
(dlc - xacc) / dp	(Debt in Current Liabilities - Total - Accrued Expenses) / Depreciation and Amortization	-0.36	-2.39	-0.11	-0.75	-6.30	0.89

Table A7: Top 50 strategies by Fama–MacBeth t -statistic

		Mean	t_μ	SR	α	t_α	t_λ
(dv - price) / size	(Cash Dividends (Cash Flow) - Price) / Market Capitalization	-0.21	-1.24	-0.05	-0.08	-0.60	11.39
(reajo - xpr) / size	(Retained Earnings - Other Adjustments - Pension and Retirement Expense) / Market Capitalization	-0.13	-0.77	-0.04	0.22	1.52	-11.01
(recta - xpr) / size	(Retained Earnings - Cumulative Translation Adjustment - Pension and Retirement Expense) / Market Capitalization	-0.10	-0.69	-0.03	0.02	0.16	-10.96
(dvt - price) / size	(Dividends - Total - Price) / Market Capitalization	-0.18	-1.14	-0.05	-0.06	-0.49	10.49
(dvp - reajo) / size	(Dividends - Preferred-Preference - Retained Earnings - Other Adjustments) / Market Capitalization	-0.10	-0.61	-0.03	-0.59	-3.47	10.45
(pstkc - reajo) / at	(Preferred Stock - Convertible - Retained Earnings - Other Adjustments) / Assets - Total	-0.12	-0.77	-0.04	-0.51	-3.02	10.28
(pstkc - reajo) / lse	(Preferred Stock - Convertible - Retained Earnings - Other Adjustments) / Liabilities and Stockholders Equity - Total	-0.12	-0.77	-0.04	-0.51	-3.02	10.28
(pstkc - reajo) / icapt	(Preferred Stock - Convertible - Retained Earnings - Other Adjustments) / Invested Capital - Total	-0.13	-0.85	-0.04	-0.52	-3.08	10.18
(re - reuna) / size	(Retained Earnings - Retained Earnings - Unadjusted) / Market Capitalization	-0.17	-1.39	-0.07	0.18	1.53	-10.15
(dvc - price) / size	(Dividends Common-Ordinary - Price) / Market Capitalization	-0.19	-1.22	-0.05	-0.07	-0.52	10.06
(cshr - reajo) / size	(Common-Ordinary Shareholders - Retained Earnings - Other Adjustments) / Market Capitalization	0.08	0.36	0.02	-0.45	-2.91	9.83
(pstkc - reajo) / at	(Preferred-Preference Stock (Capital) - Total - Retained Earnings - Other Adjustments) / Assets - Total	0.03	0.19	0.01	-0.39	-2.22	9.77
(pstkc - reajo) / lse	(Preferred-Preference Stock (Capital) - Total - Retained Earnings - Other Adjustments) / Liabilities and Stockholders Equity - Total	0.03	0.19	0.01	-0.39	-2.22	9.77
(aco - reajo) / size	(Current Assets - Other - Total - Retained Earnings - Other Adjustments) / Market Capitalization	0.20	1.30	0.07	0.07	0.44	9.62
(acox - reajo) / size	(Current Assets - Other - Sundry - Retained Earnings - Other Adjustments) / Market Capitalization	0.19	1.35	0.07	0.05	0.38	9.61
(ivst - xsga) / lse	(Short-Term Investments - Total - Selling, General and Administrative Expense) / Liabilities and Stockholders Equity - Total	-0.24	-1.42	-0.06	0.02	0.13	-9.40
(ivst - xsga) / at	(Short-Term Investments - Total - Selling, General and Administrative Expense) / Assets - Total	-0.24	-1.42	-0.06	0.02	0.13	-9.40
(pstkc - reajo) / price	(Preferred Stock - Convertible - Retained Earnings - Other Adjustments) / Price	-0.18	-1.21	-0.06	-0.51	-3.28	9.37
(invfg - recta) / price	(Inventories - Finished Goods - Retained Earnings - Cumulative Translation Adjustment) / Price	-0.11	0.70	0.04	0.17	1.07	9.33
(invfg - tstkp) / price	(Inventories - Finished Goods - Treasury Stock - Preferred) / Price	-0.12	-0.80	-0.04	-0.24	-1.58	9.32
(pstkc - recta) / icapt	(Preferred-Preference Stock (Capital) - Total - Retained Earnings - Cumulative Translation Adjustment) / Invested Capital - Total	-0.03	-0.20	-0.01	-0.21	-1.56	9.27
(pstkc - reajo) / icapt	(Preferred-Preference Stock (Capital) - Total - Retained Earnings - Other Adjustments) / Invested Capital - Total	0.06	0.41	0.02	-0.38	-2.15	9.26
(dvp - reajo) / csho	(Dividends - Preferred-Preference - Retained Earnings - Other Adjustments) / Common Shares Outstanding	-0.14	-0.87	-0.04	-0.57	-3.41	9.17
(pstkc - recta) / at	(Preferred-Preference Stock (Capital) - Total - Retained Earnings - Cumulative Translation Adjustment) / Assets - Total	-0.01	-0.05	-0.00	-0.14	-1.13	9.15
(pstkc - recta) / lse	(Preferred-Preference Stock (Capital) - Total - Retained Earnings - Cumulative Translation Adjustment) / Liabilities and Stockholders Equity - Total	-0.01	-0.05	-0.00	-0.14	-1.13	9.15
(dvp - reajo) / cshpri	(Dividends - Preferred-Preference - Retained Earnings - Other Adjustments) / Common Shares Used to Calculate Earnings Per Share - Basic	-0.12	-0.79	-0.04	-0.56	-3.33	9.15
(dvp - recta) / size	(Dividends - Preferred-Preference - Retained Earnings - Cumulative Translation Adjustment) / Market Capitalization	-0.02	-0.13	-0.01	-0.21	-1.41	9.01
(che - cstkc) / csho	(Cash and Short-Term Investments - Common Stock-Carrying Value) / Common Shares Outstanding	0.07	0.38	0.02	-0.04	-0.25	-8.97
(pstkn - reajo) / at	(Preferred-Preference Stock - Nonredeemable - Retained Earnings - Other Adjustments) / Assets - Total	-0.02	-0.12	-0.01	-0.45	-2.57	8.96
(pstkn - reajo) / lse	(Preferred-Preference Stock - Nonredeemable - Retained Earnings - Other Adjustments) / Liabilities and Stockholders Equity - Total	-0.02	-0.12	-0.01	-0.45	-2.57	8.96
(aco - recta) / size	(Current Assets - Other - Total - Retained Earnings - Cumulative Translation Adjustment) / Market Capitalization	0.29	1.89	0.10	0.17	1.09	8.94
(txw - xad) / vol	(Excise Taxes - Advertising Expense) / 1y Return Volatility	0.29	1.55	0.07	-0.01	-0.07	-8.87
(pstkr - reajo) / at	(Preferred-Preference Stock - Redeemable - Retained Earnings - Other Adjustments) / Assets - Total	-0.03	-0.17	-0.01	-0.45	-2.77	8.86
(pstkr - reajo) / lse	(Preferred-Preference Stock - Redeemable - Retained Earnings - Other Adjustments) / Liabilities and Stockholders Equity - Total	-0.03	-0.17	-0.01	-0.45	-2.77	8.86
(cstkc - invfg) / bkvlps	(Common Stock-Carrying Value - Inventories - Finished Goods) / Book Value Per Share	0.03	0.20	0.01	-0.26	-1.82	-8.84
(pstkc - txp) / cshr	(Preferred Stock - Convertible - Income Taxes Payable) / Common-Ordinary Shareholders	-0.13	-0.97	-0.05	-0.40	-2.96	8.84
(acox - txp) / seq	(Current Assets - Other - Sundry - Income Taxes Payable) / Stockholders Equity - Parent	0.01	0.05	0.00	-0.01	-0.13	8.83
(pstkc - recta) / icapt	(Preferred Stock - Convertible - Retained Earnings - Cumulative Translation Adjustment) / Invested Capital - Total	-0.05	-0.36	-0.02	-0.22	-1.70	8.83
(invfg - lifrp) / bkvlps	(Inventories - Finished Goods - LIFO Reserve - Prior) / Book Value Per Share	-0.25	-1.76	-0.09	-0.30	-2.04	8.83
(pstkrv - recta) / icapt	(Preferred Stock - Redemption Value - Retained Earnings - Cumulative Translation Adjustment) / Invested Capital - Total	-0.03	-0.24	-0.01	-0.23	-1.77	8.82
(cstkc - ivst) / ppent	(Common Stock-Carrying Value - Short-Term Investments - Total) / Property, Plant and Equipment - Total (Net)	-0.07	-0.37	-0.02	-0.37	-2.16	8.81
(acox - txp) / ceq	(Current Assets - Other - Sundry - Income Taxes Payable) / Common-Ordinary Equity - Total	-0.00	-0.02	-0.00	-0.04	-0.39	8.79
(che - cstkc) / cshpri	(Cash and Short-Term Investments - Common Stock-Carrying Value) / Common Shares Used to Calculate Earnings Per Share - Basic	0.05	0.29	0.02	-0.03	-0.19	-8.79
(pstkr - reajo) / icapt	(Preferred-Preference Stock - Redeemable - Retained Earnings - Other Adjustments) / Invested Capital - Total	-0.03	-0.17	-0.01	-0.42	-2.57	8.78
(reajo - vol) / size	(Retained Earnings - Other Adjustments - 1y Return Volatility) / Market Capitalization	0.08	0.55	0.03	0.45	3.04	-8.77
(che - tstkp) / csho	(Cash and Short-Term Investments - Treasury Stock - Preferred) / Common Shares Outstanding	0.12	0.68	0.03	0.07	0.43	-8.77
(pstkc - reajo) / lt	(Preferred Stock - Convertible - Retained Earnings - Other Adjustments) / Liabilities - Total	-0.14	-0.94	-0.05	-0.46	-2.96	8.77
(pstkl - recta) / icapt	(Preferred Stock - Liquidating Value - Retained Earnings - Cumulative Translation Adjustment) / Invested Capital - Total	-0.03	-0.19	-0.01	-0.22	-1.73	8.77
(recta - xpr) / price	(Retained Earnings - Cumulative Translation Adjustment - Pension and Retirement Expense) / Price	0.07	0.57	0.03	0.10	0.91	-8.77
(pstkc - reajo) / cshfd	(Preferred Stock - Convertible - Retained Earnings - Other Adjustments) / Common Shares Used to Calc Earnings Per Share - Fully Diluted	-0.11	-0.64	-0.03	-0.51	-2.97	8.75

Table A8: 17 strategies that survive hurdles

		Mean	t_μ	SR	α	t_α	t_λ
(cstk - reajo) / xad	(Common-Ordinary Stock (Capital) - Retained Earnings Other Adjustments) / Advertising Expense	-0.67	-2.33	-0.12	-1.20	-4.37	-3.55
(lo - sppe) / tstkn	(Liabilities Other Total - Sale of Property) / Treasury Stock Number of Common Shares	0.40	3.00	0.13	0.55	3.97	3.22
(ap - txfed) / dvc	(Accounts Payable Trade - Income Taxes Federal) / Dividends Common-Ordinary	-0.49	-2.99	-0.13	-0.61	-3.82	-3.54
(csho - xsga) / xint	(Common Shares Outstanding - Selling, General and Administrative Expense) / Interest and Related Expense	-0.77	-3.44	-0.15	-0.95	-3.96	-4.82
(cshpri - xsga) / dd3	Total (Common Shares Used to Calculate Earnings Per Share Basic - Selling, General and Administrative Expense) / Debt Due in 3rd Year	-0.66	-3.19	-0.15	-0.87	-3.95	-4.02
(cshpri - xsga) / xint	(Common Shares Used to Calculate Earnings Per Share Basic - Selling, General and Administrative Expense) / Interest and Related Expense Total	-0.64	-2.78	-0.12	-1.01	-4.22	-4.84
(dcvsub - xrent) / dd2	(Debt Subordinated Convertible - Rental Expense) / Debt Due in 2nd Year	-0.49	-3.32	-0.15	-0.71	-4.67	-3.16
(dcvt - mrc5) / dltd	(Debt Convertible - Rental Commitments Minimum 5th Year) / Long-Term Debt Total	-0.44	-2.99	-0.14	-0.58	-3.88	-3.66
(dltd - pstkr) / mrc1	(Long-Term Debt Issuance - Preferred-Preference Stock Redeemable) / Rental Commitments Minimum 1st Year	-0.48	-2.64	-0.13	-0.85	-4.58	-3.12
(dltd - pstkr) / mrc2	(Long-Term Debt Issuance - Preferred-Preference Stock Redeemable) / Rental Commitments Minimum 2nd Year	-0.47	-2.57	-0.13	-0.85	-4.38	-3.96
(dltd - pstkr) / mrc3	(Long-Term Debt Issuance - Preferred-Preference Stock Redeemable) / Rental Commitments Minimum 3rd Year	-0.51	-2.77	-0.14	-0.89	-4.58	-4.21
(dltd - pstkr) / mrc4	(Long-Term Debt Issuance - Preferred-Preference Stock Redeemable) / Rental Commitments Minimum 4th Year	-0.57	-3.04	-0.15	-0.91	-4.46	-3.46
(dltd - pstkr) / mrc5	(Long-Term Debt Issuance - Preferred-Preference Stock Redeemable) / Rental Commitments Minimum 5 Year	-0.50	-2.81	-0.14	-0.92	-5.12	-3.57
(rectr - xsga) / xint	Total (Receivables Trade - Selling, General and Administrative Expense) / Interest and Related Expense Total	-0.60	-2.82	-0.13	-1.04	-4.90	-3.60
(esubc - txdi) / dpvieb	(Equity in Net Loss Earnings - Income Taxes Deferred) / Depreciation (Accumulated) Ending Balance (Schedule VI)	0.64	3.45	0.15	1.08	6.36	3.94
(txdi - xpr) / dpvieb	(Income Taxes Deferred - Pension and Retirement Expense) / Depreciation (Accumulated) Ending Balance (Schedule VI)	-0.45	-2.86	-0.13	-0.68	-3.97	-4.98
(pstkc - txdi) / ppevb	(Preferred Stock Convertible - Income Taxes Deferred) / Property, Plant, and Equipment Ending Balance (Schedule V)	0.38	2.75	0.12	0.67	4.93	3.86