The Effect of Exogenous Information on Voluntary Disclosure and Market Quality*

Sivan Frenkel, Tel Aviv University[†]

Ilan Guttman, New York University[‡]

Ilan Kremer, Hebrew University and University of Warwick[§]

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Abstract

We analyze a model in which information may be either voluntarily disclosed by a firm and/or by a third party such as a financial analyst. Under plausible assumptions, analyst coverage crowds out corporate voluntary disclosure. Due to its strategic nature, corporate voluntary disclosure differs from the information provided by analysts. Nevertheless, we show that an increase in analyst coverage increases the overall quality of public information. We base this claim on two market quality measures: price efficiency, which is statistical in nature, and liquidity, defined as the expected bid-ask spread in a trading stage that follows the disclosure stage.

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†Email: frenkels@post.tau.ac.il.

‡Email: iguttman@stern.nyu.edu.

§Email: ikremer@mail.huji.ac.il.

1 Introduction

There is a growing literature on voluntary disclosure that studies how agents (or firms) strategically decide whether to disclose or withhold their private information. Public companies, for example, are mandated to disclose certain information in their periodic reporting, but some information is disclosed at the discretion of the manager. For example, a firm does not have to disclose that a major customer is negotiating a deal with one of its competitors. Hence, corporate voluntary disclosure is often a major source of information in capital markets.[1] Another example is an entrepreneur who seeks funding from investors (VC funds, angels, etc.); the entrepreneur can choose whether to disclose or conceal the results of previous attempts to raise funding or acquire new customers. Examples are not limited to financial markets. An incumbent politician may obtain private information about the success or failure of policies she has supported, and can choose whether to disclose or conceal these results. In all of these examples, informed agents are reluctant to lie because of severe or even criminal punishment, or because once the information is voluntarily disclosed it can be easily verified. Instead, they can choose to withhold negative information, taking advantage of public uncertainty about whether they indeed have private information.

The current literature focuses on settings in which a single agent chooses whether to disclose or withhold private information and there is no other source that can potentially discover this information. In practice, various sources that may discover and reveal firm's private information are very prevalent. For example, financial analysts and rating agencies provide additional information about public firms, investors can gather information through their social network about an entrepreneur, and the media and independent think-tanks can asses public policies.

In this paper, we introduce such additional sources of information to a standard voluntary disclosure setting with uncertainty about information endowment. Our main question is how the possibility of information arrival from a third party affects the aggregate amount of publicly available information. In order to answer this question, we need to

[1] Beyer et al. (2010) find that approximately 66% of accounting-based return variance is generated by voluntary disclosures, 22% is due to analyst forecasts, 8% is due to earnings announcements, and 4% is due to SEC filings.

first study the reaction of the disclosing agent to the possibility of a report by the third party and then to analyze the overall information that is revealed by both sources.

Our model departs from a standard voluntary disclosure setting with uncertain information endowment (a la Dye, 1985, and Jung and Kwon, 1988). A manager of a public firm, who wishes to maximize her firm's stock price, may be endowed with private value-relevant information. The financial market prices the firm based on all publicly available information. If the manager is informed, she can credibly and costlessly disclose her information to the market. The novelty of our model is the additional external source of information, e.g., an analyst, who may discover and publish private information held by the manager. We assume the analyst may discover and publish information when the manager is informed as well as when the manager is uninformed, and allow for correlation in the manager's and analyst's endowment of information.

We first show that, as standard in this literature, the game has a unique equilibrium, in which the manager discloses the realization of her private information if and only if it is higher than an equilibrium threshold. We then study how the firm's disclosure strategy changes in response to an increase in analyst coverage, i.e., an increase in the probability that the analyst discovers and dicloses information. We show that, under plausible assumptions, analyst coverage crowds-out corporate voluntary disclosure, i.e., firms respond to an increase in analyst coverage by increasing the disclosure threshold, which decreases the amount of information that they disclose. This result, which is new to the theoretical literature, is consistent with the empirical evidence in Anantharaman and Zhang (2011), Balakrishnan et al. (2014) and Ellul and Panayides (2018) (see more details on the empirical literature below).

Given the crowding-out result, we analyze the effect of an increase in analyst coverage on the overall amount of public information – including both the information disclosed by the analyst and by the manager. This is a challenging question, due to the qualitative difference between voluntary disclosure and information provided by the analyst. While informed firm's managers tend to disclose positive information and hide negative information, exogenous sources (such as analysts, the media, etc.) provide information that may be positive or negative. Thus, more exogenous information does not only affects the *amount* of information that becomes available but also the *type* of information, and specifically the balance between positive and negative information. Formally, information in environments with varying levels of analyst coverage cannot be ranked using the Blackwell informativeness criterion.

We use two separate measures to capture the overall information available to the market. First, we consider a quadratic loss function, which equals the expected squared difference between the firm's actual and perceived value. This measure has a natural interpretation in terms of price efficiency or ex-post return volatility. It can also represent the utility function of an information "receiver", such as an investor, and is consistent with the assumption that such receiver sets prices to be equal to the expected value, conditional on all available information.

Our second measure of information quality is more specific to the capital market example and can be directly linked to empirical findings. We use the expected bid-ask spread as a measure that reflects the extent of information asymmetry in the market. We augment the disclosure model by introducing a trading stage a la Glosten and Milgrom (1985) that follows the disclosure stage. The trade and pricing in this stage are affected by the information that was revealed by the manager and the analyst. Our main result is that both price efficiency and liquidity increase as a result of an increase in analyst coverage, that is, the overall effect of an increase in third party disclosure on market quality is always positive.

The intuition for our result is that an increase in analyst coverage changes the balance between negative and positive information that is being disclosed: more negative news are now disclosed, while positive information is less affected (since it is disclosed by the firm as well as the analyst). Thus, the overall quality of information improves. The change in the balance between negative and positive information, due to an increase in third party disclosure, should be reflected in the skewness of returns. While firms with little coverage will exhibit strong positive skewness of the disclosed information, an increase in analyst coverage should make the distribution of public information more symmetric. Support for this can be found in Acharya et al. (2011), who find that larger firms exhibit a more symmetric return distribution. This seems to be consistent with our findings, since smaller firms receive less attention by exogenous information sources such as financial analysts and the media.

We further show that our main results are robust to changes in the modeling assumptions. We first discuss the case where an informed analyst's signal is less precise than the signal of an informed manager. This assumption is realistic to various types of private information, e.g., internal firm information. We show our result continue to hold given some additional assumptions on the information production technology of the analyst. Next, we discuss endogenous information acquisition in the trading stage. We show that analyst coverage does not only crowd out corporate disclosure, but also crowds out private information acquisition by traders.

Our results can be used to assess certain policies that aim to increase market transparency in voluntary disclosure settings. Such policies often focus on improving the information provided by one market participant without considering its effect on other market participants and the overall information available to the market.[2] Financial analysts have an important role in revealing firms' private information to the capital market, but there are other sources of exogenous revelation, such as news media, social media, competitors, suppliers and the government. Our results show that an improvement in one information source may crowd-out information from another source, and that different parties affect the overall public information differently. Our model suggests that to the extent that increasing the likelihood of such information discovery is not too costly, it is beneficial in terms of price efficiency and liquidity.

Unlike the theoretical literature (which is reviewed in the next subsection), the empirical literature has studied the effect of analyst coverage on a firm's voluntary disclosure and on the liquidity of the firm's stock. Empirical evidence supports the predictions of our model. For example, Kelly and Ljungqvist (2012) show that following an exogenous decrease in analyst coverage, due to mergers and closings in the brokerage industry, information about affected firms became more asymmetric, and the liquidity of these firms' stocks decreased. Ellul and Panayides (2018) use a statistical model to identify exogenous terminations of analyst coverage. They show that stocks of firms who have lost complete

[2] Examples of regulations that focus on information provision include: the Sarbanse-Oxley Act attempts to increase the mandated reporting of firms; the Williams Act of 1968 limits the ability of investors to trade anonymously on their private (optimistic) information; the regulation on analyst certification (Reg AC) requires analysts to disclose possible conflicts of interests and prevent biased reports; the Dodd-Frank Act includes several measures aimed at improving the transparency and viability of credit ratings. See also the discussion at Goldstein and Yang (2017).

analyst coverage experience a decrease in both liquidity and price efficiency.[3]

Anantharaman and Zhang (2011) and Balakrishnan et al. (2014) use the same exogenous negative shock to analyst coverage that is used in Kelly and Ljungqvist (2012) to establish the effect of a decrease in analyst coverage on firms' voluntary disclosure. Balakrishnan et al. (2014) show that one quarter following the decrease in analyst coverage, the affected firms increased their voluntary disclosure (earning guidance) to mitigate the increase in information asymmetry and the decrease in liquidity. This increased disclosure partially reverses the decrease in liquidity, although the overall effect remains negative, consistent with both predictions of our model. Ellul and Panayides (2018) divide their sample of firms that experienced unexpected coverage termination to those that increased the number of news releases in the post-termination period and those that kept the number unchanged or decreased it. They show that liquidity deteriorates less for the former group, suggesting again that firms disseminate more information to the market in order to mitigate the effect of the decrease in analyst coverage. Ellul and Panayides (2018) also find that termination of analyst coverage increases the informed trading in a stock, and that informed trades become more profitable.

There is a more extensive empirical literature that studies how disclosure and transparency affect the informational environment in general and the bid-ask spread in particular. While the results are mixed, many papers find that increased disclosure increases informativeness and decreases the bid-ask spread (e.g. Welker 1995; Healy et al. 1999; Leuz and Verrecchia 2000; Heflin et al. 2005; Leuz and Wysocki 2016).

Following a review of the related theoretical literature, we describe in Section 2 the setting of our model. Our objective is to address three questions pertaining to the voluntary disclosure setting with the possibility of an exogenous signal. First, how the introduction of an exogenous signal affects the equilibrium of the disclosure game, in particular the likelihood of voluntary disclosure and the price given no disclosure. This analysis is presented in Section 3. Second, since the presence of an exogenous signal affects the manager's disclosure strategy, how does a change in the probability of an exogenous sig-

[3]Kelly and Ljungqvist (2012) use several measures of liquidity, including the bid-ask spread, which is the measure we use in Section 5. Ellul and Panayides (2018) use a measure of price efficiency that follows Hasbrouck (1993), and is close in nature to our theoretical measure in Section 4. Their measure of liquidity is also the bid-ask spread.

nal, e.g., through a change in analyst coverage, affect price efficiency. We answer this in Section 4. Finally, we study how changes in analyst coverage affect the liquidity of the firm's stock, as captured by the expected bid-ask spread. To do that, in Section 5 we introduce, and analyze, and extended model that includes a stylized trading stage a la Glosten and Milgrom (1985). Section 6 presents two possible extensions of the model: Section 6.1 discusses a framework where the analyst observes information that is noisier than the information of the manager, and Section 6.2 discusses the effect of information acquisition in the Glosten-Milgorm model. Section 7 briefly concludes.

1.1 Related Theoretical Literature

Our study of voluntary disclosure in the presence of potentially informed traders contributes to two streams of the theoretical literature. The first is the voluntary disclosure literature. To the best of our knowledge, only a few theoretical papers study voluntary disclosure in the presence of a potentially informed trader/receiver. Langberg and Sivaramakrishnan (2008, 2010) offer two models with a firm that can voluntarily disclose information and strategic analysts that can disclose additional information. In these papers, the analyst's information is orthogonal to the information of the firm; in our model, analysts and the firm potentially learn the same information. This makes the analysis very different. Moreover, in these papers, by construction, greater firm disclosure encourages the analysts to obtain more information. Einhorn (2018) also explores the effect of additional information sources on voluntary disclosure. In her model, in contrast to the present paper, there is a difference between the information that can be disclosed and the fundamental value of the firm, and she focuses on an equilibrium where this difference determines the firm's disclosure strategy. The closest work with an informed receiver is Ispano (2016), whose model, while very different, can be seen as a simplified version of our model with three possible firm values and a specific analyst technology. He shows, in his discrete example, that the utility of the receiver – which is equivalent to price efficiency in our setting – is increasing with the probability that the receiver is informed. He does not discuss liquidity. Quigley and Walther (2018) present a model of costly disclosure with an additional source of information. The cost prevents high types from disclosing, as they prefer to rely on the outside signal. More precise outside information crowds-out disclosure by high types in Quigley and Walther (2018), while in our model an increase in the probability of obtaining outside information crowds out disclosure by low types. Dutta and Trueman (2002) study a setting in which a firm's manager can credibly disclose verifiable private information, but cannot disclose additional information about how to interpret this information. The manager, not knowing whether the market will interpret the disclosed information as good news or bad news, faces uncertainty about the market reaction. In this setting, Dutta and Trueman (2002) show that the equilibrium disclosure strategy is not necessarily a threshold strategy. Banerjee and Kim (2017) explore a model where a manager may disclose information to the public and may also use private cheap-talk communication to contact her employees. With some probability, private communication is "leaked" and becomes public. It turns out that the possibility of private communication becoming public has very different implications than the possibility of the manager's information becoming public, which is the case in our model. Finally, several papers deal with disclosure of two strategic firms/experts (Bhattacharya and Mukherjee, 2013; Bhattacharya et al., 2018; Kartik et al., 2017). In those papers, as in ours, agents consider the possibility of additional information due to disclosure by their peers, but these papers do not focus on changes in overall public information.[4]

The second stream of literature studies how changes in one source of information affect the incentives for acquiring information among other parties. Several papers have considered the acquisition of private information by investors in a setting where public information is available (see, e.g., Verrecchia (1982); Diamond (1985); Demski and Feltham (1994); Kim and Verrecchia (1994); McNichols and Trueman (1994)). Such public information can be interpreted either as corporate mandatory disclosure or as disclosure by a third party, such as an analyst. A key result in this literature is that better public information crowds out incentives to acquire private information. Goldstein and Yang (2017) specifically examine, in such a setting, the overall effect on public information. They show that when traders' information is kept constant, a more precise public signal improves market liquidity and price efficiency. However, when the crowding out of private information acquisition is taken into account, the overall effect is ambiguous and depends

[4]The effect of an informed receiver on the sender's strategy is also explored in the literature that follows the "Bayesian Persuasion" paradigm, that is, where the sender commits ex-ante to a disclosure strategy. See, for example, Rayo and Segal (2010); Kolotilin (2018); Azarmsa and Cong (2018).

on the parameter values and the particular measure of market quality. By contrast, the present paper combines both a third party disclosure and an endogenous, voluntary disclosure decision. Section 6.2 shows that, as in Goldstein and Yang (2017), better analyst coverage decreases the incentives of investors to acquire private information. The decrease in informed trading decreases asymmetric information and increases liquidity.[5]

2 Setting

Our model builds on the voluntary disclosure literature initiated by Grossman (1981), Milgrom (1981), and Dye (1985). We consider a firm that is involved in a project, e.g., drug development or oil exploration, which will either succeed or fail. We denote the terminal value of the firm by $x \in \{0, 1\}$ where $x = 1$ following success and $x = 0$ following failure. The ex-ante probability of success is $\mu_0 \equiv \Pr(x = 1)$ and the probability of failure is $1 - \mu_0 \equiv \Pr(x = 0)$.

Information Structure With probability $q \in (0, 1)$, the manager of the firm observes additional information about the possible outcome of the project, in the form of a signal s. With probability $1-q$ the manager does not observe a signal. Information endowment is independent of the realization of s, and therefore the ex-ante expected value of s (or x) conditional on an information event also equals μ_0. The signal may represent, for example, the results of a clinical trial or an oil exploration, information about competing projects/firms or information about relevant macroeconomic conditions. We assume that all players in the game are risk neutral, and thus it is without loss of generality to assume that the signal s is simply the updated probability of success, that is, $\Pr(\tilde{x} = 1|s) = E(\tilde{x}|s) = s$. Hence, we assume that $\tilde{s} \in [0, 1]$, with a PDF $f(s)$, a continuous CDF $F(s)$, and $E[\tilde{s}] = \mu_0$.

Remark 1 (Alternative State Space). Most of our results extend to arbitrary continuous distributions of $\tilde{x}$ and $\tilde{s}$ (with bounded or unboanded support). This include all the results in Sections 3 and 4. The binary structure is only used to simplify the trading stage in

[5] Another related paper is by Gao and Liang (2013), who study how a firm's commitment to disclosure affects investors' incentives to acquire information. Their focus is on the feedback effect, whereby the firm's manager learns from prices

Section 5.

Disclosure and Pricing If the manager observes the private signal s, she can voluntarily disclose it to the market. Disclosure is assumed to be costless and credible (verifiable at no cost). As standard in the voluntary disclosure literature, if the manager does not obtain the private signal, she cannot credibly convey that she is not informed. The manager seeks to maximize the market value, or price, of the firm.[6] For now, assume that risk neutral investors set the market price, P, equal to the expected value conditional on all the available public information, I. That is, $P = E[\tilde{s} \mid I] = E[\tilde{x} \mid I]$. Later, in Section 5, we introduce a trading stage that follows Glosten and Milgrom (1985) where prices are set by a centralized market maker.

The setting introduced so far is similar to a standard voluntary disclosure setting with uncertainty about information endowment, which has been studied extensively. The main innovation of our setting is the possibility that the signal s will be made public by an external third party.

Analyst (Exogenous Signal) We use financial analysts as our main motivating example, however, any mechanism that induces stochastic public supply of the firm's information, such as news media, competitors, suppliers, social media, regulators etc., will have a similar effect in our model.

To study the interaction between firm's voluntary disclosure and the potentially informed market, we add to the above setting a financial analyst, who may also learn the realization of the updated probability of success, s. We abstract from strategic considerations of the analyst, and assume that whenever analysts discover information they publish it truthfully.[7] In the baseline model we assume that, if informed, both the analyst and the manager observe the same information. In Section 6.1 we discuss the case where the analyst's information is less precise than that of the manager.

[6] As standard in the literature, we take the performance-based compensation of the manager as given. Such compensation may be an optimal contract when the manager has additional activities, which are left unmodeled, that demand effort (as in Holmström, 1979, 1999). Such compensation is also optimal when the market / receiver wishes to price the firm "correctly" (Hart et al., 2017).

[7] It is immediately obvious that all of our results are robust to an analyst's reporting strategy that is potentially biased, as long as the analyst always issues a report when obtaining information and the analyst's forecast follows a separating strategy. For an example and additional references see Beyer and Guttman (2011).

The likelihood of the analyst to discover information may depend on whether the manager is informed or not. For example, if the information s is the result of a clinical drug trial, it is unlikely that the analyst will discover this information before the manager does. However, if the signal s is information about market conditions, the analyst may discover this information even when the manager is uninformed. To allow for both types of information, we assume a relatively non-restrictive analyst's information production technology. In particular, assume that the analyst's information production technology is reflected by a pair of conditional probabilities $(g_I(r), g_U(r))$, where $g_I(r) \in [0, 1)$ and $g_U(r) \in [0, 1)$ are the probabilities that the analyst discovers s conditional on the manager being informed and uninformed, respectively. We introduce the parameter r to capture the overall quality and/or quantity of analysts that cover the firm. We refer to r as "analyst coverage." An increase in analyst coverage weakly increases the probability that the analyst becomes informed when the manager is informed and when the manager is uninformed. For simplicity, we assume that g_I and g_U are differentiable, and thus assume $g'_U(r) \geq 0$ and $g'_I(r) \geq 0$, with at least one strict inequality. Note that the ex-ante probability that the analyst issues a report is $q \cdot g_I(r) + (1 - q) \cdot g_U(r)$.

Timeline To summarize our disclosure game, the timeline is as follows.

1. With probability q the manager privately learns the signal s.

2. If the manager is informed, she decides whether to publicly disclose s or not.

3. Analysts learn the signal s with probabilities $g_I(r)$ or $g_U(r)$, depending on the outcome of stage 1. An informed analyst immediately discloses s to the market.

4. Following the disclosure or lack of disclosure by both the manager and the analyst, market participants update their beliefs about the expected value of the firm/project.

5. The price of the firm is determined, and the manager is compensated accordingly. We first assume risk neutral pricing, and in Section 5 we specify a market mechanism that generates the price.

The setting and all the parameters of the model are common knowledge.

Remark 2 (Alternative Timing). The information that the manager and the analyst may learn and disclose is identical. Thus, the manager's disclosure is relevant only in the case the analyst has not published a report. This implies that even if the manager knows whether the analyst has published, or about to publish, a report before making her disclosure decision (that is, even if stage 3 is before stage 2), the equilibrium is essentially the same: following a disclosure by the analyst the manager is indifferent whether to disclose or not, and following no analyst report the manger's strategy is identical to her strategy in the current model.

3 Analysis of the Disclosure Decision

3.1 Equilibrium Disclosure Strategy

Given the realized signal, an informed manager chooses a disclosure strategy that maximizes the expected firm price. If s is publicly disclosed either by the manager or by the analyst – an event we denote by "D" – the price of the firm equals its expected value, i.e.,

$$P^{\mathrm{D}}(s) \equiv E[\tilde{x}|s] = s.$$

Denote by "ND" the event that neither the manager nor the analyst disclosed s, and by P^{ND} the price following such an event. P^{ND} is the market's belief about the firm's expected value following no disclosure., i.e., $P^{\mathrm{ND}} \equiv E[\tilde{x}|\mathrm{ND}]$.

The manager's disclosure decision affects the price only when s is not disclosed by the analyst. Thus, though an informed manager does not know whether the analyst will be informed or not, she conditions her decision only on the event that the analyst will not be informed. When the analyst is not informed, an informed manager's optimal strategy is to disclose s if and only if $P^{\mathrm{D}}(s) > P^{\mathrm{ND}}$. While $P^{\mathrm{D}}(s)$ is increasing in s, P^{ND} is independent of the manager's type. Therefore, any equilibrium disclosure strategy is characterized by a threshold signal - which we denote by σ - such that an informed manager discloses her signal if and only if $s \geq \sigma$.

The price following no disclosure by the manager or the analyst, P^{ND}, depends on the market's belief about the manager's disclosure strategy. If the market believes the manager uses a disclosure threshold σ, then the price following no disclosure is given by

$$P^{\mathrm{ND}}(\sigma) \equiv E\left[\tilde{x}|\mathrm{ND},\sigma\right] = \frac{(1-q)\cdot(1-g_U(r))\,E\left[\tilde{s}\right] + qF\left(\sigma\right)\cdot(1-g_I(r))\cdot E\left[\tilde{s}|s<\sigma\right]}{(1-q)\left(1-g_U(r)\right) + qF\left(\sigma\right)\left(1-g_I(r)\right)}. \tag{1}$$

The price is a weighted average of the prior mean and the mean conditional on withholding signals below σ, with weights representing the conditional probabilities that the manager is informed and uninformed, given that no analyst report was published. Thus, for any exogenously given disclosure threshold $\sigma \in (0,1)$ the price given no disclosure is lower than the prior mean, that is, $P^{\mathrm{ND}}(\sigma) < E\left[\tilde{s}\right] = \mu_0$.

Our disclosure model generalizes Dye (1985) and Jung and Kwon (1988) to a setting that contains an additional stochastic public revelation mechanism. Formally, those models are a particular case of our setting in which $g_I\left(r\right) = g_U\left(r\right) = 0$. It is easy to extend the analysis in Jung and Kwon (1988) to our setting and show that a threshold equilibrium exists, and that it is unique.

Fact 1. *There exists a unique equilibrium to the disclosure game, in which an informed manager discloses if and only if the signal s is greater than a disclosure threshold σ^*. σ^* is the signal that makes the manager indifferent between disclosing or withholding. The disclosure threshold is given by the unique solution of the condition*

$$\sigma^* = P^{\mathrm{ND}}(\sigma^*). \tag{2}$$

An additional useful property of voluntary disclosure games that also holds in our model is the Minimum Principle property, first described by Acharya et al. (2011). This property shows that $P^{\mathrm{ND}}(\sigma)$ is minimized under the equilibrium threshold.

Fact 2 ("The Minimum Principle," Acharya et al. 2011, Proposition 1)**.** *The equilibrium threshold σ^* is the unique disclosure threshold that minimizes the price given no disclosure, that is, $\sigma^* = \min_\sigma P^{\mathrm{ND}}(\sigma)$.*

An immediate corollary of the minimum principle is that a change in any parameter that increases or decreases the function $P^{\mathrm{ND}}(\sigma)$ for any threshold σ, also increases or decreases the equilibrium threshold σ^*. If, for example, a change in r increases the price following no disclosure for any exogenously given disclosure threshold, then, by the minimum principle, it must also increase the equilibrium threshold (that is, decrease disclosure). This is formalized in the following corollary.

Corollary 1. *The equilibrium disclosure threshold σ^* is increasing (decreasing) in r, if and only if $P^{\mathrm{ND}}(\sigma)$ is increasing (decreasing) in r.*

3.2 The Effect of Analyst Coverage on the Disclosure Strategy

In this section we analyze the main comparative static of the disclosure game – how the level of analyst coverage, r, affects the manager's equilibrium disclosure threshold, σ^*.

Based on Corollary 1, to study the effect of analyst coverage on corporate disclosure, we can study how analyst coverage affects the price given no disclosure for an exogenous disclosure threshold σ, i.e., $\frac{\partial P^{\mathrm{ND}}(\sigma)}{\partial r}$. Note from (1) that, for any exogenous disclosure threshold σ, $\frac{\partial P^{\mathrm{ND}}(\sigma)}{\partial g_I(r)} > 0$ and $\frac{\partial P^{\mathrm{ND}}(\sigma)}{\partial g_U(r)} < 0$. Greater $g_I(r)$ means that the analyst is more likely to discover and publish s when the manager is informed. Thus, no disclosure when $g_I(r)$ is greater implies that it is less likely that the manager is informed and withholds negative information. Therefore, an increase in $g_I(r)$ increases P^{ND}. In contrast, greater $g_U(r)$ means that the analyst is more likely to discover and disclose s when the manager is uninformed. Thus, no disclosure when $g_U(r)$ is greater implies that it is more likely that the manager is informed and withholds negative information. Therefore, an increase in $g_U(r)$ decreases P^{ND}. The overall effect of an increase in r on the price given no disclosure is

$$\frac{\partial P^{\mathrm{ND}}(\sigma)}{\partial r} = \frac{\partial P^{\mathrm{ND}}(\sigma)}{\partial g_I(r)} g_I'(r) + \frac{\partial P^{\mathrm{ND}}(\sigma)}{\partial g_U(r)} g_U'(r).$$

Since both $g_I(r)$ and $g_U(r)$ increase in r, the overall effect of changes in r on P^{ND} is not clear. Without further assumptions about the functions $g_I(r)$ and $g_U(r)$, one cannot conclude whether an increase in analyst coverage increases or decreases the equilibrium disclosure threshold. Next, we provide the condition that determines the effect of a change in r on the disclosure strategy, and thus on corporate disclosure.

3.2.1 Condition for the Crowding Out Effect of Analyst Coverage

In order to study the effect of analyst coverage on the equilibrium disclosure strategy, it is useful to consider the following function

$$m(r) \equiv \frac{\Pr\left(\text{analyst is uninformed } \middle| \text{manager is uninformed}\right)}{\Pr\left(\text{analyst is uninformed } \middle| \text{manager is informed}\right)} = \frac{1 - g_U(r)}{1 - g_I(r)}. \tag{3}$$

$m(r) \in [0, \infty)$ is the ratio between the likelihood that the analyst does not discover and discloses s when the manager is uninformed and the likelihood that the analyst does not disclose s when the manager is informed. For convenience, we henceforth refer to $m(r)$ as the *"informed analyst ratio."*

Denote by σ_D^* the disclosure threshold in a model with no analyst, i.e., where $g_U = g_I = 0$. This is the classic Dye (1985) model. We first show that the size of $m(r)$ determines whether the presence of an analyst increases or decreases voluntary disclosure compared to the Dye (1985) model.

Lemma 1. *The firm discloses less information compared to the case where an analyst is not available if and only if the informed analyst ratio is greater than one; that is*

$$\sigma^*(r) > \sigma_D^* \iff m(r) > 1.$$

Proof. Using (1), $P^{\mathrm{ND}}(\sigma, r)$ can be rewritten as

$$P^{\mathrm{ND}}(\sigma, r) = \frac{(1-q)E\left[\tilde{x}\right] + q \cdot m(r)^{-1} \cdot F(\sigma)\, E\left[\tilde{x} \mid s < \sigma\right]}{1 - q + q \cdot m(r)^{-1} \cdot F(\sigma)}. \tag{4}$$

By (4) and the fact that $E\left[\tilde{s}\right] > E\left[\tilde{s} \mid s < \sigma\right]$, it is clear that $P^{\mathrm{ND}}(\sigma, r)$ is increasing in $m(r)$. By (3), $m = 1$ when $g_I = g_U = 0$. Thus, $P^{\mathrm{ND}}(\sigma, r) > P^{\mathrm{ND}}(\sigma, r)\mid_{g_I = g_U = 0}$ if and only if $m(r) > 1$. The lemma then follows from Corollary 1. $\square$

We now turn to the effect of *changes* in analyst coverage on the level of voluntary disclosure, i.e., on the disclosure threshold. The following proposition shows that this effect depends on the directional change in $m(r)$ as r changes.

Proposition 1. *In equilibrium, analyst coverage crowds out corporate voluntary disclosure* *if and only if $m'(r) > 0$, that is,*

$$\frac{\partial \sigma^*}{\partial r} > 0 \iff m'(r) > 0.$$

Proof. By (4) and the fact that $E[\tilde{s}] > E[\tilde{s} \mid s < \sigma]$, it is clear that $P^{\mathrm{ND}}(\sigma, r)$ is increasing in $m(r)$. Thus, $\frac{\partial P^{\mathrm{ND}}(\sigma)}{\partial r} > 0$ iff $m'(r) > 0$. The lemma then follows from Corollary 1. □

Greater $m(r)$ means that the analyst is *relatively* more likely to be uninformed when the manager is uninformed than when the manager is informed. Thus, if the analyst does not report, this signals that the manager is more likely to be uninformed. Formally, as shown by (4),

$$\Pr(\text{manager is uninformed} \mid \mathrm{ND}) = \frac{1-q}{1-q+q \cdot m(r)^{-1} \cdot F(\sigma)}.$$

Therefore, higher $m(r)$ gives the manager a higher payoff in the case that the analyst does not publish a report, and thus a higher incentive to withhold. Note that, as discussed above, the probability that the analyst becomes informed does not enter the manager's payoff function in any way except through P^{ND}.

3.2.2 Information Structure Examples

Since the effect of analyst coverage on voluntary disclosure depends on $m(r)$, i.e., on the analyst's information production function, we offer two relatively simple examples of information structures, that we find appealing and realistic.

Example 1 (Private Inquiry and Leaks)**.** Suppose that the manager learns $\tilde{s}$ with probability q. The analyst has two potential sources of information, one within the firm and the other external. Examples for external sources could be information about the industry or macro economic conditions. Further assume that the probability that the analyst learns s from an external source is r and this probability is independent of whether the manager is informed or not. One interesting case of this example is $r = 0$, which may represent the results of a clinical trial or oil and gas drilling, that are unlikely to be available to the analyst and not to the manager.

The inside source of information captures information that is “leaked” to the analyst from within the firm.[8] Such information can be observed by the analyst only when the manager is informed. Suppose that the probability that the analyst learns s from insiders, conditional on the manager being informed, is $\delta(r) \in (0,1)$. Naturally we assume that an increase in analyst coverage increases the probability of leaks. For simplicity, we assume that $\delta(r)$ is differentiable, and $\delta'(r) > 0$. In this example, we obtain $g_U(r) = r$ and $g_I(r) = r + (1-r)\delta(r)$.

Example 2 (Conditionally Independent Information Endowment)**.** Suppose that with probability $\omega \in (0,1)$ some information event occurs and with probability $1-\omega$ no information event occurs. If no information event occurs, the firm's expected value remains the prior mean (μ_0). However, if an information event occurs, it generates a new probability of success s, which equals to the updated expected value of the firm.

Conditional on an information event occurring, the probability that the analyst discover s is r, and the probability of the manager discovering s is $\frac{q}{\omega}$ (so the overall probability that the manager discovers s is q). Assume that the information endowment events of the manager and the analyst are independent, conditional on an information event. This structure implies

$$g_U(r) = \frac{\omega(1-\frac{q}{\omega})r}{1-q} = \frac{\omega - q}{1-q} \cdot r \text{ and } g_I(r) = \frac{\omega\frac{q}{\omega}r}{q} = r.$$

One can easily verify that $m'(r) > 0$ in both examples.[9] Thus, by Proposition 1, the manager's disclosure threshold increases in analyst coverage ($\frac{\partial \sigma^*}{\partial r} > 0$). In other words, in both of these examples an increase in analyst coverage crowds out voluntary disclosure.

3.3 Assumption about Analyst's Information Production

Following the two examples above, in what follows we focus our attention on the case where $m'(r) > 0$, i.e., analyst coverage crowds out disclosure. That is, we assume the following regarding the analyst's information production technology $(g_I(r), g_U(r))$:

[8] Green et al. (2014) show that access to management remains an important source of information for analysts even following Regulation Fair-Disclosure (Reg FD).

[9] Note that $m'(r) > 0$ if and only if $\frac{g'_U(r)}{1-g_U(r)} < \frac{g'_I(r)}{1-g_I(r)}$.

Assumption 1. *The informed analyst ratio $m(r)$, as calculated in (3), is increasing in r.*

Note that this assumption is supported by the empirical literature presented above (Anantharaman and Zhang, 2011; Balakrishnan et al., 2014). Moreover, in the case where $m(r)$ is decreasing in r, and thus voluntary disclosure is increasing in analyst coverage, the main results of the paper regarding price efficiency and liquidity trivially continue to hold.

4 Price Efficiency

An increase in analyst coverage, r, by definition increases the probability that the signal will be discovered and disclosed by analysts, and thus has a direct effect of increasing the available public information. However, as established above, an increase in analyst coverage also affects the firm's voluntary disclosure. In particular, given Assumption 1, an increase in r decreases corporate voluntary disclosure (Proposition 1). As such, the overall effect of changes in analyst coverage on investors' information, or price informativeness, is not clear. In this section we asses the overall effect of an increase in analyst coverage. This effect can be decomposed into two parts:

- A change in the probability that the signal s is made public, either by the manager and/or by the analyst. The probability of this event is given by

  $$q \cdot g_I(r) + q(1 - g_I(r))(1 - F(\sigma^*)) + (1 - q)\, g_U(r).$$

  As mentioned before, since the manager's equilibrium disclosure threshold, σ^*, is increasing in analyst coverage r, it is not clear whether this probability increases or decreases following an increase in r.

- Market uncertainty regarding s in case it does not become public. An increase in r affects the distribution of types given no disclosure, and hence the uncertainty given no disclosure.

Due to the effect on disclosure strategy one cannot use the Blackwell informativeness criterion as a way to measure the effect of an increase in analyst coverage on the amount of public information. This is because more coverage increases the probability that the value of some types will be disclosed (low types that are disclosed only by the analyst), but decreases this probability for other types (types between the previous and the new disclosure thresholds, who are now being withheld). In the next section we suggest a measure of price efficiency, which is the inverse of the expected squared distance of prices from the fundamental value. We then show that an increase in analyst coverage always increases price efficiency according to this measure.

4.1 A Measure of Price Efficiency

In our model, when information is made public either by the manager or by the analyst, the price perfectly reflects all the information, i.e., the price is $P^{\mathrm{D}} = E[\tilde{x}|s] = s$. When information is not made public the price is on average correct, but it is a noisy measure of the signal (that the manager may either not know or actively withholding), $P^{\mathrm{ND}} = E[s \mid \mathrm{ND}]$.

To measure how efficiently prices reflect information about future cash flows, we adopt the commonly used expected squared deviation between the market price and the signal s. Our price efficiency measure, which we refer to as PEF, is given by

$$\mathrm{PEF} \equiv -E\left[(s-P)^2\right]. \tag{5}$$

PEF may represent the "social" benefit from having a price that is close to the fundamental, or the externalities and gains that are obtained from the informativeness of prices. Note that this measure is in line with our assumption of risk neutral pricing: a social planner who wishes to maximize efficiency will choose $P = E[\tilde{s} \mid I]$, where I is all the available information.

Another interpretation of PEF is that it is the variance of the noise in the price relative to the true underlying value s. Thus, higher price efficiency means a decrease in the residual uncertainty of prices (the future movement of prices when the real cash flows x will be realized or revealed).

4.2 Analyst Coverage and Price Efficiency

We have discussed above the challenge in determining even the directional effect of changes in analyst coverage, r, on price efficiency. One of our main results is that an increase in analyst coverage always increases price efficiency.

Proposition 2. *Price efficiency increases in analyst coverage, i.e.,*

$$\frac{d\mathrm{PEF}(r)}{dr} > 0.$$

The formal proof of the Proposition is quite involved, and hence is relegated to the appendix. The intuition for the result is as follows. In equilibrium, whenever the manager obtains a signal below the disclosure threshold, $s < \sigma^*$ she does not disclose, and if the analyst does not reveals s, the resulting price is $P^{\mathrm{ND}} = \sigma^*$.

Consider a change from r to $r + \Delta$ for some small $\Delta > 0$. This will lead to a change in the disclosure threshold from σ^* to $\sigma^* + \Delta'$, where Δ' reflects the effect on the manager's disclosure strategy following this increase in r. Given Assumption 1, $\Delta' > 0$. One can examine the total effect on price efficiency by deviding it to two effects: (i) the effect of changing r to $r + \Delta$ without changing σ^* (ii) changing σ^* to $\sigma^* + \Delta'$ without changing r.

Our claim follows from the fact that the first effect is positive and is a first order effect of magnitude $O(\Delta)$, while the second effect is negative but of a second order with a magnitude of at most $O(\Delta'^2)$. The fact that $\Delta' = O(\Delta)$ implies that the overall effect is positive and so the derivative is positive. The first effect is clear: an increase in the probability that s is revealed by the analyst increases the probability that the price is equal to the true type, s. This has a first order effect – $O(\Delta)$.

The second effect is the increase in the manager's disclosure threshold, that is, a decrease in corporate voluntary disclosure. This increase in the disclosure threshold means that signals $s \in (\sigma^*, \sigma^* + \Delta')$, which originally were disclosed and priced correctly when the manager was informed, are now withheld, and thus receive, with some positive probability, a price P^{ND}. For the moment, suppose that P^{ND} does not change and remains σ^*. There is a decrease in price efficiency because types $s \in (\sigma^*, \sigma^* + \Delta')$ are not always priced correctly when the manager is informed. However, this is a $O(\Delta'^2)$ effect, because even when types $s \in (\sigma^*, \sigma^* + \Delta')$ are not priced correctly, they obtain a price of σ^* that is still very close to their fundamental value. Moreover, the price following no disclosure, P^{ND}, changes, and thus the pricing of all types whose value is not disclosed changes. By definition, the price following no disclosure $P^{\mathrm{ND}} = E[\tilde{s} \mid \mathrm{ND}]$ maximizes price efficiency following no disclosure. Thus, the new P^{ND}, that reflects the additional types who do not disclose increases the overall price efficiency compared to keeping the old price. This means that the negative effect is even smaller.

Proposition 2 implies that although analyst coverage has an adverse effect on corporate voluntary disclosure, the overall effect of analyst coverage on public information, as captured by our price efficiency measure, is positive. These results are supported by Ellul and Panayides (2018), who measure price efficiency using the methodology of Hasbrouck (1993). This methodology uses VAR to statistically estimate the difference between trading prices and the stock's estimated fundamental price, and measure price inefficiency as the standard deviation of this difference. Our measure of PEF is evidently the same, though it is developed within a much simpler, static, model. Ellul and Panayides (2018) find that price efficiency decreases following the termination of analyst coverage, and that the decrease is more moderate for firms that increased the number of news releases in the post-termination period, and for firms that issue earning guidance. These results are in line with the predictions of this section.

5 Informed Trading and Liquidity

The results in the previous section examine the effect of analyst coverage on a theoretical measure of price efficiency. While price efficiency is a very appealing theoretical construct, empirically measuring or estimating it is not easy[10]. In this section, we study the effect of analyst coverage on liquidity, which is a measure of information asymmetry that is common in the empirical literature and can be measured directly. Our measure of liquidity is the bid-ask spread, which is relatively easy to estimate. We analyze how the expected bid-ask spread, which reflects the information asymmetry that remains after the disclosure game, is affected by analyst coverage. Note that the bid-ask spread in our model reflects difference in information quality among market participants, while the price efficiency

[10]In particular when there are no traded options of the firm's stock, which can be used to estimate the implied volatility.

measure analyzed in the previous section reflects the uncertainty of the market overall about the fundamentals. Although these two measures are related, the two constructs capture different aspects of the information environment.

We extend our disclosure model by adding a stylized trading stage. Trading occurs after the manager's potential voluntary disclosure decision and after the potential release of the analyst's report. Let I be the public information by the end of the disclosure stage, then $\mu \equiv \Pr(x = 1 \mid I)$ is the public belief about the firm's terminal value at the beginning of the trading stage.

The trading stage is a static version of the Glosten and Milgrom (1985) model (henceforth GM). The trading stage involves a competitive market maker and a single trader. The trader can either buy or sell one unit (share) of the firm's stock. With probability $1-p$ the trader is a "*liquidity trader*", who sells or buys independently of the firm's value (for example, due to a liquidity shock). The liquidity trader chooses to sell or to buy one unit with equal probabilities (our results are robust to changes in the probabilities). With probability $p \in (0,1)$ the trader is *strategic* and trades to maximize his trading profit given his information (we assume this trader obtains a payoff of zero in case he does not trade). With probability $\chi \in (0,1]$ the strategic trader is *informed*, and knows the firm's terminal value, x. In Section 6.2 below we endogenize χ, but for now we just treat it as an exogenous parameter, that is, assume that the trader is strategic and informed with probability $p\chi$. With probability $p(1-\chi)$ the trader is strategic but does not have additional information, that is, has a belief of μ.

The risk neutral market maker does not have private information about the firm value or the type of the trader. The market maker operates in a competitive market (which is not modeled), and sets prices that lead to zero expected profit. Given the initial belief μ, the bid price, $b(\mu)$, is set to equal the expected value of the asset conditional on the trader selling a share. Similarly, the ask price, $a(\mu)$, is set to equal the expected value of the asset conditional on the trader buying a share. The term $a(\mu) - b(\mu)$ is the bid-ask spread, and we show below it is always positive.

5.1 Prices and the Bid-Ask Spread

In this section we provide a short derivation of the bid and ask prices and the resulting bid-ask spread in a standard static GM setting. Readers who are familiar with this derivation can skip directly to Lemma 2.

First note that a strategic uninformed trader never trades. Such a trader understands that since the market maker breaks even, and an informed trader gains an information rent, an uninformed trade is expected to generate a loss. Moreover, a strategic informed trader always buys if $x = 1$ and sells if $x = 0$. This is because the public belief in the beginning of the trading stage, μ, is between zero and one, and thus the bid and ask prices are also between zero and one.[11] Given that the informed strategic trader always trades, it is clear that no trade does not convey additional information on the asset's value. Therefore, the posterior beliefs following no trade is $E\left[\tilde{x} \mid \mu, \text{no trade}\right] = \mu$.

Let "purchase" and "sale" denote the events where the trader purchases or sells one unit, respectively. For a given public belief μ, the probability of a "purchase" event is $p\chi\mu + (1-p)\frac{1}{2}$. Conditional on a purchase event, the probability that the trader is informed is $\Pr\left(\text{informed} \mid \text{purchase}\right) = \frac{p\chi\mu}{p\chi\mu+(1-p)\frac{1}{2}}$. Thus, the market maker sets an ask price that equals

$$\begin{aligned} a(\mu) \equiv E\left[\tilde{x} \mid \mu, \text{purchase}\right] &= \frac{p\chi\mu}{p\chi\mu + (1-p)\frac{1}{2}} \cdot 1 + \frac{(1-p)\frac{1}{2}}{p\chi\mu + (1-p)\frac{1}{2}} \cdot \mu \\ &= \frac{1-p+2p\chi}{1-p+2p\chi\mu}\mu. \end{aligned} \tag{6}$$

A similar calculation result in a bid price of

$$b(\mu) \equiv E\left[\tilde{x} \mid \mu, \text{sale}\right] = \frac{1-p}{1-p+2p\chi(1-\mu)}\mu. \tag{7}$$

It is easy to see that $b(\mu) < \mu < a(\mu)$ for any $\mu \in (0,1)$ and $p \in (0,1)$, and that both $a(\mu)$ and $b(\mu)$ are strictly increasing in μ.

The bid-ask spread, which we denote by $\Psi(\mu)$, is the difference between the ask and

[11] For simplicity, assume that in the zero probability events that there is no uncertainty about x in the beginning of the trading stage, that is, $s = \mu = 1$, and $s = \mu = 0$, the informed trader still chooses to buy and sell, respectively, for a fair price.

the bid prices above, that is,

$$\Psi(\mu) \equiv a(\mu) - b(\mu), \tag{8}$$

where $a(\mu)$ and $b(\mu)$ are defined in (6) and (7), respectively. The following Lemma provides some properties of the bid-ask spread.

Lemma 2. *The bid-ask spread, $\Psi(\mu)$, has the following properties:*

1. *It is a strictly concave inverse U-shape function of μ.*

2. $\Psi(0) = \Psi(1) = 0$.

3. *For any $\mu \in (0, 1)$,the spread is increasing in p and χ.*

The proof is trivial and merely involves differentiation of (8) and thus is omitted. The main characteristic of the bid-ask spread that we will be using is the concavity in the beliefs, μ.

5.2 Disclosure Decision in the Extended Model

In this section we analyze the manager's disclosure strategy when she knows that a trading stage occurs following her disclosure decision. The basic model in Section 3 assumes risk neutral pricing based on all publicly available information, that is, assumes $P = \mu$. In the extended model, however, there are three possible prices: an ask price $a(\mu)$ when the trader buys one unit (a "purchase"), a bid price $b(\mu)$ when the trader sells one unit (a "sale"), and μ when there is no trade. From an outsider's point of view, such as the market maker, the expected price is always μ. This can be easily seen using the law of iterated expectation:

$$\begin{aligned} E\left[P;\mu\right] =& \Pr\left(\text{purchase};\mu\right)\cdot a(\mu) + \Pr\left(\text{sale};\mu\right)\cdot b(\mu) + \Pr\left(\text{no trade};\mu\right)\cdot \mu \\ =& E\left[\tilde{x} \mid \mu\right] = \mu. \end{aligned}$$

If an informed manager chooses to disclose her signal s, then this leads to a public belief $\mu = s$. Following disclosure, because the manager has the same information as the market maker and the public regarding the value of the firm, the informed manager expected price, or payoff, is also $U^{\mathrm{D}}(s) \equiv E[P; s] = s$. This is not the case, however, if neither the manager nor the analyst disclose. In such a case an informed manager has a better prediction than the market maker about the information of the informed trader, and thus about the probabilities of purchase and sale events. A manager with a better signal s, is more optimistic about the possibility that the trader will purchase and the price will be $a(\mu)$, and gives a lower probability to a price of $b(\mu)$. Thus, in contrast to the basic model, the informed manager's payoff conditional on no disclosure is increasing in her type.

Nevertheless, one can show that the extended model has a threshold equilibrium and, moreover, this threshold is the same as the one in the basic model. The following proposition describes the equilibrium of the extended two-stage model.

Proposition 3. *The unique equilibrium of the extended model has a threshold disclosure strategy, σ^*. The threshold σ^* is the unique solution of the indifference condition (2), as in the basic model.*

Proposition 3 entails that the threshold is independent of χ, the probability that the trader is informed, and is the same as the threshold in a disclosure game where prices simply equal to the expected fundamental. Therefore, all the results of Section 3, including Proposition 1 about the effect of changes in analyst coverage, hold in this model as well. The proof is in the Appendix, but to see the intuition behind the result recall that in the basic model the public belief following no disclosure is $\mu = \sigma^* = P^{\mathrm{ND}}(\sigma^*)$ (Fact 1). To see that σ^* is the threshold also in the extended model note that type σ^* has the same beliefs as the market following disclosure as well as no-disclosure. Thus, for the same argument as in the previous paragraph, this type expects an average price of $\mu = \sigma^*$ following disclosure as well as following no-disclosure.

5.3 Disclosure and Liquidity

The public information in the trading stage is a result of information that is disclosed by the manager and the analyst at the disclosure stage. We now study how the parameters of the disclosure game affect illiquidity that results from information asymmetry.

Our measure of illiquidity, $\mathrm{IL}(q,r) > 0$, which depends on the parameters of the disclosure game, q and r (as well as the parameters of the trading stage, p and χ, which are treated as given), is the expected bid-ask spread, and is given by

$$\mathrm{IL}(q,r) \equiv E\left[\Psi(\mu) \mid q,r\right].$$

When we refer to liquidity we refer to $\mathrm{L}(q,r) \equiv \mathrm{IL}(q,r)^{-1}$.

We can identify three mutually exclusive events that lead to different amounts of public information following the disclosure stage:

1. With probability $q \cdot g_I(r) + (1-q)\, g_U(r)$ the analyst observes and publishes s. In this case all realizations of the signal become public.

2. With probability $q\,(1-g_I(r))\,(1-F(\sigma^*))$ the analyst is uninformed, but the manager is informed and discloses all realized signals above σ^*.

3. With probability $1-(q \cdot g_I(r) + q\,(1-g_I(r))\,(1-F(\sigma^*)) + (1-q)\, g_U(r))$ there is no disclosure; the analyst is uninformed, and the manager is either uninformed, or informed but withholds signals that are below σ^*. The expectation of public belief in this case is σ^* (Proposition 3).

Given these events, and the resulting distribution of beliefs, we can write the expected bid-ask spread as

$$\begin{aligned}\mathrm{IL}(q,r) = &\left[1-(q \cdot g_I(r) + q\,(1-g_I(r))\,(1-F(\sigma^*)) + (1-q)\, g_U(r))\right] \cdot \Psi(\sigma^*) \\ &+ \left[q \cdot g_I(r) + (1-q)\, g_U(r)\right] \cdot E\left[\Psi(s)\right] \\ &+ q\,(1-g_I(r))\,(1-F(\sigma^*)) \cdot E\left[\Psi(s) \mid s \geq \sigma^*\right].\end{aligned} \tag{9}$$

We are interested in the effect of analyst coverage, r, on liquidity. The difficulty in the analysis is similar to the one described in Section 4, and stems from the fact that an increase in r has an ambiguous effect on the probability that the signal becomes public, as well as the effect of the underlying uncertainty following no disclosure. IL, however, captures a different economic construct than PEF. In particular, expected liquidity is not a linear function of PEF, and hence Proposition 2 does not imply that the expected liquidity increases in r. For example, if a certain signal s is disclosed with higher probability following an increase in r, then this clearly has a positive effect on price efficiency because disclosure results in $P = s$. However, since the spread is non-monotone (Lemma 2), disclosure of s may actually decrease liquidity if $\Psi(\sigma^*) < \Psi(s)$. Thus, the direct effect of an increase in coverage on IL is more nuanced than the effect on PEF. Nevertheless, it is possible to show that analyst coverage always has a total positive effect on liquidity:

Proposition 4. *The expected bid-ask spread,* $\mathrm{IL}(q, r)$*, is decreasing in* r *for any* $q \in (0, 1)$*, that is*

$$\frac{d\mathrm{IL}\,(q, r)}{dr} < 0.$$

The proof of proposition 4 is in the Appendix. The key part of the proof is to show that an increase in r has a direct effect of decreasing illiquidity, despite the fact that $\Psi(\sigma^*) < \Psi(s)$ for some values s, as described above. This proof relies on the concavity of the bid-ask spread function (Lemma 2). The proof uses similar intuition as in the proof of Proposition 2 to show that the change in disclosure threshold plays a second order effect where the direct effect is of first order.

The result of Proposition 4, which provides additional motivation for the informational benefit of analyst coverage, is consistent with the empirical findings of the papers we have presented in the introduction. Kelly and Ljungqvist (2012) and Ellul and Panayides (2018) find that following an exogenous negative shock to analyst coverage, there is a decrease in the liquidity of the affected firms. Balakrishnan et al. (2014) as well as Ellul and Panayides (2018) find evidence that the decrease in liquidity is partially reversed by an increase in voluntary disclosure (in form of earning guidance and press releases), but overall liquidity still decreases, in line with the results of this section.

6 Extensions

In this section we discuss two extensions of the model. First, we discuss how changes in the information structure may affect our main results. Second, we discuss how the trading and liquidity results of Section 5 are affected when the strategic trader chooses how much information to acquire.

6.1 Analyst Observes a Noisier Signal

So far we have assumed that the analyst and the firm manager have a potential to learn the same information. A natural extension is the case in which the analyst's information is less precise than the manager's information. Formally, assume a model similar to the one presented in Section 2, where the information endowment of the analyst and the manager is uncertain and possibly correlated, except that now the analyst only observe a noisy signal of s, which we denote by s^a. Following an analyst report that is not accompanied by a manager's disclosure, in contrast to the basic setup, some uncertainty about s remains. This uncertainty is captured by a posterior $f(s \mid s^a)$.

Note first that in this model, in contrast to the basic model (see Remark 2), the order of moves matters. If the manager discloses before the analyst and does not know the analyst's signal s^a, then she is uncertain regarding her payoff in case she does not disclose. Because the manager's expectations about this payoff depend on her private information, the analysis is convoluted and the model becomes intractable. Hence, we focus here on the case in which the manager discloses after the analyst. In such a case, a threshold equilibrium exists and it is unique. In this equilibrium, the manager discloses according to a threshold that depends on the analyst's report, $\sigma^*(s^a)$, and discloses following no report using a threshold $\sigma^*(\emptyset)$. For brevity, we shall not provide a formal characterization of the model and instead discuss some of its properties.

First, because there are multiple thresholds, it is more difficult to measure how much information is disclosed by the manager, and the effect of a change in analyst coverage cannot necessarily be described as "crowding out" or "crowding in". To see why, note that in this model an analyst's report informs the market not only of the fundamental value of the firm s, but also of the information endowment of the manager. An increase in coverage (and thus in the probabilities g_I and g_U) may decrease the probability that the manager is informed given that no analyst report is published, thus increasing $\sigma^*(\emptyset)$, and at the same time increase the probability that the manager is informed given an analyst report, thus decreasing $\sigma^*(s^a)$ for any s^a.[12]

Second, our results regarding the effect of analyst coverage on market quality continue

[12] In this model the quality of outside information depends on the probabilities g_I and g_U that the analyst observes s^a, as well as on the precision of s^a, captured by $f(s \mid s^a)$. For comparability with the basic setup, we treat $f(s \mid s^a)$ as given, and assume that changes in coverage affect only g_I and g_U.

to hold given additional assumptions on the analyst's information production technology (that is, g_I, g_U, and s^a). As in the basic model, the direct effect of an increase in analyst coverage on market quality continues to be of first order compared to the effect of the change in corporate disclosure. This can be proven using a similar, albeit more complex, analysis as in Sections 4 and 5. Our results continue to hold as long as we make additional assumptions to assure that public information is sufficiently better following an analyst report compared to no report. Appendix B provides a more formal treatment of price efficiency in a model with a noisy analyst signal. Though a full analysis is complex, we show that in the simple case of $g_U = g_I$, that is, when the information endowment of the manager and the analyst are uncorrelated, price efficiency never deteriorates when coverage increases, and strictly increases in a normal distribution example. Though a similar analysis regarding liquidity is more complex, we believe similar results can be obtained.

6.2 Information Acquisition

In this section we consider information acquisition by traders and how it is affected by changes in analyst coverage, and the resulting change in corporate disclosure. We introduce information acquisition to the model presented in Section 5 by endogenizing the probability that a strategic trader is informed, χ. We assume that the strategic trader chooses, at the beginning of the game, whether to pay a cost and learn the terminal value x before trading. As in Grossman and Stiglitz (1980) and Diamond (1985), we assume an exogenous and constant acquisition cost of size c.[13] We allow for mixed strategies, so that χ is the probability that the strategic trader chooses to be informed. The model now contains three periods:

$t = 0$ The strategic trader chooses a probability χ for paying c and becoming informed.

$t = 1$ Disclosure stage, as described in Section 2.

[13] An alternative assumption is that the strategic trader has a random information acquisition cost c, observed by the strategic trader before information acquisition. In this case, information acquisition is characterized by a threshold c^*, such that the trader acquires information if and only if $c \leq c^*$, and the value of c^* determines χ. All of our results continue to hold under this assumption.

$t = 2$ Trading stage, as described in Section 5. If the trader is strategic and informed then he observes x before trading.

The strategic trader's information acquisition decision is based on his beliefs about the expected profit from trade at period $t = 2$. In equilibrium, this decision affects the bid-ask spread and the profit from trading. Note that the disclosure strategy in $t = 1$ is independent of the parameters of the trading stage, including χ (Proposition 3).

One can show that *the model admits a unique equilibrium*. In this equilibrium, the probability of becoming informed, χ, is weakly decreasing in the acquisition cost c. For low and high levels of c the strategic trader acquires information with probability $\chi = 1$ and $\chi = 0$, respectively. For intermediate levels of c, the strategic trader chooses a mixed strategy, that is, $\chi \in (0, 1)$. In this case, the expected trading profit of the informed trader equals to c. The uniqueness of the acquisition strategy can be proven by showing that the profit from trade decreases in χ. All other aspects of the equilibrium are similar to the one in Section 5.

A second result is that *an increase in analyst coverage weakly decreases information acquisition by traders*. That is, an increase in r decreases χ. We briefly outline the proof by analyzing the expected profit from trade. Because in equilibrium the market maker breaks even on average, so does the trader, that is, $(1-p)\Pi^\ell + p\chi\Pi^s + p(1-\chi)0 = 0$, where Π^ℓ and Π^s is the expected profit of a liquidity trader and an informed strategic trader, respectively. For a given prior μ, the liquidity trader's expected profit is

$$\Pi^\ell \equiv 0.5\left(\mu - a(\mu,\chi)\right) + 0.5\left(b(\mu,\chi) - \mu\right) = -0.5\Psi(\mu,\chi), \tag{10}$$

and thus, due to the break-even condition, the profit of the strategic informed trader is

$$\Pi^s = \frac{1-p}{2p\chi}\Psi(\mu,\chi).$$

At the beginning of the game, the expected trading profit of a strategic trader from acquiring information is

$$E\left[\Pi^s \mid q, r, \chi\right] = \frac{1-p}{2p\chi} E\left[\Psi(\mu) \mid q, r, \chi\right],$$

where the expectation is with respect to μ, the expected value following the disclosure stage. In equilibrium the expected profit of an informed trader is increasing in the expected spread, which is our measure of illiquidity (Section 5.3).

Proposition 4 entails that, for a given χ, an increase in analyst coverage decreases the expected spread. Thus, following an increase in coverage, informed trading is less profitable. In the case where $\chi \in (0, 1)$, so that the expected trading profit of the informed trader is c, this leads to a new equilibrium with a strictly lower χ.

These results are in line with the findings of Ellul and Panayides (2018). They use a statistical method to identify informed trading, and find that informed trading has increased in stocks that have experienced a termination of analyst coverage. Moreover, the profitability of informed trades has also increased following termination of coverage.

A corollary of the above is that *an increase in analyst coverage improves liquidity even when information acquisition is endogenous*, that is, Proposition 4 continues to hold. An increase in coverage weakly decreases the fraction of strategic informed traders χ, which in turn may further decrease the bid-ask spread (property 3 in Lemma 2) and improve liquidity. Thus, information acquisition amplifies the effect that is captured by Proposition 4: an increase in r decreases the expected spread more than in a model where the trader's information is fixed.

Our results contribute to the theoretical literature on the relation between corporate disclosure and information acquisition by traders. In contrast to the works of Diamond (1985) and Gao and Liang (2013), who find that increased disclosure crowds out information acquisition, in our setup corporate disclosure and information acquisition are positively correlated – they are both crowded out together by analyst coverage. The difference is due to the introduction of an additional source of information, and due to the fact that corporate disclosure in our paper depends on the type of information that the management has – positive or negative.

7 Concluding Remarks

The vast theoretical literature on voluntary disclosure has focused on settings with a single information provider. In practice, however, the corporate disclosure environment is complex and often characterized by several agents who may acquire private information. Financial analysts are one example of such agents. In this paper we have studied how the possibility that the firm's private information may be revealed by a third party (such as an analyst, the media, a regulator, social media, competitors, suppliers, rating agencies) affects the firm's voluntary disclosure policy and the overall information available to the market. We found that for plausible information structures, an increase in analyst coverage crowds out corporate voluntary disclosure.

We have developed two measures of market quality: the first is the future volatility of prices, which has a natural interpretation of price efficiency in our model as it reflects the extent to which current prices reflect the fundamentals. The second measure is the expected bid-ask spread, which measures illiquidity that arises from information asymmetry. In order to calculate the former measure, we presented a trading stage a la Glosten and Milgrom (1985) that follows the disclosure game. We have shown that an increase in analyst coverage increases market efficiency and liquidity despite the crowding out effect, so that the total effect of public information is positive. The relative importance of corporate versus third party disclosure affects the balance between negative and positive information, which in turn determines the quality of public information and other properties such as the skewness of returns. We have demonstrated the robustness of the results to settings in which the analyst's information is less precise than the manger's information and to settings in which trader's information acquisition is determined endogenously.

Our results provide potential regulatory implication, by implying that if the regulator can increase the probability of discovery of a firm's information by various mechanisms, such as analyst coverage, it always has a positive effect on the information environment. Therefore, as long as actions that facilitate more discovery of firm's private information by a third party are not too costly, they are desired.

A Proof Appendix

Proof of Proposition 2

Proof. Denote by $P^{\mathrm{ND}}(\sigma, r)$ the price given no disclosure by the firm or the analyst, as a function of a given disclosure threshold, σ, and a given analyst coverage r. $P^{\mathrm{ND}}(\sigma, r)$ is given by (1). In addition, define $G(r,\sigma)$ as the PEF function (Equation (5)) for a given disclosure threshold σ and analyst coverage r:

$$\begin{aligned} G(r,\sigma) =& - E\left[(s-P(\sigma,r))^2\right] \\ =& - (1-q)(1-g_U(r))\, E\left[\left(s-P^{\text{ND}}(\sigma,r)\right)^2\right] \\ & - q(1-g_I(r))\, F(\sigma) E\left[\left(s-P^{\text{ND}}(\sigma,r)\right)^2 \mid s\le\sigma\right]. \end{aligned}$$

Note that in equilibrium the manager's disclosure threshold is $\sigma=\sigma^*(r)$ and hence, $\text{PEF}(r)=G(r,\sigma^*(r))$.

We need to show that in equilibrium, PEF is increasing in r, that is $\frac{d\text{PEF}}{dr}>0$. This equals to

$$\frac{d\text{PEF}}{dr}=\frac{dG(r,\sigma^*(r))}{dr}=\frac{\partial G(r,\sigma)}{\partial r}\,|_{\sigma=\sigma^*(r)}+\frac{\partial G(r,\sigma)}{\partial\sigma}\,|_{\sigma=\sigma^*(r)}\,\frac{d\sigma^*(r)}{dr}.$$

A sufficient condition for $\frac{\partial\text{PEF}}{\partial r}>0$ is that (1) $\frac{\partial G}{\partial r}\,|_{\sigma=\sigma^*(r)}>0$ and (2) $\frac{\partial G}{\partial\sigma}\,|_{\sigma=\sigma^*(r)}=0$. We prove those two properties below.

1. Proof that $\frac{\partial G}{\partial r}\,|_{\sigma=\sigma^*(r)}>0$:

   $\frac{\partial G(r,\sigma)}{\partial r}$ is given by

$$\begin{aligned} \frac{\partial G(r,\sigma)}{\partial r} =& (1-q)\, g_U'(r) E\left[\left(s-P^{\text{ND}}(\sigma,r)\right)^2\right] \\ & + q\cdot g_I'(r)\cdot F(\sigma)E\left[\left(s-P^{\text{ND}}(\sigma,r)\right)^2 \mid s\le\sigma\right] \\ & + 2(1-q)(1-g_U(r))\, E\left[s-P^{\text{ND}}(\sigma,r)\right]\frac{\partial P^{\text{ND}}(\sigma,r)}{\partial r} \\ & + 2q(1-g_I(r))\, F(\sigma)E\left[s-P^{\text{ND}}(\sigma,r)\mid s\le\sigma\right]\frac{\partial P^{\text{ND}}(\sigma,r)}{\partial r}. \end{aligned}$$

   Using (1) one can assure that

$$(1-q)(1-g_U(r))\, E\left[s-P^{\text{ND}}(\sigma,r)\right]+q(1-g_I(r))\, F(\sigma)E\left[s-P^{\text{ND}}(\sigma,r)\mid s\le\sigma\right]=0,$$

and thus the last two lines sum to zero. At $\sigma = \sigma^*(r)$ we therefore obtain

$$\frac{\partial G(r,\sigma)}{\partial r}|_{\sigma=\sigma^*(r)} = (1-q)\, g_U'(r) E\left[(s-\sigma^*(r))^2\right] + q \cdot g_I'(r) \cdot F(\sigma^*(r)) E\left[(s-\sigma^*(r))^2 \mid s \leq \sigma^*(r)\right]$$

Since, by definition, $g_U'(r) \geq 0$ and $g_U'(r) \geq 0$, with at least one strict inequality, we obtain $\frac{\partial G(r,\sigma)}{\partial r}|_{\sigma=\sigma^*(r)} > 0$.

2. Proof that $\frac{\partial G}{\partial \sigma}\mid_{\sigma=\sigma^*(r)} = 0$:

   We can rewrite $G(r,\sigma)$ as

$$G(r,\sigma) = -(1-q)(1-g_U(r)) \int_0^1 \left(s - P^{\text{ND}}(\sigma,r)\right)^2 f(s)\,\mathrm{d}s - q(1-g_I(r)) \int_0^\sigma \left(s - P^{\text{ND}}(\sigma,r)\right)^2 f(s)\,\mathrm{d}s.$$

   Differentiating with respect to σ we obtain

$$\frac{\partial G(r,\sigma)}{\partial \sigma} = -2(1-q)(1-g_U(r)) \int_0^1 \left(s - P^{\text{ND}}(\sigma,r)\right) f(s)\,\mathrm{d}s \cdot \left(-\frac{\partial P^{\text{ND}}(\sigma,r)}{\partial \sigma}\right) - q(1-g_I(r))\, 2 \int_0^\sigma \left(s - P^{\text{ND}}(\sigma,r)\right) f(s)\,\mathrm{d}s \cdot \left(-\frac{\partial P^{\text{ND}}(\sigma,r)}{\partial \sigma}\right) - q(1-g_I(r)) \left(\sigma - P^{\text{ND}}(\sigma,r)\right)^2. \tag{11}$$

   To obtain $\frac{\partial G}{\partial \sigma}\mid_{\sigma=\sigma^*(r)}$ observe that: (i) by Fact 1, $\sigma^*(r) = P^{\text{ND}}(\sigma^*(r), r)$. Thus, the third term in (11) equals zero; and (ii) by the minimum principle, $\frac{\partial P^{\text{ND}}(\sigma,r)}{\partial \sigma}\mid_{\sigma=\sigma^*(r)} = 0$. Therefore, the first two terms in (11) also equal zero. Thus $\frac{\partial G}{\partial \sigma}\mid_{\sigma=\sigma^*(r)} = 0$.

□

Proof of Proposition 3

Proof. Let the public expectation of $\tilde{x}$ given no disclosure be some exogenous belief $\mu = P^{\text{ND}}$. I prove the proposition using the following steps:

1. **Type $s = P^{\text{ND}}$ is indifferent:** A manager that observes a signal s and expect a public belief of P^{ND}, expects a payoff of

$$U^{\text{ND}}(s, P^{\text{ND}}) \equiv \Pr(\text{purchase}; s) \cdot a(P^{\text{ND}}) + \Pr(\text{sale}; \text{s}) \cdot b(P^{\text{ND}}) + \Pr(\text{no trade}; s) \cdot \mu. \tag{12}$$

   Due to the law of iterated expectations, $U^{\text{ND}}(s, s) = s = U^{\text{D}}(s)$, so an informed manager with a signal $s = P^{\text{ND}}$ is indifferent whether to disclose or not.

2. **Equilibrium involves a threshold strategy:** From the analysis in Subsection 5.1 we know that a manager with a signal s expects the following probabilities of events:

$$\Pr(\text{purchase}; s) = p\chi s + \frac{1-p}{2}$$
$$\Pr(\text{sale}; s) = p\chi(1-s) + \frac{1-p}{2}$$
$$\Pr(\text{no trade}) = p(1-\chi)$$

   Substituting these probabilities in (12) we can easily see that

$$\frac{\partial U^{\text{ND}}(s, \cdot)}{\partial s} = p\chi\Psi\left(P^{\text{ND}}\right),$$

   where $\Psi\left(P^{\text{ND}}\right)$ is defined using (8). Because, by definition, $\Psi \leq 1$, then $\frac{\partial U^{\text{ND}}(s,\cdot)}{\partial s} \in (0, 1)$ for any s. Thus, given step 1, $U^{\text{ND}}(s, P^{\text{ND}}) \lesseqqgtr U^{\text{D}}(s) = s$ if and only if $s \gtreqqless P^{\text{ND}}$. That is, there is a threshold equilibrium. Moreover, let σ^* denote the equilibrium threshold, then $\sigma^* = P^{\text{ND}}$.

3. **Threshold calculated using the same condition as in the basic model:** Finally, given that there is a threshold equilibrium, in equilibrium the belief following no disclosure by the manager or the analyst satisfies (1), and given step 2 the threshold type is a solution to the fixed point condition (2). This is the same condition as in the basic model and therefore the threshold is the same.

$\square$

Proof of Proposition 4

Proof. For a given and constant value of q, define a function $H(r,\sigma)$ that equals the expected spread conditional on analyst coverage r and a given disclosure threshold σ (which may not be the equilibrium threshold), as follows:

$$H(\sigma,r) \equiv \Pr\mathrm{ND}(r,\sigma)\,\Psi\left(P^{\mathrm{ND}}(\sigma,r)\right) + ((1-q)g_U(r) + q\cdot g_I(r))\cdot E[\Psi(s)] \\ + q\cdot(1-g_I(r))\int_{\sigma}^{1}\Psi(s)\cdot f(s)\,\mathrm{d}s, \tag{13}$$

where

$$\Pr\mathrm{ND}(r,\sigma) \equiv (1-q)(1-g_U(r)) + q(1-g_I(r))F(\sigma) \tag{14}$$

is the probability of no disclosure, and$P^{\mathrm{ND}}(\sigma,r)$, given in (1), is the price following no-disclosure by the manager or the analyst. When evaluated at the equilibrium disclosure threshold, $H(\sigma,r)$ is our measure of illiquidity, that is, $\mathrm{IL}(q,r) = H(\sigma^*(r),r)$. Thus, the total derivative of $\mathrm{IL}(q,r)$ with respect to r is:

$$\frac{d\mathrm{IL}(q,r)}{dr} = \frac{\partial H(\sigma,r)}{\partial r}\,|_{\sigma=\sigma^*(r)} + \frac{\partial H(\sigma,r)}{\partial\sigma}\,|_{\sigma=\sigma^*(r)}\,\frac{d\sigma^*(r)}{dr}. \tag{15}$$

To obtain $\frac{d\mathrm{IL}(q,r)}{dr} < 0$ it is sufficient to show that $\frac{\partial H(\sigma,r)}{\partial r}\,|_{\sigma=\sigma^*(r)} < 0$ and $\frac{\partial H(\sigma,r)}{\partial\sigma}\,|_{\sigma=\sigma^*(r)} = 0$. We establish these sufficient conditions in the two lemmas below.

Lemma 3. $\frac{\partial H(\sigma,r)}{\partial r}\,|_{\sigma=\sigma^*(r)} < 0$.

Proof. We show that $\frac{\partial H(\sigma,r)}{\partial r} < 0$ for any given σ, and hence it also holds for $\sigma = \sigma^*(r)$. Given the continuity of $H(r,\sigma)$ in r, it is sufficient to show that $H(r_h,\sigma) < H(r_l,\sigma)$ for any $r_h > r_l$ and any σ.

1. Using (18), we compute $H(r_l, \sigma) - H(r_h, \sigma)$:

$$\begin{aligned} H(r_l, \sigma) - H(r_h, \sigma) = &\Pr \mathrm{ND}\left(r_l, \sigma\right) \Psi\left(P^{\mathrm{ND}}(\sigma, r_l)\right) - \Pr \mathrm{ND}\left(r_h, \sigma\right) \Psi\left(P^{\mathrm{ND}}(\sigma, r_h)\right) \\ &+ \left[(1-q)\left(g_U(r_l) - g_U(r_h)\right) + q \cdot \left(g_I(r_l) - g_I(r_h)\right)\right] \cdot E\left[\Psi(s)\right] \\ &+ q \cdot \left(g_I(r_h) - g_I(r_l)\right) \int_\sigma^1 \Psi(s) \cdot f(s) \, \mathrm{d}s \\ = &\Pr \mathrm{ND}\left(r_l, \sigma\right) \Psi\left(P^{\mathrm{ND}}(\sigma, r_l)\right) - \Pr \mathrm{ND}\left(r_h, \sigma\right) \Psi\left(P^{\mathrm{ND}}(\sigma, r_h)\right) \\ &- (1-q)\left(g_U(r_h) - g_U(r_l)\right) \cdot E\left[\Psi(s)\right] \\ &- q \cdot \left(g_I(r_h) - g_I(r_l)\right) F(\sigma) \cdot E\left[\Psi(s) \mid s < \sigma\right] \end{aligned}$$

We can therefore establish that $H(r_l, \sigma) - H(r_h, \sigma) > 0$ if and only if

$$\begin{aligned} \Pr \mathrm{ND}\left(r_l, \sigma\right) \Psi\left(P^{\mathrm{ND}}(\sigma, r_l)\right) > &\Pr \mathrm{ND}\left(r_h, \sigma\right) \Psi\left(P^{\mathrm{ND}}(\sigma, r_h)\right) \\ &+ (1-q)\left(g_U(r_h) - g_U(r_l)\right) \cdot E\left[\Psi(s)\right] \\ &+ q \cdot \left(g_I(r_h) - g_I(r_l)\right) F(\sigma) \cdot E\left[\Psi(s) \mid s < \sigma\right]. \end{aligned} \tag{16}$$

2. Now observe from (1) that

$$\Pr \mathrm{ND}\left(r, \sigma\right) \cdot P^{\mathrm{ND}}(\sigma, r) = (1-q)\left(1 - g_U(r)\right) E\left[s\right] + qF\left(\sigma\right)\left(1 - g_I(r)\right) \cdot E\left[s \mid s < \sigma\right].$$

This equation, applied to r_l and r_h, together with some some algebra, leads to

$$\begin{aligned} \Pr \mathrm{ND}\left(r_l, \sigma\right) \cdot P^{\mathrm{ND}}(\sigma, r_l) = &\Pr \mathrm{ND}\left(r_h, \sigma\right) \cdot P^{\mathrm{ND}}(\sigma, r_h) \\ &+ (1-q)\left(g_U(r_h) - g_U(r_l)\right) E\left[s\right] \\ &+ q\left(g_I(r_h) - g_I(r_l)\right) F\left(\sigma\right) \cdot E\left[s \mid s < \sigma\right]. \end{aligned} \tag{17}$$

Observe the similarity between the LHS and RHS of (16) and (17); in the next step we use (17) to prove that (16).

3. We can use (14) to rewrite (17) explicitly as

$$P^{\mathrm{ND}}(\sigma, r_l) = A \cdot P^{\mathrm{ND}}(\sigma, r_h) + B \cdot E\left[s\right] + (1 - A - B) \cdot E\left[s \mid s < \sigma\right]$$

where $A=\frac{\Pr\mathrm{ND}(r_h,\sigma)}{\Pr\mathrm{ND}(r_l,\sigma)}$ and $B=\frac{(1-q)[(g_U(r_h)-g_U(r_l))]}{\Pr\mathrm{ND}(r_l,\sigma)}$. This representation presents $P^{\mathrm{ND}}(\sigma,r_l)$ as an average of $P^{\mathrm{ND}}(\sigma,r_h)$ and various signals. In order to obtain (16) remember that $\Psi(\cdot)$, is a strictly concave function (Lemma 2). Thus, by definition,

$$\Psi\left(P^{\mathrm{ND}}(\sigma,r_l)\right)<A\cdot\Psi\left(P^{\mathrm{ND}}(\sigma,r_h)\right)+B\cdot E\left[\Psi\left(s\right)\right]+(1-A-B)\cdot E\left[\Psi\left(s\right)\mid s<\sigma\right].$$

This inequality is simply (16), and thus implies that $H(r_l,\sigma)>H(r_h,\sigma)$.

□

Lemma 4. $\frac{\partial H}{\partial\sigma}\mid_{\sigma=\sigma^*(r)}=0$.

Proof. Differentiating (13) with respect to σ we obtain

$$\frac{\partial H}{\partial\sigma}=q\left(1-g_I(r)\right)f(\sigma)\left[\Psi\left(P^{\mathrm{ND}}(\sigma,r)\right)-\Psi(\sigma)\right]+\Pr ND\left(r,\sigma\right)\Psi'(\cdot)\frac{\partial P^{\mathrm{ND}}}{\partial\sigma}. \tag{18}$$

To obtain $\frac{\partial H}{\partial\sigma}\mid_{\sigma=\sigma^*(r)}$ observe that: (i) by Fact 1, $\sigma^*(r)=P^{\mathrm{ND}}\left(\sigma^*(r),r\right)$. Thus, the first term in (18) equals zero; and (ii) by the minimum principle, $\frac{\partial P^{\mathrm{ND}}(\sigma,r)}{\partial\sigma}\mid_{\sigma=\sigma^*(r)}=0$. Therefore, the second term in (18) also equals zero. Thus $\frac{\partial H}{\partial\sigma}\mid_{\sigma=\sigma^*(r)}=0$. □

This completes the proof of Proposition 4. □

B Price Efficiency in a Model with Noisy Analyst's Signal

Consider the model described in Section 6.1, in which the analyst's information is less precise than the manager's information. Specifically, assume that the analyst may observe a noisy signal s^a about s, and that s^a, if observed, is published before an informed manager decides whether to disclose s or not. In what follows, we treat the probability that the manager is informed q, as well as the distributions of s and s^a as given and fixed, and consider only a change in the conditional probabilities that the analyst is informed, g_I and g_U.

General analysis of price efficiency. The purpose of this section is to analyze how price efficiency, as defined in Equation (5), behaves in this model. First, consider a game in which $g_U = g_I = 0$, that is, there is no analyst. This game is the model of Dye (1985). For a given probability that the manager is informed q, Let $\mathrm{PEF}_0(q)$ be the price efficiency in this game. Now consider the game in which $g_U = g_I = 1$, that is, s^a is always publicly available. Following a given realization of s^a, the game is similar to the model by Dye (1985) with a posterior probability $f(s \mid s^a)$. The manager, if informed, decides whether to disclose using a threshold strategy $\sigma^*(s^a)$. Let $\mathrm{PEF}_1(q)$ be the ex-ante price efficiency in this game, that is, $\mathrm{PEF}_1(q)$ is a weighted average of price efficiencies that are calculated for any given signal s^a.

In a model with general g_U and g_I, denote by $\Pr s^a \equiv q \cdot g_I + (1-q)g_U$ the overall probability that the analyst observes s^a and publishes a report, and by

$$\begin{aligned}
\hat{q}_1 &= \frac{q \cdot g_I}{(1-q)g_U + q \cdot g_I}, \text{ and} \\
\hat{q}_0 &= \frac{q(1-g_I)}{(1-q)(1-g_U) + q(1-g_I)}
\end{aligned}$$

the probabilities that the manager is informed conditional on an analyst report, and conditional on no analyst report, respectively. Price efficiency equals to

$$\mathrm{PEF}(q, g_I, g_U) = (1 - \Pr s^a)\,\mathrm{PEF}_0\left(\hat{q}_0\right) + \Pr s^a \cdot \mathrm{PEF}_1\left(\hat{q}_1\right). \tag{19}$$

This is simply a result of the law of iterated expectation.

Using (19) we can analyze how price efficiency is affected by a small increase in coverage, that is, an increase in g_I and/or g_U (remember we assume that $g_U'(r) \geq 0$ and $g_I'(r) \geq 0$). The effect of an increase in coverage can be decomposed into two parts:

1. A direct change: an increase in the probability of an analyst report $\Pr s^a$, that increases the relative weight of $\mathrm{PEF}_1(\hat{q}_1)$ and decreases the weight of $\mathrm{PEF}_0(\hat{q}_0)$.

2. An indirect change: changes in $\hat{q}_1$ and $\hat{q}_0$ that affect the manager's disclosure strategy and change $\mathrm{PEF}_1(\hat{q}_1)$ and $\mathrm{PEF}_0(\hat{q}_0)$, respectively.

Uncorrelated information endowment ($g_U = g_I$) If $g_I(r) = g_U(r) = g(r)$, then $\hat{q}_0 = \hat{q}_1 = q$ and $\Pr s^a = g$. Thus, a change in r affects price efficiency only through a direct change in $\Pr s^a$ (effect 1 above). Overall price efficiency therefore increases if and only if $\text{PEF}_1(q) > \text{PEF}_0(q)$.

We first show this is always the case when s^a and s follow a joint normal distribution. Without loss of generality assume that both have zero mean, that is, $s^a = s + u$ where $s \sim N(0, \sigma_s^2)$, $, u \sim N(0, \sigma_u^2)$ and $\text{cov}(s, u) = 0$. Thus, $s \mid s^a \sim N(as^a, b^2\sigma_s^2)$ where $a = \frac{\sigma_s^2}{\sigma_s^2 + \sigma_u^2}$ and $b = 1 - a < 1$. Let z^* be the disclosure threshold in a Dye model when the prior is $N(0, 1)$. Proposition 2 in Acharya et al. (2011) shows that when the prior is distributed $N(\mu, \sigma^2)$, the disclosure threshold is $\mu + \sigma z^*$. Thus, for normal distributions, price efficiency (as defined in (5)) satisfies $\text{PEF}_{N(\mu,\sigma^2)} = \sigma^2\text{PEF}_{N(0,1)}$. An immediate implication is that $\text{PEF}_1(q) = b^2\text{PEF}_0(q) > \text{PEF}_0(q)$.

In the general case we can show that $\text{PEF}_1(q) \geq \text{PEF}_0(q)$ using an argument that follows Hart et al. (2017). We describe the argument informally and point the reader to Hart et al. (2017) for the formal treatment. Consider the game where the analyst always publish a report, that is, $g_I = g_U = 1$, and suppose that, instead of risk neutral pricing, the market ("receiver") can commit at the beginning of the game on any pricing function. Specifically, suppose that the market chooses to ignore the signal s^a: $P = s$ following a disclosure by the manager, and $P = E[s \mid \text{ND}]$ following no such disclosure, where this price is the same as the price in a game without an analyst. Clearly, following such commitment the manager will choose the same disclosure strategy as in a game without an analyst, and price efficiency will be PEF_0. The main result of Hart et al. (2017) is that such a commitment cannot decrease the quadratic loss; that is, price efficiency without such commitment, PEF_1, is equal or greater than PEF_0.

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