Bayesian Persuasion, Incentive Contracting, and Performance Manipulation*

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June 2019

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Abstract

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We study how the design of an information system about an agent's productivity implemented by a principal affects the agent's incentives to create value and manage earnings. We find that, in equilibrium, the principal's choice of an information system and the incentive contract that she offers to the agent are determined by the severeness of the multi-task problem and the real effects of earnings management. Specifically, we identify conditions for the existence of a unique threshold value for the relative marginal productivities of the agent's actions above (below) which the principal strictly prefers an uninformative (a perfectly informative) system. Surprisingly, we also find that the principal always strictly prefers a perfectly informative system if inflated earnings fully revert as cash outflows in future periods. In an extension, we examine the interplay between the firm's information and internal control systems and find that the firm is more inclined to implement a perfectly informative system, if the information it generates is also used to determine the optimal level of internal control.

Keywords: Bayesian persuasion, performance manipulation, earnings management, optimal contracting

1 Introduction

We study the effects of ex ante information design on an agent's incentives to create value and manage accounting income. Our paper is based on an agency model where a principal ("she") must compensate an agent ("he") on the basis of short-term earnings, because the firm's terminal value is realized after the current reporting period and is not contractible. Current earnings reflect the agent's contribution to firm value (i.e., the agent's value-creating activities) but are also affected by some unproductive "window dressing." The manipulation effort inflates current accounting income and diminishes firm value. Both activities are unobservable and personally costly to the agent. The marginal productivities of the agent's value-creating effort and his window dressing are unknown to the parties and are correlated, in a sense that they both depend on a common parameter, representing the underlying firm characteristics. In this study we ask whether the principal can improve the contractual solution of the agency problem by implementing an (optimally chosen) information system that provides the agent with a signal about the unknown common parameter before taking his actions.

As a benchmark case, we first study the role of the tasks productivity for the solution of the contracting problem, assuming that the productivity is known. We find that the agent's relative productivity in producing output and performing window dressing uniquely determines the optimal allocation of effort to the two tasks. Particularly, we find that the optimal task allocation ratio is independent of the incentive rate of the optimal compensation contract. Quite intuitively, we also observe that the severeness of the multi-task problem is increasing in the common parameter, whenever the marginal productivity of the agent's window dressing is more sensitive to changes in the common characteristics than the agent's value-creating activity and vice versa.

In the main part of the paper, we study the principal's joint problem of choosing an optimal information system and determining the optimal compensation contract, assuming the principal commits to an information system at the contracting stage. The information system generates a potentially noisy signal of the unknown productivity parameter that is observed by the agent before he decides on the effort levels. Our

analysis suggests that the design of the information system critically depends on the agent's relative productivity in performing his tasks, on the magnitude of the incentive rate in his compensation contract, and on the degree to which his current earnings management reverts, in terms of negative cash flows, in later periods.

We first show that the principal's problem of designing the optimal information system reduces to the choice between a perfectly informative system and an uninformative one. In the former case, the information system reveals the true value of the common parameter to the agent before he chooses his actions. In the latter case, the agent receives an uninformative signal (or equivalently no signal at all) and allocates his tasks according to his prior expectation about the unknown parameter. We also identify conditions for the existence of a unique threshold value for the agent's incentive rate above which the principal strictly prefers an uninformative information system over a perfectly informative one and vice versa. Perhaps most surprisingly, we find that the firm strictly prefers to provide the agent with a perfectly informative system, if the consequences of earnings management are most severe. In our model, this is the case if an increase of current earnings by a given amount fully reverts as a decline in cash receipts in future periods.

The impetus behind our results is that, for a given incentive rate, both the equilibrium output and the agent's compensation are increasing and strictly convex in the agent's conditional expectation of the common parameter. Since the expected outcome benefits the principal but the expected compensation represents a cost for her, the principal's objective function comprises both a strictly convex and a strictly concave component. As long as the expected benefits are sufficiently large relative to the expected compensation cost, the principal's objective is strictly convex in the agent's conditional expectation of the unknown parameter. In this case, a perfectly informative system benefits the principal from an ex ante perspective (Kamenica and Gentzkow 2011). However, if the agent's incentive rate is sufficiently large, the negative effect of the expected compensation costs dominates, and the principal's objective function can become concave in the conditional expectation of the common parameter. In this case,

the principal incurs an expected loss if she provides the agent with information about the unknown parameter. To avoid the loss, the principal optimally leaves the agent uninformed. And, since a full reversal of earnings in the future can only render the provision of positive effort incentives profitable if the agent is relatively more productive in creating value than in manipulating the firms' earnings, the firm always benefits from providing the agent with perfect information about the unknown firm characteristics in this case.

Comparative static analysis shows that the principal's choice relates closely to the severeness of the multi-task problem. In fact, in the absence of the multi-task problem, the firm always benefits from informing the agent, because this policy allows the agent to tailor his effort choice to his true productivity which unambiguously benefits the firm from an ex ante perspective. However, in the presence of the multi-task problem, the firm faces a trade-off because a more precise information signal also allows the agent to match his window-dressing effort with his actual productivity in manipulating the firm's earnings. The firm thus faces a trade-off between the benefits from allowing the agent to tailor his productive effort and the loss from allowing him to tailor his manipulation effort to his true productivity in performing these tasks. Taking the information system as given, we find that, whenever the marginal productivity of the agent's manipulation effort is more prone to changes in the unknown parameter than the marginal productivity of the agent's value-creating effort, the firm offers the agent a lower incentive rate with a perfectly informative system than with an uninformative one and vice versa. The optimal information system considers this trade-off and the real consequences of earnings management. Specifically, we identify a unique threshold value for the difference between the tasks marginal productivities beyond which the provision of perfect information is no longer optimal for the principal. Lastly, we extend our results by analyzing the interplay between the firm's information and internal control systems. We find that the firm is more inclined to implement a perfect information system if the signal it generates is used to determine the optimal level of internal control.

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This paper belongs to the growing literature on Bayesian persuasion. Kamenica and Gentzkow (2011) formalize the idea that a sender can persuade a receiver to take a preferable action by committing to an information system that informs the receiver about an underlying state. Arya, Glover and Sivaramakrishnan (1997), Göx and Wagenhofer (2009) and Rayo and Segal (2010) also consider ex ante commitment to information dissemination. Several studies have extended the work of Kamenica and Gentzkow (2011) to settings with multiple receivers (Alonso and Camara 2016; Michaeli 2017; Kolotilin, Mylovanov, and Zapechelnyuk 2018), while others have considered settings with multiple senders (Gul and Pesendorfer 2012; Bhattacharya and Mukherjee 2013; Chang and Szydlowski 2016; Gentzkow and Kamenica 2017) and the interaction between ex-ante commitment to public information dissemination and ex-post disclosure of privately observed information (Friedman, Hughes and Michaeli 2018).

Only few papers study the role of Bayesian persuasion in the context of a moral hazard model. Georgiades and Szentes (2018) propose a continuous time agency model, where the principal can acquire informative signals to monitor the agent's effort choice. Boleslavsky and Kim (2018) study a three player sender-receiver game, in which the outcome distribution is jointly determined by the agent's effort and the receivers' actions. To the best of our knowledge, our paper is the first to consider the role of Bayesian persuasion in the context of an earnings management model.

This paper also relates to the literature on performance measure manipulation in moral hazard settings (e.g., Demski 1998; Dutta and Gigler 2002; Beyer, Guttman and Marinovic 2014) which considers various extensions of the standard moral hazard model (Mirrlees 1974; Holmström 1979) assuming that the agent's compensation contract cannot be based on the firm's output but on a performance measure that can be manipulated by the agent. In part of this literature the manager privately observes the firm's actual performance and issues a potentially distorted performance report to the principal who then compensates the agent on the basis of his report (Arya, Glover and Sunder 1998; Dye 1988; Demski 1998).[1] Another part of this literature is based

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[1]In these models, the manager has an incentive to misreport his private information only if the revelation principle does not apply.

on the multi-task version of the moral hazard problem (Holmström and Milgrom 1990; Feltham and Xie 1994) and considers earnings management in single-period settings (Dutta and Gigler 2002; Crocker and Slemrod 2007; Beyer, Guttman and Marinovic 2014) or in multi-period settings (Demski, Frimor and Sappington 2004; Liang 2004; Dutta and Fan 2014). These models commonly assume that the performance measure used for evaluating the agent is not only affected by his productive effort but also by unproductive "window dressing" that the agent can do at a personal cost.

In our model the manager can also inflate current accounting earnings at a personal cost but our focus is on the ability of the principal to improve the contractual solution of the agency problem by implementing an accounting information system that provides the agent with information about the unknown productivity of his tasks. This question relates to the literature studying the consequences of private post-contract, pre-decision information for the optimal solution of a standard moral hazard model (Penno 1984; Baiman and Sivaramakrishnan 1991; Bushman, Indjejikian and Penno 2000). As in our model, these paper's study the consequences of the agent's information about the productive environment, before he decides on his productive effort. However, different from our model, the agent's pre-decision information is privately observed by the agent and is not a signal provided by the information system that is also observed by the principal. Furthermore, these models typically focus on single-task models and do not allow for window dressing.

Our model also relates to the signal jamming models in which a manager can issue a biased accounting report to boost market expectations about the firm's fundamental value (Stein 1989; Fischer and Verrecchia 2000; Dye and Sridhar 2004; Ewert and Wagenhofer 2005). Unlike our model, this literature focuses on the role of the manager's incentives to manipulate the stock price and the market's ability to correct the stock price for the manager's reporting bias, based on the information available to investors. The models in this literature typically do not study the role of productive effort and take the structure of the agent's compensation contract as given. An exception is the model of Goldman and Slezak (2006), who allow for productive effort and endogenous

contracts, in the context of a signal jamming model, but assume that the marginal productivity of the agent's productive effort and his biasing activities are common knowledge and unrelated. Lastly, our paper relates to the literature on internal controls (Ewert and Wagenhofer 2018; Schantl and Wagenhofer 2018; Pae and Yoo 2001).

The paper proceeds as follows. Section 2 describes the economic setting. Section 3 considers a benchmark case with known effort productivity. Section 4 studies how the information system affects the contractual solution of the agency problem. Section 5 extends our results by considering internal controls. Section 6 concludes. All proofs are in the Appendix.

2   Economic setting

A risk neutral manager (the agent, "he") runs a firm on behalf of a risk neutral owner (the principal, "she"). The agent can improve the distribution of the firm's random terminal value x by his productive effort a_r. In line with the multi-task agency literature (Holmström and Milgrom 1990; Feltham and Xie 1994), we assume that the terminal value is not contractible and is realized after the relevant contracting horizon.[2] However, there is a potentially biased performance measure y available for contracting. Subsequently, we refer to y as the firm's accounting earnings.

The distribution of the firm's earnings is affected by the agent's productive effort a_r but also by an unproductive activity a_t that we interpret as performance manipulation or the agent's earnings management. As in the models of Goldman and Slezak (2006) or Ewert and Wagenhofer (2005), we allow that earnings management has a real effect on firm value. Specifically, we assume that a fraction $\alpha \in [0, 1]$ of the difference between the firm's accounting earnings y and the value x reduces the firm's future cash flow. Put differently, the parameter α measures the degree to which the agent's window dressing has real consequences. If $\alpha = 0$, the agent's earnings manipulation has pure accounting

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[2]To keep the analysis of the Bayesian persuasion game tractable, our focus is on the interplay between the firm's information system and the multi-task agency problem. Therefore, we follow Beyer, Guttman and Marinovic (2014) and assume that all players are risk neutral.

consequences but no impact on the firm's cash flow. In contrast, if $\alpha = 1$, increasing accounting income by one dollar today reduces the firm's future cash flow by one dollar.

Both the productive and the manipulation activities are personally costly to the agent. To avoid that differences between the costs are pivotal for our results, we assume that the curvature of the cost function is the same for both activities, so that identical levels of productive and manipulation effort cost the same. Specifically, we assume that the cost $C(\cdot)$ of either activity is increasing and strictly convex. To facilitate the derivation of closed form solutions, we restrict our attention to the case where $C(\cdot)$ is quadratic with $C(0) = 0$ and $C''(\cdot) = c$. Let $s(y)$ denote the agent's compensation as a function of realized earnings y. To keep the model tractable, we follow the literature on the multi-task agency model and assume that the principal relies on an affine compensation contract $s(y) = w + vy$, where w is a lump sum transfer and v is the incentive rate.[3] The ex-post utility of the agent is then $u = s(y) - C(a_r) - C(a_t)$ and the ex-post payoff of the principal sums up to $\pi = x - \alpha(y - x) - s(y)$.

We assume that the firm's fundamental value and its earnings take the form

$$x = r(\theta_r) \cdot a_r + \widetilde{\varepsilon}_x, \tag{1}$$

$$y = r(\theta_r) \cdot a_r + t(\theta_t) \cdot a_t + \widetilde{\varepsilon}_y, \tag{2}$$

and depend on the agent's efforts and two productivity parameters. The productivity parameters are realizations of two random variables $\widetilde{\theta}_r$ and $\widetilde{\theta}_t$ that we define in more detail below. Furthermore, $\widetilde{\varepsilon}_x$ and $\widetilde{\varepsilon}_y$ are noise terms distributed according to some known distribution, with mean zero and unbounded support. Here, $r(\theta_r)$ and $t(\theta_t)$ are affine functions of θ_i so that $r(0) = t(0) = 1$ and $r'(\theta_r) = k_r \geq 0$, $t'(\theta_t) = k_t \geq 0$. It follows that x is increasing in a_r and nondecreasing in θ_r, whereas y is increasing in a_r and a_t and nondecreasing in θ_r and θ_t. Thus the values of the productivity parameters θ_i determine the marginal productivity of the agent's effort levels and thereby his ability to alter the distributions of the firm's terminal value and its accounting earnings. Ceteris paribus, a higher value of $r(\theta_r)$ makes y (and x) a better measure of the

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[3]Since the size of constant is immaterial for the subsequent analysis, we also simply refer to $s(y)$ as a "linear" contract.

agent's productive effort a_r, whereas a higher value of $t(\theta_t)$ makes y a better measure of the agent's earnings management effort a_t. To assure that the firm always finds it worthwhile to induce a positive level of productive effort, we restrict the relevant parameters of our model, so that given the principal's expectations of θ_i it holds that $E[r(\theta_r)] > \alpha \cdot E[t(\theta_t)]$. This condition ensures that the expected marginal benefit of the agent's productive effort cannot exceed the negative real consequences of his earnings management. Since the agent is risk neutral, these assumptions are sufficient to determine the optimal compensation contract.

The productivity parameters θ_r and θ_t capture the aggregate impact of exogenous and firm-specific factors that determine the productivity of the agent's tasks. For the productive task a_r, these factors could represent product or input market conditions or the efficiency of the firm's production process. In contrast, for the unproductive task a_t these factors could represent the legal and the regulatory environment as well as the organization of the firm's accounting and internal control system. Many of these factors are certainly task specific, but others, such as the firm's organizational structures and procedures, its internal reporting and communication system, or its corporate culture could affect the productivity of both tasks. To distinguish between task-specific and nontask-specific factors, we assume that $\widetilde{\theta}_i = \widetilde{\eta}_i + \widetilde{\theta}$, where $\widetilde{\eta}_i, i \in \{r, t\}$ represents the task-specific factors of task i and $\theta \in \Theta = [0, \overline{\theta}]$ represents all firm-specific variables that affect the productivity of both tasks. We assume that $\widetilde{\eta}_r$ and $\widetilde{\eta}_t$ are distributed according to some known distribution $h(\eta_i)$ and normalize their mean to zero. We assume that η_r and η_t are unknown to the players.

We analyze the role of θ for the solution of the agency problem in two steps. First, we solve the agency model assuming that the value of the scale parameter is common knowledge and study how changes of θ affect the structure of the optimal compensation contract, the optimal levels and the allocation of the agent's tasks, and the principal's expected profit. In a second step, we consider a setting, where none of the players observes θ before entering the agency relationship. We use this setting to analyze how information about θ affects the contractual solution to the agency problem and ask how
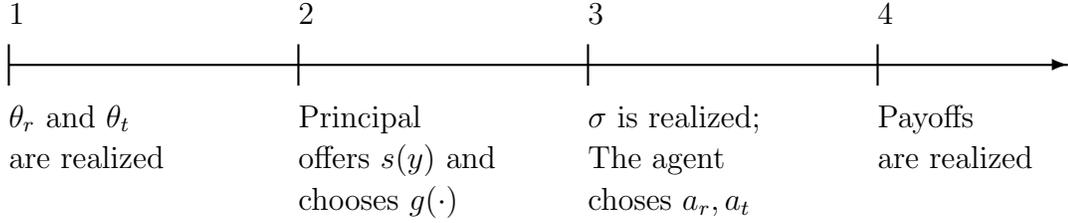
Figure 1: Timeline

the principal can improve efficiency by managing the agent's beliefs about θ.

To this end, we assume that θ is drawn from some (commonly known) probability distribution $f \in \Delta(\Theta)$ with mean $E[\theta]$ and variance $Var(\theta)$. The principal can implement an information system that will provide a publicly observable signal $\sigma \in \Theta$ about the state of nature θ. Each signal realization leads to a posterior belief $\beta \in \Delta(\Theta)$. The information system creates a (commonly known) distribution over posterior beliefs $g \in \Delta(\Delta(\Theta))$. Throughout this section, we refer to g as the design of the information system. Let $supp(f) \subseteq \Delta(\Theta)$ be the set of all possible posteriors. The distribution of posteriors g has to be Bayes-plausible, i.e., $f = \int \beta g(\beta) d\beta$. Let $F(f)$ denote the set of all Bayes-plausible information systems. Then $F(f) = \left\{ g \in \Delta(\Delta(\Theta)) : f = \int \beta g(\beta) d\beta \right\}$. Note that $F(f)$ includes, as special cases, a perfectly informative system with $\sigma = \theta$ as well as an uninformative system. In the former case, the information system perfectly reveals the true value of the state to the agent, whereas in the latter case the agent takes his effort decisions on the basis of his prior information about θ. We assume that whenever indifferent, the principal implements a perfectly informative system.

The timeline of events is presented in Figure 1. At date 1, the states θ_r and θ_t are realized but not observed by the parties. At date 2, the principal offers a contract $s(y)$ to the agent and implements an information system by choosing the distribution $g(\cdot)$ of the posterior beliefs to maximize her expected payoff $\Pi(\cdot) \equiv E[\pi(\cdot)]$. At date 3, the

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information system generates a publicly observable signal σ about θ, the agent updates his beliefs and chooses efforts a_r and a_t to maximize his expected utility conditional on the signal realization $\widehat{U}(\cdot) \equiv E[u(\cdot)|\sigma]$. At date 4, the ex-post payoffs v and π are realized.

3   Benchmark with known effort productivity

We first consider as a benchmark the case where θ is a common knowledge (and therefore no implementation of information system is needed). The solution to the principal's contracting problem, for a given value of θ, is found by maximizing the difference between the expected net terminal value and the agent's expected compensation

$$\Pi(a_r, a_t, \theta) = E[\widetilde{x}] - \alpha(E[\widetilde{y}] - E[\widetilde{x}]) - E[s(\widetilde{y})] \tag{3}$$

subject to the incentive constraints in (4) and (5) and the participation constraint in (6):

$$\frac{\partial E[s(\widetilde{y})]}{\partial a_r} - C'(a_r) = 0, \tag{4}$$

$$\frac{\partial E[s(\widetilde{y})]}{\partial a_t} - C'(a_t) = 0, \tag{5}$$

$$E[s(\widetilde{y})] - C(a_r) - C(a_t) \geq 0. \tag{6}$$

The incentive constraints in (4) and (5) assure that the principal anticipates the agent's optimal effort choices in designing the compensation contract. The participation constraint in (6) guarantees that the agent weakly prefers the principal's contract proposal over alternative employment opportunities, where we normalize the agent's reservation utility to zero without loss of generality. Proposition 1 summarizes the optimal solution of the agency problem with full information about θ.

Proposition 1 *Suppose that the scale parameter θ is publicly observable. The agent optimally allocates his tasks according to the ratio*

$$\frac{a_t}{a_r} = \frac{t(\theta)}{r(\theta)} := \rho(\theta). \tag{7}$$

where $i(\theta) = E_{\eta_i}[i(\theta_i)]$ for $i \in \{r, t\}$. The principal optimally implements a productive effort level

$$\bar{a}_r(\theta) = \frac{1}{c} \cdot \frac{r(\theta)(1 - \alpha \cdot \rho(\theta)^2)}{1 + \rho(\theta)^2} \tag{8}$$

and window dressing effort $\bar{a}_t(\theta) = \bar{a}_r(\theta) \cdot \rho(\theta)$ by offering the agent an optimal incentive rate of

$$v(\theta) = \frac{1 - \alpha \rho(\theta)^2}{1 + \rho(\theta)^2} \in [0, 1] \tag{9}$$

and a fixed component $w(\theta)$ that satisfies (6) as an equality. The optimal incentive rate $v(\theta)$ is decreasing in α and $\rho(\theta)$.

The optimal solution of the agency problem is found by maximizing the expected surplus of the agency considering the agent's incentive constraints

$$a_r = v \cdot \frac{r(\theta)}{c}, \quad a_t = v \cdot \frac{t(\theta)}{c}. \tag{10}$$

The structure of the constraints in (10) illustrates the nature of the multi-task problem. For given effort productivities $r(\theta)$ and $t(\theta)$ the agent's effort levels are proportional to the incentive rate v. However, since y is an increasing function of a_r and a_t, the contract slope only determines how the agent's compensation varies with y but not how the agent allocates his efforts between his productive task and window dressing. The optimal task allocation is determined by the ratio $\rho(\theta) = t(\theta)/r(\theta)$. This ratio measures the relative marginal impact of the agent's activities on the expected value of the performance signal. For a given productive effort $a_r(\theta)$ implemented by the principal, the agent picks a manipulation effort of $a_t(\theta) = a_r(\theta) \cdot \rho(\theta)$ and thereby determines the expected value of the firm's earnings.

The optimal incentive weight $v(\theta)$ assures that the agent's effort maximizes the social surplus, considering the cost of earnings management, whereas the fixed component $w(\theta)$ is used to transfer the expected surplus, net of the agent's reservation utility, to the principal. The optimal incentive rate $v(\theta)$ balances the expected benefits from the agent's productive effort against the agency cost arising from the agent's opportunity to manipulate the performance measure.

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The agency cost comprises two components. On the one hand, the agent's unproductive effort reduces the firm's terminal value by the amount $\alpha \cdot t(\theta)a_r$. Accordingly, the optimal incentive rate is decreasing in α. On the other hand, the multi-task problem raises the total cost of inducing productive effort. Using the task allocation ratio in (7), the total cost of inducing productive effort a_r can be expressed as

$$C(a_r) + C(a_t(a_r)) = C(a_r) \cdot (1 + \rho(\theta)^2), \tag{11}$$

where the factor $1 + \rho(\theta)^2$ scales the actual cost of providing productive effort by the cost of the additional earnings management activities that the agent adopts in equilibrium. The higher $\rho(\theta)$, the higher the effective cost of providing productive effort. This explains why the optimal incentive rate is decreasing in $\rho(\theta)$. Evidently, there are no agency costs only if the agent cannot manipulate the performance signal $(t(\theta) = 0)$, because it is only in this case that a contract based on earnings y is essentially a contract on the firm's fundamental value x. Substituting the agent's equilibrium effort into the principal's objective function yields

$$\Pi(\theta) \equiv \Pi(\overline{a}_r(\theta), \overline{a}_t(\theta), \theta) = \frac{1}{2c} \cdot \frac{r(\theta)^2 \left(1 - \alpha\rho(\theta)^2\right)^2}{1 + \rho(\theta)^2}, \tag{12}$$

which allows us to conclude that the principal's expected equilibrium profit is monotonically increasing in the agent's marginal productivity but decreasing in α and $\rho(\theta)$.

Corollary 1 *The curvature of the optimal task allocation ratio $\rho(\theta)$ is determined by the sign of the expression*

$$\kappa = r'(\theta)/t'(\theta) = k_r/k_t. \tag{13}$$

Suppose that $r'(\theta) = k_r > 0$. If $\kappa < 1$, the ratio $\rho(\theta)$ is increasing and strictly concave in θ; if $\kappa > 1$, the ratio $\rho(\theta)$ is decreasing and strictly convex in θ. If $\kappa = 1$, the ratio is constant and independent of θ.

As summarized in Corollary 1, the productivity parameter θ is pivotal for the severeness of the multi-task problem, as measured by the task allocation ratio $\rho(\theta)$. Since $\rho(\theta)$ is increasing in $t(\theta)$ and decreasing in $r(\theta)$, the agent exerts relatively more productive

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effort as $r(\theta)$ increases and engages more in earnings management as $t(\theta)$ increases. The role of θ in governing this relation is twofold. Whenever the marginal productivity of manipulation effort a_t is more prone to changes of θ than the marginal productivity of productive effort a_r (i.e. $\kappa < 1$), higher values of θ exacerbate the multi-task problem and make y an inferior measure of the agent's productive effort. The opposite is true if changes of θ have a higher impact on the marginal productivity of a_r than on the marginal productivity of a_t (i.e. $\kappa > 1$). Clearly, in the latter case, the principal benefits from higher values of θ whereas in the former case, she suffers as θ gets larger.

4 The role of ex ante information design

In this section, we study how information about θ that the principal makes available to the agent via an information system affects the contractual solution of the agency problem. To this end, we expand the analysis of the previous section assuming that no party observes $\theta \in \Theta$ before entering the agency relationship. We solve the model by backward induction. At date 3, for given compensation contract and signal realization σ, the agent's effort choices maximize $\widehat{U}(\sigma, a_r, a_t)$, i.e.,

$$a_r, a_t \in \arg\max \ v \cdot E[y|\sigma] - C(a_r) - C(a_t), \tag{14}$$

where, using (2), it holds that $E[y|\sigma] = E[r(\theta)|\sigma] \cdot a_r + E[t(\theta)|\sigma] \cdot a_t$. Therefore, the optimal effort levels take the form

$$a_r^o(v, \sigma) = \frac{v}{c} \cdot E[r(\theta)|\sigma] \quad \text{and} \quad a_t^o(v, \sigma) = \frac{v}{c} \cdot E[t(\theta)|\sigma]. \tag{15}$$

The agent's efforts in (15) are increasing in the incentive weight v and in the posterior expectations of $r(\theta)$ and $t(\theta)$. Furthermore, the productivity parameters k_r and k_t determine how the agent's activities are affected by his expectations about θ. Specifically, changes of $E[\theta|\sigma]$ have a higher impact on the agent's productive effort than on his earnings management activities if $k_r > k_t$ and vice versa if $k_r < k_t$. Moreover, in line with Proposition 1, the optimal task allocation ratio is given by $a_t/a_r = E[t(\theta)|\sigma]/E[r(\theta)|\sigma]$ for an arbitrary positive incentive rate v.

At date 3 the expected outcome and the performance measure boil down to

$$E[x(a_r^o(v,\sigma))|\sigma] = \frac{v}{c}(E[r(\theta)|\sigma])^2, \tag{16}$$

$$E[y(a_r^o(v,\sigma), a_t^o(v,z))|\sigma] = \frac{v}{c}\left[(E[r(\theta)|\sigma])^2 + (E[t(\theta)|\sigma])^2\right]. \tag{17}$$

Let $\widehat{\Pi}(\sigma) \equiv E[\pi(\cdot)|\sigma]$ denote the principal's objective function conditional on the realized signal value σ,

$$\widehat{\Pi}(\sigma) = E[(1+\alpha)x(a_r^o(v,\sigma)) - \alpha y(a_r^o(v,\sigma), a_t^o(v,z)) - C(a_r^o(v,\sigma)) - C(a_t^o(v,\sigma))|\sigma].$$

Our next result establishes how the structure of the agent's incentive contract affects the shape of the principal's expected profit at date 3 if the incentive rate v is exogenously given.

Lemma 1 *There exists a critical value $\widehat{v}$ so that $\widehat{\Pi}(\sigma)$ is strictly convex in $E[\theta|\sigma]$ if $v < \widehat{v}$ and strictly concave in $E[\theta|\sigma]$ if $v > \widehat{v}$, where*

$$\widehat{v} = \frac{2\left(k_r^2 - \alpha k_t^2\right)}{k_r^2 + k_t^2}. \tag{18}$$

The observation in Lemma 1 is crucial for the principal's choice of the information structure. Intuitively, the existence of a threshold value $\widehat{v}$ reflects the fact that $E[r(\theta)|\sigma]$ not only determines the agent's effort levels but also the marginal contribution of the productive effort to firm value. Since the agent's equilibrium efforts in (15) are increasing in $E[\theta|\sigma]$, the expected terminal value $a_r^o(v,\sigma)E[r(\theta)|\sigma]$, the real effect of earnings management $\alpha \cdot a_t^o(v,\sigma)E[t(\theta)|\sigma]$, and the agent's effort costs are all monotonically increasing and strictly convex in $E[\theta|\sigma]$. However, since the real effect of earnings management and the agent's effort cost enter the principal's objective with a negative sign, $\widehat{\Pi}(\sigma)$ is essentially the sum of a convex output and three concave cost components. In equilibrium, the difference between the first two components determines the expected net terminal value and the effort cost components are equivalent to the agent's expected compensation. Clearly, if the net terminal value has a stronger impact on the curvature of the principal's objective function than the compensation cost, the objective function

is strictly convex, otherwise it is strictly concave. The critical value $\widehat{v}$ for the contract slope above which the principal's objective function becomes concave is determined by the sign of the second derivative of $\widehat{\Pi}(\sigma)$ which is proportional to the term

$$k_r^2 - \alpha k_t^2 - \frac{v}{2}\left(k_r^2 + k_t^2\right) \tag{19}$$

The term in (19) represents the relative importance of the cash and compensation components on the curvature of the principal's objective function at date 3. The sum of the first two terms in (19) is proportional to the change of the marginal terminal value with respect to an increase of the conditional expectation $E[\theta|\sigma]$. This difference must be positive because otherwise the agent's effort would destroy firm value and the principal would optimally set no effort incentives. The sum of the last two components is proportional to the change of the principal's marginal compensation cost in response to a marginal increase of $E[\theta|\sigma]$. As argued above, this term is negative because the compensation cost is strictly convex in $E[\theta|\sigma]$ but enter the principal's objective with a negative sign.

Thus, evaluating the expressions in (19) and (18) shows that for a given incentive rate v the objective function is more likely to be convex if higher values of $E[\theta|\sigma]$ have a stronger impact on the marginal productivity of the agent's value creating activity (higher k_r) but more likely to be concave if $E[\theta|\sigma]$ has a stronger impact on the marginal productivity of the agent's window dressing activities (higher k_t). Likewise, a stronger real effect on the firm's terminal cash flow (higher α) and a higher incentive rate v render it more likely that the objective function at date 3 is concave in the conditional expectation of θ.

At date 2, the principal simultaneously chooses the contract $s(y)$ and the information system $g(\cdot)$. Of course, these choices are interdependent. On the one hand, the magnitude of the incentive rate determines how information about the unknown parameter θ affects the curvature of the principal's objective function. On the other hand, the information about θ determines her choice of the agent's incentive rate. Put differently, the principal's choice of the information system has a direct effect on her expected profit and an indirect effect on her choice of the optimal incentive rate. The

interplay between these two effects determines the optimal solution of the principal's simultaneous choice problem.

To understand the economic forces driving the simultaneous solution to the principal's choice problem, it is instructive to first focus on the direct effect and study the optimal choice of the information system for a given contract and then solve for the optimal contract in a second step. The principal's choice of $g(\cdot)$ for a given incentive rate v can be presented as

$$\max_{g(\cdot)} \quad \Pi = E_\sigma[\widehat{\Pi}(\sigma)] \tag{20}$$

$$\text{s.t.} \quad f = \int \beta g(\beta) d\beta.$$

Proposition 2 *At date 2, for given v, the principal chooses a perfectly informative system if $v \leq \widehat{v}$ and an uninformative system otherwise.*

The result in Proposition 2 shows that the magnitude of the incentive rate is pivotal for the principal's choice of the information system. If the incentive rate is relatively low so that $v \leq \widehat{v}$, the principal optimally implements an information system that perfectly reveals the value of θ before the agent makes his effort decisions. In contrast, if the incentive rate is relatively high so that $v > \widehat{v}$, the principal prefers that the agent makes his action choice based on his prior expectation about θ. If we compare the results in Lemma 1 and Proposition 2, we can see that a perfectly informative system is the optimal choice if $\widehat{\Pi}(\sigma)$ is strictly convex in $E[\theta|\sigma]$ whereas an uninformative information system is optimal if $\widehat{\Pi}(\sigma)$ is strictly concave in $E[\theta|\sigma]$. This solution is consistent with Kamenica and Gentzkow (2011) and, more fundamentally, with Jensen's inequality.

In line with the expression in (19), our result implies that incentives and information are substitutes in the sense that lower-powered incentives increase the likelihood that the principal is willing to provide information about the state and vice versa. To gain further intuition for the bang-bang character of the optimal information system, note that by the Law of Iterated Expectations, the expected value of the principal's objective

function value in (20) can be presented as:

$$\Pi \equiv E_\sigma[\widehat{\Pi}(\sigma)] = \frac{v}{c} \cdot E_\sigma \left[\left(1 - \frac{v}{2}\right)(E[r(\theta)|\sigma])^2 - \left(\alpha + \frac{v}{2}\right)(E[t(\theta)|\sigma])^2 \right]$$
$$\propto \quad const + \frac{k_r^2 + k_t^2}{2}(\widehat{v} - v)\,Var(E[\theta|\sigma]), \tag{21}$$

The expression in (21) shows that the principal's choice of the information system for a given incentive rate v boils down to choosing the variance of the conditional expectation. The impact of $Var(E[\theta|\sigma])$ on the principal's expected profit depends on the relative magnitude of v vis a vis $\widehat{v}$. If the agent faces lower-powered incentives $(v < \widehat{v})$, the principal benefits from a high variance of the posterior expectation. On the other hand, if the agent faces high-powered incentives $(v > \widehat{v})$, the principal benefits from a low variance of the posterior expectation. Lastly, at the knife-edge case when $v = \widehat{v}$, the principal is indifferent. By the Law of total variance, $Var(E[\theta|\sigma]) \in [0, Var(\theta)]$. For an uninformative information system the variance of the posterior expectation is zero. In contrast, if the the system is perfectly informative, the signal σ perfectly reflects the realization of the actual scale parameter θ and so the variance of the posterior expectation $E[\theta|\sigma]$ equals the prior variance $Var(\theta)$. Taking into account our indifference assumption, the principal chooses a perfectly informative system if $v \leq \widehat{v}$ and an uninformative system if $v > \widehat{v}$. Since $v \in [0, 1]$ by construction, our next observations are immediate from Lemma 1 and equation (19) without a formal proof:

Corollary 2 *Let $\kappa = k_r/k_t$ as defined in Corollary 1.*

(i) *If $\kappa < \underline{\kappa} \equiv \sqrt{\alpha}$, the principal chooses an uninformative information system for any $v > 0$.*

(ii) *If $\kappa \geq \overline{\kappa} \equiv \sqrt{1 + 2\alpha}$, the principal chooses a perfectly informative system for any $v > 0$.*

As shown in Corollary 1, k_r and k_t measure how the marginal productivities of the agent's tasks vary with the productivity parameter θ The parameter κ measures the ratio of these marginal changes. Its magnitude determines the principal's choice of the

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optimal information system for given incentive rate v. There are two limit cases, for which the choice of the information system is independent of the incentive rate v. If κ is small so that $\kappa < \sqrt{\alpha} \leq 1$, the principal implements an uninformative information system for any incentive rate v because $\hat{v} < 0 \leq v$ in this case. In contrast, if κ is large so that $\kappa > \sqrt{1 + 2\alpha} \geq 1$, the principal implements a perfectly informative system for any incentive rate v because $\hat{v} > 1 \geq v$ in this case. Finally, if $\kappa \in K \equiv (\underline{\kappa}, \overline{\kappa})$, the optimal information system depends on the incentive rate $v \in [0, 1]$ and the principal chooses a perfectly informative system only if the optimal incentive rate satisfies $v < \hat{v}$.

Quite intuitively, the range in which the magnitude of the optimal incentive rate is pivotal for the principal's information system choice is shifted to the right if the agent's earnings management activities have real economic consequences ($\alpha > 0$). That is, if earnings management reduces the firm's terminal value, the difference between the marginal productivity parameters k_r and k_t of the value creating activity a_r and the manipulation effort a_t must be larger than in a setting where earnings management only affects accounting income ($\alpha = 0$) to justify the choice of a perfectly informative system. In the latter case, where $\alpha = 0$, it holds that $K = (0, 1)$ so that the principal always strictly prefers an uninformative information system only if $k_r = 0$ and a perfect information system whenever $k_r > k_t$. In contrast if $\alpha = 1$, the principal strictly preferences an uninformative information system if $k_r < k_t$ and a perfectly informative system only if $k_r > \sqrt{3}k_t$.

Apart from these extreme cases, the principal must jointly determine the optimal information system and the optimal incentive rate v. To find the optimal contract, the principal compares the solutions of the two mutually exclusive programs:

$\mathcal{P}_I$ (**Perfectly informative system**):

$$\max_{v \in [0,1]} \quad \Pi = \frac{v}{c} \cdot E_\sigma \left[\left(1 - \frac{v}{2}\right) (E[r(\theta)|\sigma])^2 - \left(\alpha + \frac{v}{2}\right) (E[t(\theta)|\sigma])^2 \right]$$

$$\text{s.t.} \quad v < \hat{v}$$

$$Var(E[\theta|\sigma]) = Var(\theta)$$

$\mathcal{P}_U$ **(Uninformative system):**

$$\max_{v\in[0,1]} \quad \Pi = \frac{v}{c} \cdot E_\sigma\left[\left(1-\frac{v}{2}\right)(E[r(\theta)|\sigma])^2 - \left(\alpha+\frac{v}{2}\right)(E[t(\theta)|\sigma])^2\right]$$

$$\text{s.t.} \quad v \geq \widehat{v}$$

$$Var(E[\theta|\sigma]) = 0.$$

Recall that when the agent is risk neutral and both parties observe θ, the principal provides incentives $v(\theta) = \frac{1-\alpha\rho(\theta)^2}{1+\rho(\theta)^2}$ where $\rho(\theta) = \frac{t(\theta)}{r(\theta)}$. To accommodate for the fact that the parties now only observe a (possibly) noisy signal of θ, we introduce

$$\rho_I = \sqrt{\frac{t(E[\theta])^2 + k_t^2 Var(\theta)}{r(E[\theta])^2 + k_r^2 Var(\theta)}} \quad \text{and} \quad \rho_U = \frac{t(E[\theta])}{r(E[\theta])}.$$

to derive the optimal solution of the firm's simultaneous incentive rate and information system choice problem.

Proposition 3 *For $\kappa = k_r/k_t$ as defined in Corollary 1 and $\alpha < 1$ there exists a unique threshold value $\widehat{\kappa} \in (\underline{\kappa}, \overline{\kappa})$ such that:*

(i) *If $\kappa \in [\underline{\kappa}, \widehat{\kappa})$, the principal offers the incentive rate $v_U = \frac{1-\alpha\rho_U^2}{1+\rho_U^2}$ and implements an uninformative system.*

(ii) *If $\kappa \geq \widehat{\kappa}$, the principal sets an incentive rate of $v_I = \frac{1-\alpha\rho_I^2}{1+\rho_I^2}$ and implements a perfectly informative system.*

If $\alpha = 1$ the principal offers incentives v_I for any value of $\kappa \in [\underline{\kappa}, \overline{\kappa}]$ and implements a perfectly informative system. The optimal incentive rates have the following properties:

(i) *v_I and v_U are strictly increasing in κ and it holds that $v_I \gtreqless v_U$ if $\kappa \gtreqless 1$.*

(ii) *v_I is increasing in $Var(\theta)$ if $\kappa > 1$ and decreasing in $Var(\theta)$ if $\kappa < 1$.*

The results in Proposition 3 show how the direct and the indirect effects of providing the agent with information about θ interact. As shown in Proposition 2 and Corollary 2, the principal's choice of the information system depends on the relative magnitudes

of $\widehat{v}$ and v. If $v \leq \widehat{v}$, the principal perfectly reveals the value of θ to the agent and if $v > \widehat{v}$ he provides no information about θ. The corresponding incentive rates to these solutions for given information systems are v_I and v_U. As we can see, these incentive rates are only identical if $\kappa = 1$. In this case, $k_r = k_t$ and we know from Corollary 1 that this condition implies that the optimal task allocation ratio ρ is independent of θ. Information about θ can thus only affect ρ and thereby the agent's incentive rate if $\kappa \neq 1$. In line with the result for a known parameter value θ in Corollary 1, it can be shown that $\rho(E[\theta|\sigma]) = t(E[\theta|\sigma])/r(E[\theta|\sigma])$ is concave in $E[\theta|\sigma]$ if $k_t > k_r$ but convex in $E[\theta|\sigma]$ if $k_r > k_t$. This observation implies that $\rho_I \gtreqless \rho_U$ if $k_t \gtreqless k_r$ and, since v is decreasing in ρ, it explains why $v_I > v_U$ *if* $\kappa > 1$ and $v_I < v_U$ *if* $\kappa < 1$. In other words, if information about θ exacerbates the multi-task problem ($\kappa < 1$), the agent's incentive rate with perfect information about θ is lower than the incentive rate based on the prior expectations of θ and vice versa.

Considering these indirect effects of the principal's information system choice on the optimal incentive rates, a consistent choice of v and $g(\cdot)$ requires that the condition for the choice of the information system is satisfied by the corresponding incentive rate. That is, an uninformative system can only be optimal if $v_U > \widehat{v}$ and a perfectly informative system can only be optimal if $v_I \leq \widehat{v}$. Since $\widehat{v} > v_I$ if $\kappa > 1$ but $v_I > v_U$, it is easy to see that the principal always strictly prefers a perfectly informative system if $\kappa > 1$. This solution is quite intuitive because for $\kappa > 1$, more precise information about θ not only mitigates the multi-task problem but higher values of κ also increase the principal's expected benefit from allowing the agent to tailor his productive effort decision to his actual productivity. In fact, it can be shown that the difference between $\widehat{v}$ and v_I is strictly positive and increasing in κ as long as $\widehat{v} > v_I$. Since the distance between $\widehat{v}$ and v_I scales the impact of the conditional variance on the principal's expected profit and thereby her expected benefits from providing the agent with more precise information, higher values of κ make the provision of information more attractive to the principal.

Of course, a declining value of κ must have the opposite effect on the principal's

expected profit. On the one hand, information about θ exacerbates the multi-task problem whenever $\kappa < 1$. On the other hand, lower values of κ reduce the expected benefit from allowing the agent to tailor his productive effort to his actual productivity. However, since the principal's objective function is strictly convex in $E[\theta|\sigma]$ if $\kappa = 1$ and $\alpha < 1$, the principal still prefers to implement a perfectly informative system if $k_t > k_r$ provided that $1 > \kappa > \widehat{\kappa}$ and $\alpha < 1$. If $\kappa < \widehat{\kappa}$ the principal strictly prefers to implement an uninformative information system because the benefit derived from tailoring the agent's productive effort to his true productivity is outweighed by the increasing cost of the multi-task problem. In this case, it holds that $\widehat{v} < v_U$ so that providing information to the agent reduces the principal's expected profit. Accordingly, the principal does best to implement an uninformative information system.

Somewhat surprisingly, the second solution can never be optimal if the agent's earnings management activities are fully reversed by cash outflows in future periods ($\alpha = 1$). In fact, the principal always finds it optimal to provide the agent with full information about θ despite the fact that the agent's earnings management have arguably the most severe financial consequences for the principal. Intuitively, this result stems from the fact that for $\alpha = 1$, the principal only provides effort incentives to the agent if $\kappa > 1$. Thus, it must be that $k_r > k_t$ to render the provision of effort incentives beneficial to the principal because otherwise, a positive incentive rate would destroy more value tomorrow than it creates today. However, if $\kappa > 1$, it holds that $\widehat{v} > v_I > v_U$ so that the principal's objective function is always strictly convex in $E[\theta|\sigma]$.

5   The role of internal controls

The analysis of section 4 focuses on the interplay between providing information and monetary compensation in balancing the agent's incentives to create firm value and to manage earnings. Frequently, firms also use their internal control system to increase the reliability of their financial reporting. To allow for this possibility in the context of our model, we assume that the firm can increase the marginal cost of earnings management by adopting an internal control effort e. To keep things simple, assume that a given level

of internal control effort e raises the agent's marginal cost of earnings management by the amount ce . Of course, establishing an internal control system comes at a cost $Q(e)$. In line with the other model assumptions and to facilitate the derivation of closed form solutions, we assume that $Q(e)$ is quadratic with $Q(0) = 0$ and $Q''(e) = n$. To avoid trivial solutions where the internal control system completely discourages the agent's earnings management activities, we assume that n is sufficiently large so that $a_t > 0$.[4]

To analyze how the internal control system affects the solution of the agency problem, we consider two different scenarios. First, we study the case where the firm decides on its optimal level of internal control along with its choice of the information system and the agent's incentive rate at date 2. Second, we also study the case where the firm decides on its optimal control level after implementing the optimal information system and observing the signal about the unknown parameter. Of course, if the firm decides on its optimal control level before observing the signal about the parameter θ, it must fix e based on its prior expectations about the agent's marginal productivity in managing the firm's earnings. In contrast, if the firm fixes e after observing σ, it can tailor its internal control system to its updated information about the agent's productivity.

Proposition 4 *If the firm establishes an optimal control system at date 2, it chooses an optimal control level of $\overline{e} = \alpha \cdot E[t(\theta)]/n$ independent of its optimal information system and incentive rate choices in Proposition 3.*

Proposition 4 suggests that the optimal level of internal control is independent of the firm's choices of the information system and incentive rate. Accordingly, the firm adopts the same policy as defined in Proposition 3. The reason for this result is twofold. First, the firm chooses its optimal control level before the information system generates the signal σ. Therefore, the curvature of the firm's date 3 profit $\Pi(\sigma)$ is independent of the internal control level e so that the condition for the optimal choice of the information system in Lemma 1 and Proposition 2 remain the same. Second, the choice of e is independent of v because the agent corrects his manipulation effort

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[4]In fact, to assure that $a_t > 0$ it must be that $n > \alpha c/v^*$ where $v^* \in \{v_I, v_U\}$, and v_I and v_U are the optimal incentive rates in the absence of internal control as defined in Proposition 3.

by the control level e. In fact, in equilibrium the agent chooses a manipulation effort of $a_t^o = v \cdot E[t(\theta)|\sigma]/c - e$ which is additively separable in e and v and implies that the manipulation cost $C(a_t{}^\circ + e)$ is a function of the incentive rate v only. Of course, despite the fact that the firm's internal control system is independent of its other choices, the firm benefits from its internal control system because the agent reduces his equilibrium level of earnings management. In equilibrium this reduction equals $\bar{e}$ and the firm increases its expected profit by the amount $\alpha E[t(\theta)]\bar{e} - Q(\bar{e})$.

Proposition 5 *If the firm chooses the control system at date 3, there exists a threshold level $\kappa^e < \widehat{\kappa}$ such that:*

(i) *If $\kappa < \kappa^e$, the principal chooses a monitoring level $e_U = \bar{e}$, offers the incentive rate v_U and implements an uninformative system.*

(ii) *If $\kappa > \kappa^e$, the principal chooses a monitoring level $e_I = \alpha \cdot t(\theta)/n$, offers the incentive rate v_I and implements a perfectly informative system.*

The optimal level of internal control is no longer independent of the firm's information system if the firm can wait with the choice of e until the signal σ on the productivity parameter θ is observed. Of course, if the firm finds it optimal to implement an uninformative information system at date 2, it must base its control level on its prior expectation of the agent's marginal productivity in manipulating the firm's earnings and picks the control level $\bar{e}$. In contrast, if the firm chooses a perfectly informative system at date 2, it can tailor its control level to the agent's' true productivity and implement a more effective control system. Quite intuitively, the optimal control level $e_I = \alpha \cdot t(\theta)/n$ is increasing in the marginal productivity of the agent's manipulation effort so that an agent who is more productive in manipulation the firm's earnings faces a higher level of control in equilibrium.

Since the separability of the firm's choices of v and e is preserved, the optimal incentive rates for both information systems are not affected by the change in the order of moves. However, the presence of an internal control system now makes the choice of

a perfectly informative system profitable for firm with a lower profitability ratio κ. In fact, all firms with a profitability ratio $\kappa \in [\kappa^e, \widehat{\kappa})$ that strictly prefer an uninformative system in the absence of internal control, are now better off with a perfectly informative system. This result can be intuitively explained by the fact that the late choice of e increases the firm's expected benefits from more precise information about θ. In the absence of internal control, the firm's information system choice is driven by the trade-off between the benefits derived from tailoring the incentive rate to the agent's productive effort and the costs of exacerbating the multi-task problem. In the presence of an internal control system, the firm also benefits from tailoring its optimal control level to the agent's true productivity which makes the choice of a perfect information system relatively more profitable for a firm with a given productivity ratio κ.

6   Conclusion

We consider an earnings management model where an agent is compensated on the basis of a performance measure that is affected by his productive effort and costly and unproductive "window dressing" activities. We show that the principal's choice of information design is closely related to the severeness of the multi-task problem. In the absence of a multi-task problem the firm always benefits from informing the agent. However, with the multi-task problem, the firm faces a trade-off between the benefits from allowing the agent to tailor his productive effort and the loss from allowing him to tailor his manipulation effort to his true productivities in performing these tasks. Taking the information system as given, we find that whenever the marginal productivity of the agent's manipulation effort is more prone to changes in the unknown parameter than the marginal productivity of the agent's value-creating effort, the firm offers the agent a lower incentive rate with a perfectly informative system than with an uninformative information system and vice versa. The optimal information system considers this trade-off and the real consequences of earnings management. More specifically, we identify a unique threshold value for the difference between the agent's marginal productivities beyond which the provision of perfect information is no longer optimal

for the principal.

Appendix: Proofs

Proof of Proposition 1: The principal maximizes the expected firm value $\Pi(a_r, a_{tr}, \theta)$ subject to the agent's participation constraint (6) and the incentive constraints in (4) and (5). Using (2), the latter take the form

$$\frac{\partial E[s(y)]}{\partial a_r} = v \cdot E_{\eta_r}[r(\theta_r)] = ca_r, \tag{22}$$

$$\frac{\partial E[s(y)]}{\partial a_t} = v \cdot E_{\eta_r}[t(\theta_t)] = ca_t. \tag{23}$$

Combining conditions (22) and (23) and defining $E_{\eta_r}[r(\theta_r)] = r(\theta)$ and $E_{\eta_t}[t(\theta_t)] = t(\theta)$ yields the task allocation ratio in (7). Using the agent's participation constraint to substitute for $E[s(y)]$ and substituting for $a_t(a_r, \theta) = a_r \cdot \rho(\theta)$, the optimal productive effort level a_r from the principal's perspective maximizes

$$\Pi(a_r, a_t(a_r, \theta), \theta) = r(\theta)a_r - \alpha t(\theta)a_t(a_r, \theta) - C(a_r) - C(a_t(a_r, \theta))$$

maximizing this expression with respect to a_r, using the fact that $C'(a_t(a_r, \theta)) = C'(a_r) \cdot \rho(\theta)^2$ yields the optimal effort level in (8). It is straightforward to see that the agent picks the optimal effort levels $\bar{a}_r(\theta)$ and $\bar{a}_t(\theta)$ in (8) if the principal sets the incentive rate equal to $v = (1 - \alpha \cdot \rho(\theta)^2) / (1 + \rho(\theta)^2)$.

Proof of Corollary 1: The optimal task allocation ratio in (7) is determined by the curvature of $\rho(\theta) = t(\theta)/r(\theta)$. Since $r(\theta)$ and $t(\theta)$ are affine and nondecreasing functions of θ, it holds that

$$\rho'(\theta) = \frac{1}{r(\theta)^2} \cdot k_r \left(\frac{1 - \kappa}{\kappa}\right), \qquad \rho''(\theta) = -2 \cdot \frac{k_r}{r(\theta)} \cdot \rho'(\theta),$$

where $\kappa = k_r/k_t$ It follows that $\rho'(\theta) \gtreqless 0$ if $\kappa \lesseqgtr 1$ and that $\rho''(\theta) \gtreqless 0$ if $\kappa \gtreqless 0$ and $k_r > 0$.

Proof of Lemma 1: Simplifying,

$$
\begin{aligned}
\widehat{\Pi}(\sigma) &\equiv E[r(\theta)a_r^o(v,\sigma) - \alpha t(\theta)a_t^o(v,\sigma) - C(a_r^o(v,\sigma)) - C(a_t^o(v,\sigma))|\sigma] \qquad (24)\\
&= \frac{v}{c}\left((1+k_r E[\theta|\sigma])^2 - \alpha(1+k_t E[\theta|\sigma])^2\right) - \frac{c}{2}\left(\frac{v}{c}(1+k_r E[\theta|\sigma])\right)^2\\
&\quad - \frac{c}{2}\left(\frac{v}{c}(1+k_t E[\theta|\sigma])\right)^2\\
&= \frac{v}{c}\left(\left(1-\frac{v}{2}\right)(1+k_r E[\theta|\sigma])^2 - \left(\alpha+\frac{v}{2}\right)(1+k_t E[\theta|\sigma])^2\right).
\end{aligned}
$$

Taking the second derivative,

$$
\begin{aligned}
\frac{\partial^2 \widehat{\Pi}(\sigma)}{\partial E[\theta|\sigma]^2} &\propto (2-v)\,k_r^2 - (2\alpha+v)\,k_t^2\\
&\propto \widehat{v}-v.
\end{aligned}
$$

Hence, $\widehat{\Pi}(\sigma)$ is convex in $E[\theta|\sigma]$ if $v < \widehat{v}$ and concave if $v > \widehat{v}$.

Proof of Proposition 2: Using the expression in (24), the principal maximizes

$$
\Pi \equiv E_\sigma[\widehat{\Pi}(\sigma)] = E_\sigma\left[\frac{v}{c}\left(\left(1-\frac{v}{2}\right)(1+k_r E[\theta|\sigma])^2 - \left(\alpha+\frac{v}{2}\right)(1+k_t E[\theta|\sigma])^2\right)\right]
$$

Now note that $E_\sigma[(1+k_i E[\theta|\sigma])^2] = (1+k_i E[\theta])^2 + k_i^2 Var(E[\theta|\sigma])$ for $i \in \{r,t\}$ which allows us to restate the problem as of maximizing

$$
\begin{aligned}
\Pi \cdot \frac{c}{v} &= \left(1-\frac{v}{2}\right)(1+k_r E[\theta])^2 - \left(\alpha+\frac{v}{2}\right)(1+k_t E[\theta])^2\\
&\quad + \left(\left(1-\frac{v}{2}\right)\cdot k_r^2 - \left(\alpha+\frac{v}{2}\right)\cdot k_t^2\right) Var(E[\theta|\sigma]))\\
&\propto const + \frac{k_r^2 + k_t^2}{2}\left(\widehat{v}-v\right)\cdot Var(E[\theta|\sigma])).
\end{aligned}
$$

Hence, if $v < \widehat{v}$, the principal's expected payoff is increasing in the variance of posterior expectations and the principal will choose a fully-informative system with $\sigma = \theta$. Otherwise, the principal will choose a perfectly uninformative system.

Proof of Proposition 3: Consider first the optimal incentive rates for a given information system. The solution to $\mathcal{P}_I$ is $v = v_I \equiv \frac{1-\alpha\rho_I^2}{1+\rho_I^2}$. For this solution to be feasible,

it has to be that $v_I \leq \widehat{v}$. The solution to $\mathcal{P}_U$ is $v = v_U \equiv \frac{1-\alpha\rho_U^2}{1+\rho_U^2}$. For this solution to be feasible, it has to be that $v_U > \widehat{v}$. Moreover, it is straightforward to verify that $v_U, v_I \in [0,1)$ for $\kappa \in [\underline{\kappa}, \overline{\kappa}]$. Comparing the two solutions, we observe that $v_I \gtreqless v_U$ if $\kappa \gtreqless 1$ for any value of α. Thus, since $\widehat{v} \in [0,1]$ for $\kappa \in [\underline{\kappa}, \overline{\kappa}]$, we conclude that at least one of the two conditions $v_I \leq \widehat{v}$ or $v_U > \widehat{v}$ is always met.

We can distinguish three different cases. First, if $\kappa \geq 1$ it easy to verify that $\widehat{v} > v_I$ and since $v_I > v_U$ in this case, the optimal solution is to provide full information and choose an incentive rate of v_I if $\kappa > 1$. This solution is also optimal if $\kappa < 1$ as long as $\widehat{v} > v_U > v_I$. However, there are two more cases to consider. If $v_U > v_I > \widehat{v}$, the optimal solution is v_U and the principal implements an uninformative information system. However, if $v_U > \widehat{v} > v_I$ both conditions are met. In this case, the principal chooses the incentive rate that yields the largest expected profit. Let $\Pi(v)$ denote the expected profit evaluated at the optimal incentive rate. If $v_U > \widehat{v} > v_I$ and $\Pi(v_U) > \Pi(v_I)$, the firm chooses v_U and leaves the agent uninformed, whereas if $v_U > \widehat{v} > v_I$ and $\Pi(v_U) < \Pi(v_I)$, the firm chooses a perfect information system and sets the incentive rate v_I.

The relevant solution depends on the values of $\kappa \in (\underline{\kappa}, \overline{\kappa})$ and $\alpha \in [0,1]$. Consider now the case where $\alpha = 1$. Since $\underline{\kappa} = \sqrt{\alpha} = 1$ for this case, it must be that $\widehat{v} > v_I > v_U$. Thus, if earnings management fully reverses in terms of future cash flows, the firm optimally chooses a perfectly informative system and sets incentives v_I.

Consider next the case, where $\alpha < 1$. If $\kappa \geq 1$ the solution is the same as for $\alpha = 1$ but if $\kappa < 1$, there is always a set of parameters for which the principal finds it optimal to choose the incentive rate v_U and to leave the agent uninformed. To show existence, note that $\Pi(v_U) = \Pi(v_I) = 0$ if $\alpha = 0$ and $\kappa = \underline{\kappa}(0) = 0$ because in this case it holds that $v_U = v_I = 0$. However, if we evaluate profits and incentive rates for $\alpha \in (0,1)$ and $\kappa = \underline{\kappa} = \sqrt{\alpha} > 0$. We find that

$$\frac{v_U(\underline{\kappa})}{v_I(\underline{\kappa})} = \frac{\Pi(v_U(\underline{\kappa}))}{\Pi(v_I(\underline{\kappa}))} = 1 + \frac{Var(\theta)}{k_r^2 E[\theta]^2 + \frac{2}{1+\alpha}\left(\underline{\kappa}(1+\underline{\kappa})k_r E[\theta] + \alpha\right)} > 1 \qquad (25)$$

which allows us to conclude that $v_U(\underline{\kappa}) > v_I(\underline{\kappa})$ and $\Pi(v_U(\underline{\kappa})) > \Pi(v_I(\underline{\kappa}))$ if $\alpha \in (0,1)$.

Moreover, since $\widehat{v}(\underline{\kappa}) = 0$ for any α, it holds that $v_U(\underline{\kappa}) > v_I(\underline{\kappa}) > \widehat{v}$. It follows that the firm optimally sets an incentive rate of v_U and implements an uninformative information system if $\alpha \in (0, 1)$ and $\kappa = \underline{\kappa}$.

To determine $\widehat{\kappa}$ we first note that the incentive rates and equilibrium profits are monotonically increasing in κ for $\kappa \in (\underline{\kappa}, \overline{\kappa})$. Moreover, since $v_I > v_U$ if $\kappa = 1$, we know that for $\kappa = 1$ it holds that

$$\frac{\Pi(v_I)}{\Pi(v_U)} = \frac{v_I}{v_U} \cdot \left(1 + \frac{(k_r^2 + k_t^2)Var(\theta)}{(1 + k_r E[\theta])^2 + (1 + k_t E[\theta])^2}\right) > 1. \tag{26}$$

Together with monotonicity, conditions (25) and (26) imply that there exists a unique intersection point $\kappa = \widehat{\kappa} < 1$ so that the firm strictly prefers to leave the agent uninformed and sets $v = v_U$ if $\kappa < \widehat{\kappa}$ and to inform the agent perfectly and set $v = v_I$ if $\kappa \geq \widehat{\kappa}$.

Proof of Proposition 4: With internal control, the agent's manipulation cost becomes $C(a_t + e)$ and the second incentive constraint in (15) becomes

$$a_t^o(v, e, \sigma) = \frac{v}{c} \cdot E[t(\theta)|\sigma] - e.$$

Considering the modified incentive constraint and the principal's cost of control, the principal's objective function at date 3 can be written as

$$\widehat{\Pi}(\sigma) = \widehat{\Pi}(\sigma) + \alpha(1 + k_t E[\theta|\sigma])e - Q(e). \tag{27}$$

Since the sum of the last two terms in $\widehat{\Pi}(\sigma)$ is linear in $E[\theta|\sigma]$, we can conclude that condition for the firm's information choice in Lemma 1 and Proposition 2 are not affected by the choice of e. At date 2, the firm maximizes

$$\Pi = E[\widehat{\Pi}(\sigma)] = \Pi + \alpha(1 + k_t E[\theta])e - Q(e)$$

with respect to v, $g(\cdot)$ and e. Since the objective function is additively separable in Π and the terms depending on e, the optimal choices of solution of v, and $g(\cdot)$ are the same as in Proposition 3 and the firm implement's an optimal control level so that

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$\alpha \cdot (1 + k_t E[\theta]) = Q'(e)$. With $Q(e) = ne^2/2$, this condition yields the optimal control level $\bar{e} = \alpha \cdot (1 + k_t E[\theta])/n$.

Proof of Proposition 5: If the firm chooses e after observing σ, the optimal control level maximizes $\widehat{\Pi}(\sigma)$ at date 3. This choice yields $e^o = \alpha \cdot (1 + k_t E[\theta|\sigma])/n$. However, at date 2, the expected profit becomes

$$\begin{aligned}
\Pi &= E[\widehat{\Pi}(\sigma)] = \Pi + E[\alpha(1 + k_t E[\theta])e^o - Q(e^o)|\sigma] \\
&= \Pi + \frac{\alpha^2}{2k} \cdot \left[(1 + k_t E[\theta]) + k_t^2 \cdot Var(\theta) \right].
\end{aligned}$$

Following the proof of Proposition 2, this can be rewritten as

$$\propto const + \left(\frac{v \left(k_r^2 + k_t^2 \right)}{2c} (\widehat{v} - v) + \frac{\alpha^2 k_t^2}{2k} \right) \cdot Var(E[\theta|\sigma])).$$

Since the last term is strictly positive, we conclude that the firm strictly prefers a perfectly informative system even if $v > \widehat{v}$. Following the arguments in the proof of Proposition 3, this observation is equivalent to the preference for a perfectly informative system for a range of values where $\kappa < \widehat{\kappa}$. Thus, all else equal, there must be a critical level $\kappa^e < \widehat{\kappa}$ so that the firm prefers a perfect information system if $\kappa \geq \kappa^e$ and an uninformative system if $\kappa < \kappa^e$.

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