

# Financing Durable Assets

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## Abstract

This paper studies how the durability of assets affects financing. We show that more durable assets require larger down payments making them harder to finance, because durability affects the price of assets and hence the overall financing need more than their collateral value. Durability affects technology adoption, the choice between new and used capital, and the rent versus buy decision. Constrained firms invest in less durable assets and buy used assets. More durable assets are more likely to be rented. Economies with weak legal enforcement invest more in less durable, otherwise dominated assets and are net importers of used assets.

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# 1 Introduction

Durability is an essential feature of capital. Durability varies dramatically across types of assets; the depreciation rates vary from as low as 1% for new residential structures to as high as 31% for computing equipment.<sup>1</sup> How does durability affect financing?

It is tempting to conjecture that durable assets can serve as collateral facilitating financing, as, for example, Hart and Moore (1994) conclude.<sup>2</sup> In contrast, we argue that durable assets are harder to finance. What the previous argument overlooks is that durable assets are also more expensive, exactly because they are more durable. We show that this effect dominates. On the one hand, durability does increase the resale value and hence the collateral value which supports more borrowing consistent with the previous intuition. But on the other hand, durability increases the price of the asset and thus the financing need overall and indeed increases these by more. This means that the down payment required for more durable assets is larger, making them harder to finance. This result obtains as long as the resale value of capital cannot be fully pledged. If the collateral value can be fully pledged, then durability has no effect on the ease of financing and is hence neutral. We do not assume that durable assets are illiquid in any way, and in fact assume that there are frictionless markets for all real assets; that said, we do of course assume that there are financial frictions in terms of collateral constraints due to limited enforcement as otherwise the question of the ease of financing would be moot.

There is a critical distinction between the durability of assets and their pledgeability. Durability affects both the collateral value and the price of the assets and hence the overall financing need, and the net effect of durability is to impede financing. In contrast, pledgeability increases the extent to which assets support borrowing and unambiguously facilitates financing. Unlike prior work, our model distinguishes between durability and

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<sup>1</sup>Durable assets include private and government fixed assets and consumer durables. Fixed assets comprise residential and non-residential structures and durable equipment. Consumer durables include motor vehicles and parts, furnishings and durable household equipment, recreational goods and vehicles, and other durable goods. Private and government fixed assets (not including intellectual property) and consumer durables are \$54 trillion and net of government fixed assets are \$41 trillion in 2012 according to the Bureau of Economic Analysis' *Report on Fixed Assets and Consumer Durable Goods for 2003-2012*; the net worth of households and not for profit organizations is \$70 trillion in 2012 according to the *Flow of Funds*. Thus, tangible durable assets comprise as much as 72% of the aggregate capital stock. We focus on durable assets that are tangible although our arguments apply to intangible assets as well. See Table 3 in Fraumeni (1997) for the Bureau of Economic Analysis' depreciation rate estimates.

<sup>2</sup>They argue that (page 860) “[i]ntuitively, as the assets become more durable, they provide the creditor with the security to wait longer before being repaid. ... And hence the debtor need not set aside as much of his initial borrowing to finance early debt repayments, leaving more to finance the initial investment.”

pledgeability and predicts that durability impedes financing rather than facilitating it. The prediction of our model is empirically plausible in terms of its implications for technology adoption, the choice between new and used assets, and the rent vs. buy decision.

We consider an economy with limited enforcement without exclusion in which firms can default and divert cash flows and a fraction of durable assets and cannot be excluded from markets following default as in Rampini and Viswanathan (2010, 2013, 2018).<sup>3</sup> For this class of economies, they show that the optimal dynamic contract can be implemented with one-period ahead complete markets subject to collateral constraints. The collateral constraints imply that firms' promised repayments cannot exceed a specific fraction of the resale value of capital. These collateral constraints are similar to the ones considered by Kiyotaki and Moore (1997), but these authors do not consider the effect of depreciation or durability in their model.<sup>4</sup>

Durability of capital can be modeled in two ways, either by modeling capital with different geometric depreciation rates or with a finite useful life, such that new assets are more durable than used assets which have a shorter useful life remaining. In the former version the price of capital with different depreciation rates is determined by the cost of producing each type of capital, whereas in the latter version the relative price of used capital is determined in equilibrium. We consider both versions and show that the properties that guarantee that different types of capital are used in the version with geometric depreciation obtain in equilibrium when capital has a finite useful life.

We first consider an economy in which capital is subject to geometric depreciation, which is the typical pattern of depreciation in practice and the canonical assumption. We show that if two types of capital that differ in their depreciation rate are both used in equilibrium, then the more durable type of capital must have a higher down payment per unit of capital and must be more expensive, because otherwise less durable capital would be dominated. At the same time, the more durable type of capital must have a lower frictionless user cost. More durable capital is harder to finance because of the higher required down payment.

Since durability impedes financing, financially constrained firms may adopt domi-

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<sup>3</sup>Limited enforcement is in a similar spirit to Kehoe and Levine (1993) except that we assume that firms which default cannot be excluded from markets going forward whereas they assume that default results in exclusion from intertemporal trade. Chien and Lustig (2010) consider the asset pricing implications of limited enforcement without exclusion in an endowment economy.

<sup>4</sup>Bolton and Scharfstein (1990) and Hart and Moore (1994, 1998) consider the value of the threat of liquidation in models with incomplete contracts; in these models, liquidation implies that borrowers cannot operate subsequently, that is, are effectively excluded from markets going forward, unlike in our model. Townsend (1979) and Holmström and Tirole (1997, 1998) model the limited pledgeability of cash flows based on private information and moral hazard, respectively.

nated technologies, by investing in less durable capital that is of “low quality” in the sense that it would be dominated in the absence of financial constraints. Less durable capital is attractive to constrained firms because of the lower down payment required. In fact, severely constrained firms choose the type of capital by simply comparing down payments. In contrast, financially unconstrained firms compare frictionless user costs and invest in more durable capital only. A larger fraction of investment by firms in economies with weak legal enforcement is in less durable, low quality capital. Indeed, firms in weak legal enforcement economies may adopt less durable types of capital that are dominate in economies with strong legal enforcement, that is, may adopt dominated technologies, because less durable capital is associated with smaller financing needs. When different types of capital are imperfect substitutes, constrained firms substitute away from more durable assets towards less durable assets and, for severely constrained firms, the composition of investment is determined by the relative down payments.

Examples of types of capital that differ in durability, with a more durable, more costly variety and a less durable, less costly one, include brick vs. wood houses, reliable vs. budget cars or trucks, and concrete vs. asphalt roads. In the household context, the choice between durable boots and less durable, cheaper boots is vividly described in Pratchett (1993),<sup>5</sup> and the choice between buying larger packages of storable goods at bulk discounts and buying smaller packages more frequently is studied in Orhun and Palazzolo (2016).<sup>6</sup>

We then consider an economy in which assets last for two periods, such that new assets are durable whereas used assets are non-durable as they have only one period of useful life left. In this economy, the price of used assets is determined in equilibrium and hence endogenous. Since used assets last for only one period their residual value at

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<sup>5</sup>On page 32 of his novel, the author describes “Captain Samuel Vimes [the main character’s] ‘Boots’ theory of socioeconomic unfairness:” “The reason that the rich were so rich, Vimes reasoned, was because they managed to spend less money.

Take boots, for example. He earned thirty-eight dollars a month plus allowances. A really good pair of leather boots cost fifty dollars. But an *affordable* pair of boots, which were sort of OK for a season or two and then leaked like hell when the cardboard gave out, cost about ten dollars. Those were the kind of boots Vimes always bought, and wore until the soles were so thin that he could tell where he was in Ankh-Morpork on a foggy night by the feel of the cobbles.

But the thing was that *good* boots lasted for years and years. A man who could afford fifty dollars had a pair of boots that’d still be keeping his feet dry in ten years’ time, while a poor man who could only afford cheap boots would have spent a hundred dollars on boots in the same time *and would still have wet feet.*”

<sup>6</sup>These authors find that low income households are less likely to buy toilet paper in bulk and also buy on sale less often. The difference to high income households is smaller in the first week of the month, suggesting that liquidity constraints play a key role.

the end of the period is zero and therefore they cannot serve as collateral; thus, the firm has to pay the entire price of used assets up front, that is, the down payment on used assets equals the price. In contrast, new assets can be sold as used assets at the end of the period and have hence positive collateral value allowing firms to partially finance new asset purchases by borrowing. This seems to suggest that new assets are easier to finance than used assets. This is incorrect, however, as it misses the fact that the price of new assets must be higher than the price of used assets precisely because new assets last for two periods whereas used assets for only one. Indeed, the down payment required to purchase a unit of new assets strictly exceeds the purchase price of a unit of used assets, as long as the resale value cannot be fully pledged. A buyer of new assets has to pay up front both the cost of using the assets for one period and the fraction of the resale value that cannot be credibly pledged. This exceeds what a buyer of used assets has to pay up front which is just the cost of using the assets for one period. Therefore, constrained firms buy used assets which require fewer internal funds per unit of capital, whereas unconstrained firms prefer to buy new assets (at least weakly) and sell them when they are used, consistent with the data. The pricing of used assets in equilibrium depends on whether the marginal investor in used assets is unconstrained or constrained; in the latter case used capital trades at a premium and unconstrained firms strictly prefer buying new assets. When economies differ in terms of legal enforcement, there is trade in used capital across economies and weak legal enforcement economies are net importers of used assets. Indeed, it is possible that used assets are dominated in strong legal enforcement economies and all used assets are shipped to weak legal enforcement economies.

More durable assets are more likely to be rented given the larger financing need. Renting or leasing, which we use as synonyms, means that the financier retains ownership of the asset affording a repossession advantage as ownership is the exclusion of others from use. This ease of repossession implies that by renting an asset out the owner can effectively extend more credit than a secured lender can. The cost of renting is modeled simply as a cost of monitoring to prevent abuse of the asset. More constrained firms thus rent assets as Eisfeldt and Rampini (2009) and Rampini and Viswanathan (2013) argue. Here we show that constrained firms rent more durable assets such as structures first and that only more severely constrained firms rent less durable assets such as equipment. Moreover, our theory implies that the increase in durability of cars over the last few decades can explain the increase in car leasing as well as the fact that it is predominantly new cars, which are more durable, that are leased rather than used ones.

Our theory predicts that financial constraints are especially consequential for investment in more durable assets. To the best of our knowledge, this basic prediction about

the relation between durability and financing has not been directly tested to date. Nor have the predictions regarding the composition of investment in terms of durability across economies with different legal enforcement been investigated empirically. That said, the empirical evidence on investment in new and used capital and trade in used assets discussed in the related literature is consistent with the predictions of our theory.

Section 2 studies the effect of durability in a model in which depreciation is geometric and the depreciation rate varies across types of assets and analyzes technology adoption. Section 3 considers the choice between new, durable assets which last for two periods and used, non-durable assets with only one period of useful life remaining. Trade in used capital across economies with different legal enforcement is also analyzed. Section 4 considers how durability affects the decision to rent assets instead of buying them. Section 5 discusses the related literature. Section 6 concludes. Throughout the paper we focus on firms' investment and financing decisions, but the same basic insights apply to households' choice of consumer durables, as discussed in Appendix A.

## 2 A model of durable asset financing

We analyze the effect of the durability of assets on financing in a model with collateral constraints due to limited enforcement. We consider an economy in which entrepreneurs have a choice between two types of durable assets which differ in their geometric depreciation rate.<sup>7</sup> We show that, if both more and less durable assets are used in equilibrium, more durable assets require larger down payments of internal funds, making them harder to finance. Constrained firms may deploy less durable types of capital that are dominated from the vantage point of unconstrained firms. Moreover, the fraction of investment in less durable types of capital is larger in economies with weak legal enforcement. Indeed, firms in such economies may adopt less durable types of assets that are dominated in economies with strong legal enforcement. Finally, we show that when the different types of capital are imperfect substitutes, analogous results obtain for the composition of investment.

### 2.1 Environment

Consider an economy in discrete time with an infinite horizon, with time  $t = 0, 1, 2, \dots$ . There is a continuum of entrepreneurs. In each period, measure  $\rho \in (0, 1)$  of new entrepreneurs are born and are endowed with net worth  $w_0$ . Entrepreneurs survive to the

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<sup>7</sup>Empirically, depreciation of most types of assets is approximately geometric (see, for example, Hulten and Wykoff (1981a, 1981b) and Fraumeni (1997)).

next period with probability  $1 - \rho$  and hence the measure of entrepreneurs alive in every period is 1. Entrepreneur, which we at times refer to simply as firms, are risk neutral and have preferences  $\sum_{t=0}^{\infty} \beta^t d_t$  where  $\beta \in (0, 1)$  and  $d_t$  is the dividend at time  $t$ , with  $d_t \geq 0$ , that is, dividends are non-negative. We assume that the entrepreneurs' time preference discount factor is  $\hat{\beta}$  and let  $\beta \equiv \hat{\beta}(1 - \rho) \in (0, 1)$ . Thus, the economy has stochastic overlapping generations of entrepreneurs and we consider a stationary competitive general equilibrium model with production.

There are three types of goods, output goods (or cash flows) and two types of capital used for production, which are described in more detail below; output goods are the numeraire. Firm can choose between two types of capital that are perfect substitutes in production but differ in durability, one that is more durable and one that is less durable. For ease of reference, we denote these with a subscript  $d$  for durable and  $nd$  for non-durable, although that is a slight abuse of notation. The two types of capital depreciate at geometric depreciation rates  $\delta_d < \delta_{nd}$  each period, with  $\delta_j \in (0, 1)$ , for  $j \in J \equiv \{d, nd\}$ . The law of motion for type  $j$  capital is  $k_{j,t} = k_{j,t-1}(1 - \delta_j) + i_{j,t}$ , where  $i_{j,t}$  is type  $j$  investment (measured in terms of investment goods). Capital of type  $j$  can be produced from output goods using a linear technology; it costs  $q_j$  units of output good to make a unit of type  $j$  capital and capital production is instantaneous. The price of type  $j$  capital is hence constant over time and equals the cost of producing the capital  $q_j$ . Output is produced as follows: An amount of total capital  $k_t \equiv \sum_{j \in J} k_{j,t}$  invested by an entrepreneur at time  $t$  produces output of  $Af(k_t)$  at time  $t + 1$  where  $A > 0$  denotes the total factor productivity and  $f$  is strictly increasing, strictly concave, twice continuously differentiable, and satisfies the Inada condition  $\lim_{k \rightarrow 0} f_k(k) = +\infty$ . The fact that output is a function of total capital  $k_t$  only reflects the assumption that the two types of capital are perfect substitutes in production.

The economy has limited enforcement. Entrepreneurs can default on promises and retain all cash flows and a fraction  $1 - \theta$  of each type of capital where  $\theta \in [0, 1)$ . Importantly, entrepreneurs cannot be excluded from borrowing and saving or the market for capital following default. Our model of limited enforcement is in the spirit of Kehoe and Levine (1993) but we assume limited enforcement without exclusion whereas they assume that default triggers exclusion from intertemporal markets. For our environment, Rampini and Viswanathan (2010, 2013, 2018) show that the optimal long-term dynamic contract can be implemented with one-period ahead complete markets subject to collateral constraints.<sup>8</sup> This equivalent problem with collateral constraints is rather tractable

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<sup>8</sup>For simplicity, we consider an economy with deterministic productivity here and hence have markets for one-period ahead collateralized debt, but our results on the effect of durability can be extended to

and the collateral constraints provide a straightforward decentralization of the optimal contract.<sup>9</sup> Entrepreneurs can thus borrow up to fraction  $\theta$  of the depreciated value of their capital at interest rate  $R$ , which is determined in equilibrium and is constant in the stationary equilibrium we consider, as discussed below.

A stationary competitive equilibrium, which we define formally in the next subsection, is a policy function for each entrepreneur, an interest rate  $R$ , prices of the two types of capital  $q_j$ , for all  $j \in J$ , and a distribution of net worth across entrepreneurs, such that the policy function solves each entrepreneur's problem and the market for loans, goods, and the two types of capital clear. Two equilibrium properties are worth noting here, however: First, in a stationary equilibrium, well-capitalized entrepreneurs, that is, entrepreneurs that have accumulated sufficient net worth and are unconstrained, provide financing and hence the equilibrium interest rate on one-period loans is constant at  $R = \beta^{-1}$ , where  $R$  is the gross interest rate and we define the net interest rate to be  $r \equiv R - 1$ . Second, the prices of the two types of capital  $q_j$  are constant and equal the cost of producing them.

## 2.2 Entrepreneur's problem and equilibrium

We formulate the entrepreneur's problem recursively. Given net worth  $w$ , the entrepreneur chooses current dividend  $d$ , more and less durable capital  $k_d$  and  $k_{nd}$ , borrowing  $b$ , and net worth next period  $w'$ , to solve

$$v(w) \equiv \max_{\{d, k_d, k_{nd}, b, w'\} \in \mathbb{R}_+^3 \times \mathbb{R}^2} d + \beta v(w') \quad (1)$$

subject to the budget constraints

$$w + b \geq d + \sum_{j \in J} q_j k_j, \quad (2)$$

$$Af(k) + \sum_{j \in J} q_j k_j (1 - \delta_j) \geq Rb + w', \quad (3)$$

and the collateral constraint

$$\theta \sum_{j \in J} q_j k_j (1 - \delta_j) \geq Rb, \quad (4)$$

where  $k \equiv \sum_j k_j$ . The budget constraint in the current period (2) ensures that current net worth plus borrowing covers the current dividend plus the cost of investment in the

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an environment with stochastic productivity along the lines of their model as we show in Appendix B.

<sup>9</sup>Our decentralization is in a similar spirit to the one provided by Alvarez and Jermann (2000) except that their solvency constraints are history dependent whereas the collateral constraints in our model do not require history-dependence, facilitating the decentralization.

two types of capital. The budget constraint next period (3) implies that output plus the resale value of both types of capital purchased this period can be spent on loan repayments or carried over as net worth for the next period. The collateral constraint (4) states that the firm can borrow, including interest, up to fraction  $\theta$  of the resale value of the depreciated capital of both types of capital. As argued above, this constraint is induced by limited enforcement, as firms can abscond with all cash flows and fraction  $1 - \theta$  of the depreciated assets and cannot be excluded from financial or real asset markets following default. Notice that we have substituted out investment  $i_j$  and define net worth as output plus assets minus liabilities, that is, the resale value of depreciated capital net of debt repayments, so  $w' \equiv Af(k) + \sum_{j \in J} q_j k_j (1 - \delta_j) - Rb$ .<sup>10</sup> Net worth  $w$  is the entrepreneur's net worth at the beginning of the current period, before the current dividends are paid. We do not assume that entrepreneurs are constrained, and the entrepreneurs do in fact retain earnings by choosing not to pay dividends to accumulate net worth over time. Before characterizing the firm's problem, we define a stationary competitive equilibrium and observe two key properties.

A *stationary equilibrium* is a policy function  $x(w) = [d(w), k_d(w), k_{nd}(w), b(w), w'(w)]$ , an interest rate  $R$ , prices for the two types of capital  $q_d$  and  $q_{nd}$ , and a stationary distribution  $p(w)$  of net worth, such that (i) the policy function  $x(w)$  solves the entrepreneurs' problem in equations (1) to (4) given  $R$ ,  $q_d$ , and  $q_{nd}$ ; (ii) the loan market clears, that is,  $\sum_w p(w)b(w) = 0$ ; (iii) the goods market clears, that is, total output of the surviving entrepreneurs plus the endowments of new entrepreneurs equals total dividends plus net investment

$$\begin{aligned} (1 - \rho) \sum_w p(w) Af(k(w)) + \rho w_0 \\ = \sum_w p(w) d(w) + \sum_w p(w) \sum_{j \in J} q_j k_j(w) \delta_j + \rho \sum_w p(w) \sum_{j \in J} q_j k_j(w) (1 - \delta_j), \end{aligned}$$

where  $k(w) \equiv \sum_{j \in J} k_j(w)$ ; and (iv) the stationary net worth distribution  $p(w)$  is induced by the entrepreneurs' policy function  $x(w)$ .

Net investment includes the replacement of both capital that has depreciated and the depreciated value of capital of entrepreneurs that do not survive. Note also that given that output goods can be turned into capital goods with a linear technology, there are no separate market clearing conditions for the two types of capital as these are subsumed

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<sup>10</sup>The flow budget constraint requires that output plus net new borrowing exceed the current dividend plus investment, that is,  $Af(k_{t-1}) + (b_t - Rb_{t-1}) \geq d_t + \sum_{j \in J} q_j i_{j,t}$ ; substituting for investment using the law of motion for capital  $i_{j,t} = k_{j,t} - k_{j,t-1}(1 - \delta_j)$  and using the definition of net worth  $w_t = Af(k_{t-1}) + \sum_{j \in J} q_j k_{j,t-1}(1 - \delta_j) - Rb_{t-1}$ , we obtain (2).

by goods market clearing. Loans are in zero net supply, that is, entrepreneurs with high net worth are the lenders in the economy and have negative debt.

There are two key properties of equilibrium, which we anticipated above, that are worth noting: First, the interest rate  $R$  is constant and equals  $\beta^{-1}$  in equilibrium, as it is determined by the rate of time preference of well-capitalized entrepreneurs. Second, the prices of the two types of capital equal the cost of producing them, and are hence constant and equal to  $q_j$ .

We are now ready to characterize the firm's problem. First, observe that the problem in (1) to (4) defines a well-behaved dynamic program. The return function is (weakly) concave and the constraint set convex. The operator defined by (1) to (4) satisfies Blackwell's sufficient conditions, implying that there exists a unique value function  $v$  that solves the firm's problem. This value function is strictly increasing and (weakly) concave.

Denote the multipliers on (2) and (3) by  $\mu$  and  $\beta\mu'$  and on the collateral constraint (4) by  $\beta\lambda'$  and let  $\nu$  and  $\nu_j$  be the multipliers on the non-negativity constraints for  $d$  and  $k_j$ , respectively. The first-order conditions are

$$\mu = 1 + \nu, \tag{5}$$

$$\mu q_j = \beta\mu'[Af_k(k) + q_j(1 - \delta_j)] + \beta\lambda'\theta q_j(1 - \delta_j) + \nu_j, \quad \forall j \in J, \tag{6}$$

$$\mu = \mu' + \lambda', \tag{7}$$

$$\mu' = v_w(w'). \tag{8}$$

The envelope condition implies that  $v_w(w) = \mu$ , which is the marginal value of net worth in the current period, and note that  $\mu'$  is the marginal value of net worth next period.

The analysis of the dynamics of firm investment, financing, and dividend policy is facilitated by the fact that the firm's problem is deterministic, conditional on survival. We start by characterizing the firm's behavior in the long run, in which the firm is unconstrained and pays dividends. The first-order condition for borrowing (7) implies that  $\mu \geq \mu'$ , that is, the marginal value of net worth is non-increasing and hence firm net worth is non-decreasing. Moreover, once the firm starts to pay dividends, equation (5) implies that  $\mu = 1$ , and hence  $\mu' = 1$  and  $\lambda' = 0$  from then on. Therefore, the firm reaches a steady state in which it pays dividends and is unconstrained; until it reaches that level of net worth, the firm is constrained and retains all its earnings.

### 2.3 Trade-off between user cost and down payment

To characterize the trade-off between more and less durable capital it is useful to define two terms, the frictionless user cost of capital and the down payment. The frictionless

one-period rental rate or (frictionless) *user cost* in the language of Jorgenson (1963) for type  $j$  capital is

$$u_j \equiv R^{-1}q_j(r + \delta_j). \quad (9)$$

Notice that we assume the rental rate is paid at the beginning of the period, which is of course of no consequence in the frictionless case, but turns out to be appropriate in the economy with limited enforcement.

To assess the financing need of a unit of capital, consider the minimal amount of net worth that the firm needs to purchase a unit of type  $j$  capital which we call the *down payment*. A unit of type  $j$  capital cost  $q_j$  to purchase and the firm can borrow  $R^{-1}\theta q_j(1 - \delta_j)$  against it, and so the down payment per unit of type  $j$  capital is

$$\wp_j \equiv q_j - R^{-1}\theta q_j(1 - \delta_j). \quad (10)$$

The down payment equals the price of the asset minus the present value of the fraction of the resale value of the depreciated capital that the firm can pledge. Let us briefly analyze the relation between the frictionless user cost and the down payment. Using the frictionless user cost defined in equation (9), the down payment per unit of capital (10) can be rewritten as

$$\wp_j = u_j + R^{-1}(1 - \theta)q_j(1 - \delta_j), \quad (11)$$

that is, the down payment is the sum of the frictionless user cost plus the present value of the fraction of the resale value that cannot be pledged. Thus, the financing need of capital exceeds the frictionless user cost because the residual value is only partially pledgeable.

Combining the first-order conditions for capital  $k_j$ , equation (6), and borrowing  $b$ , equation (7), we obtain the investment Euler equation for type  $j$  capital,  $j \in J$ ,

$$1 \geq \beta \frac{\mu' Af_k(k) + (1 - \theta)q_j(1 - \delta_j)}{\mu}, \quad (12)$$

with equality if  $k_j > 0$  ( $\nu_j = 0$ ), where  $\beta \frac{\mu'}{\mu}$  is the firm's discount factor and the ratio on the right is the levered return on investment. This investment Euler equation holds for at least one type of capital with equality whether or not the firm is constrained, since  $k > 0$  and hence  $k_j > 0$  for at least one  $j \in J$ .

Alternatively, using equation (7) to substitute for  $\mu$  in equation (6), the first-order conditions for capital  $k_j$ ,  $j \in J$ , reduces to

$$u_j + \frac{\lambda'}{\mu'} \wp_j \geq \beta Af_k(k), \quad (13)$$

with equality if  $k_j > 0$  ( $\nu_j = 0$ ); the discounted marginal product of capital equals the frictionless user cost plus a penalty for the down payment when the collateral constraint

binds. Thus, the discounted marginal product of capital can exceed the frictionless user cost due to what one might call a collateral wedge. The key insight here is that in the presence of collateral constraints investment is determined by the frictionless user costs and, when the firm is financially constrained, the down payments.

To understand the durability choice, we are interested in considering the case in which both types of capital are used in equilibrium, that is, the case in which neither type of capital is dominated. Suppose  $u_j \geq u_{j'}$  and  $\wp_j \geq \wp_{j'}$ ,  $j \neq j'$ ; then equation (13) shows that type  $j$  capital is at least weakly dominated (and analogously for type  $j'$  capital). Thus, if neither type of capital is dominated,  $u_j > u_{j'}$  and  $\wp_j < \wp_{j'}$ ,  $j \neq j'$ . Suppose that  $u_d > u_{nd}$ ; then  $R^{-1}q_d(r + \delta_d) > R^{-1}q_{nd}(r + \delta_{nd})$  and thus  $q_d > q_{nd}$  which in turn implies that  $\wp_d = u_d + R^{-1}(1 - \theta)q_d(1 - \delta_d) > u_{nd} + R^{-1}(1 - \theta)q_{nd}(1 - \delta_{nd}) = \wp_{nd}$ , implying that type  $d$  capital is dominated, a contradiction. Therefore,  $u_d < u_{nd}$  and  $\wp_d > \wp_{nd}$ , and thus  $q_d(1 - R^{-1}\theta(1 - \delta_d)) > q_{nd}(1 - R^{-1}\theta(1 - \delta_{nd}))$ , which implies that  $q_d > q_{nd}$ ; the effect of durability on the price must exceed the effect on the collateral value. Thus we have proved the following proposition:

**Proposition 1** (User costs and down payments). *Suppose there are two types of capital  $j \in \{d, nd\}$  with depreciation rates  $\delta_d < \delta_{nd}$ , that is, different durability, that are perfect substitutes in production. If both types of capital are used in equilibrium, then more durable capital has a lower frictionless user cost  $u_d < u_{nd}$  but a higher down payment  $\wp_d > \wp_{nd}$ . Moreover, the more durable type of capital must also be more expensive  $q_d > q_{nd}$ .*

This result obtains since otherwise one type of capital would be at least weakly dominated. We emphasize that the prices  $q_d$  and  $q_{nd}$  are determined by technology, specifically the linear costs of producing the two types of capital, and are thus exogenous parameters. However, the relative price of the two types of capital can be endogenized by introducing a vintage structure as we show in Section 3, where we obtain an analogous result with an endogenous relative price.

## 2.4 Choice between more and less durable capital

To analyze the choice between the two types of capital, we define a notion of the user cost of capital for a potentially constrained firm. Rewriting the investment Euler equation (12) as  $u_j(w) \geq \beta \frac{\mu'}{\mu} A f_k(k)$ , we define the user cost of type  $j$  capital  $u_j(w)$  for a potentially constrained firm with net worth  $w$  as

$$u_j(w) = u_j + \beta \frac{\lambda'}{\mu} (1 - \theta) q_j (1 - \delta_j). \quad (14)$$

The user cost for a constrained firm equals the frictionless user cost plus the multiplier on the collateral constraint  $\lambda'$  (scaled by the marginal value of net worth  $\mu$ ) times the fraction of the residual value the firm cannot pledge.<sup>11</sup> Note that the user cost would equal the frictionless user cost for non-durable capital, that is, when  $\delta_j = 1$  as then  $u_j(w) = u_j$ , and would hence be independent of the firm's financial condition in that case. Moreover, if collateral were fully pledgeable, that is,  $\theta = 1$ , the user cost would again equal  $u_j$  and would also be independent of financial conditions. That said, we assume that  $\delta_j < 1$ , for all  $j \in J$ , and  $\theta < 1$  throughout.

For an unconstrained firm, the multiplier on the collateral constraint  $\lambda' = 0$  and the user cost  $u_j(w)$  in (14) equals the frictionless user cost  $u_j$ . Hence, unconstrained firms never use the less durable, “low quality” capital since  $u_d < u_{nd}$  given Proposition 1. Unconstrained firms evaluate the two types of capital based on their frictionless user costs. The less durable type of capital is clearly dominated from the perspective of an unconstrained firm.

Severely constrained firms in contrast adopt the dominated technology, that is, choose to invest in the less durable type of capital despite the higher frictionless user cost. To see this, we rewrite (14), the user cost of type  $j$  capital for a potentially constrained firm, as

$$u_j(w) = \wp_j - \beta \frac{\mu'}{\mu} (1 - \theta) q_j (1 - \delta_j); \quad (15)$$

the user cost equals the down payment paid in the current period minus the part of the resale value of the depreciated capital the firm was not able to pledge, which is recovered next period, and hence depends on the firm's discount factor,  $\beta\mu'/\mu$ .

When  $w$  goes to 0, the budget constraint (2) implies that so must  $k_j$ ,  $j \in J$ , and hence  $k = \sum_{j \in J} k_j$ ; using the investment Euler equation (12) and the Inada condition, we conclude that  $\beta \frac{\mu'}{\mu}$  goes to 0. But then from equation (15) we conclude that  $u_j(w) \rightarrow \wp_j$ . As the firm becomes severely constrained, it evaluates the two types of capital simply based on the required down payments, because it discounts the part of the residual value it recovers next period completely. For a severely constrained firm, the user cost is the down payment. In contrast, the fact that investment decisions are at times based simply on a comparison of down payments is at times interpreted as evidence of mistakes in decision making or behavioral biases. Since by Proposition 1 the less durable capital requires a lower down payment,  $\wp_d > \wp_{nd}$ , such firms choose to adopt otherwise dominated

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<sup>11</sup>We can also write (14) as  $u_j(w) = \frac{\mu'}{\mu} (u_j + \frac{\lambda'}{\mu'} \wp_j)$ , where the term in parenthesis is the expression in equation (13); the user cost for a financially constrained firm is simply the sum of the frictionless user cost plus the (scaled) multiplier on the collateral constraint times the down payment (both scaled by the ratio of the marginal value of net worth next period vs. this period).

technologies. We emphasize that our model implies that there is new investment in dominated technologies. Financially constrained firms may invest in less durable plants and buildings, and may buy less durable types of equipment, despite the fact that these are dominated, because they involve smaller financing needs in terms of internal funds.<sup>12</sup>

We summarize our conclusions regarding the composition of investment as follows:

**Proposition 2** (Durability choice). *Suppose both types of capital are used in equilibrium.*

**(i)** *Unconstrained firms purchase only the more durable capital. (ii) Severely constrained firms purchase only the less durable capital despite it being otherwise dominated; for such firms, the user cost of capital equals the down payment.*

An analogous result obtains when productivity is stochastic as we show in Appendix B. For investment dynamics, this implies that if young firms start out severely constrained, then young firms invest in less durable capital and switch to more durable capital only once they have accumulated sufficient net worth.

The model therefore provides a theory of optimal durability based on financial constraints. In a frictionless economy and in an economy in which residual value can be fully pledged, only the more durable type of capital is used. In contrast, in an economy with financial constraints, the less durable type of capital is also used because it alleviates financial constraints.<sup>13</sup> The optimal durability moreover varies with legal enforcement, as we show next, and may vary with aggregate conditions, that is, the business cycle, as well, with the composition of investment shifting to less durable types of capital in downturns

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<sup>12</sup>This result captures the intuition in Pratchett’s (1993) “boots’ theory” (see Footnote 5); severely constrained firms optimally purchase less durable capital, despite the higher frictionless user cost, because it requires a lower down payment.

<sup>13</sup>So far we consider only two types of capital with fixed durability. We can extend the argument to a continuum of types of capital of differing durability. Suppose capital with depreciation rate  $\delta$  can be produced at cost  $\phi(\delta)$  per unit of capital where  $\phi_\delta(\delta) < 0 < \phi_{\delta\delta}(\delta)$ , that is, the cost of producing a unit of capital is increasing and convex in durability  $1 - \delta$ . Assuming the cost is linear in the quantity of capital produced, the price of capital with durability  $1 - \delta$  is  $q(\delta) = \phi(\delta)$ . In a frictionless economy and in an economy in which the residual value can be fully pledged (that is, when  $\theta = 1$ ), all firms evaluate capital by its frictionless user cost. It is therefore optimal to choose the durability to minimize the frictionless user cost  $u(\delta) = R^{-1}\phi(\delta)(r + \delta)$ . Assuming  $u(\delta)$  is convex in  $\delta$ , there exists a user cost minimizing depreciation rate  $\delta^* \in \arg \max_{\delta \in [0,1]} u(\delta)$  which is interior if  $u_\delta(0) < 0 < u_\delta(1)$ . For example,  $\phi(\delta) = 1 + \phi(1 - \delta)^{\gamma_\phi}$  with  $\phi > 0$  and  $\gamma_\phi > r^{-1}(1 + \max\{\phi^{-1}, R\})$  satisfies these conditions. In a frictionless economy (or when  $\theta = 1$ ) only one type of capital with durability  $1 - \delta^*$  is produced, whereas in an economy with financial frictions, that is,  $\theta < 1$ , also less durable types of capital are produced, possibly a continuum of them. In contrast, it is never optimal to produce capital that is more durable than  $1 - \delta^*$ . Thus, the model provides a theory of optimal durability based on financial constraints, providing a rather different perspective on low durability capital than the theory of durability based on planned obsolescence discussed in the section on the related literature.

when firms are more constrained and to more durable types of capital in expansions when firms' constraints are relaxed.

## 2.5 Technology adoption and legal enforcement

To analyze the effect of legal enforcement, suppose the world economy consists of two types of economies, economies with weak legal enforcement and economies with strong legal enforcement. We model weak vs. strong legal enforcement simply in terms of the fraction of the resale value of durable assets that can be collateralized, that is,  $\theta_L < \theta_H$  with  $\theta_i \in [0, 1)$ , for  $i \in \{H, L\}$ . The two types of economies are otherwise exactly as the one we analyzed so far, with two types of capital with  $\delta_d < \delta_{nd}$ . The interesting case is again the case in which both types of capital are used in equilibrium at least in weak legal enforcement economies; Proposition 1 implies that then it must be the case that  $u_d < u_{nd}$  (which do not depend on legal enforcement) while  $\varphi_d(\theta_L) > \varphi_{nd}(\theta_L)$  (and  $q_d > q_{nd}$ ).

The down payment for type  $j$  capital depends on legal enforcement, that is,  $\varphi_j(\theta_i)$ , and is decreasing in  $\theta_i$ , so  $\varphi_j(\theta_L) > \varphi_j(\theta_H)$ ,  $\forall j \in J$ . Moreover, the difference between the down payments on durable and non-durable capital  $\varphi_d(\theta_i) - \varphi_{nd}(\theta_i)$  is also decreasing in  $\theta_i$  and indeed is negative for  $\theta_i$  sufficiently close to 1 as  $\varphi_d(\theta_i) - \varphi_{nd}(\theta_i) \rightarrow u_d - u_{nd} < 0$  when  $\theta_i \rightarrow 1$ . This has important implications for technology adoption: The fact that the less durable capital is used in equilibrium in weak legal enforcement economies does not necessarily imply that this type of capital is used in strong legal enforcement economies. In fact, if  $\theta_H$  is sufficiently close to 1, then  $\varphi_d(\theta_H) < \varphi_{nd}(\theta_H)$  and the less durable type of capital is dominated in strong legal enforcement economies. Firms in weak legal enforcement economies may adopt technologies that are dominated in strong legal enforcement economies, simply because they are less durable and hence have a lower financing need.

If legal enforcement is such that the less durable type of capital is used in equilibrium in both types of economies, we can show that a larger fraction of entrepreneurs adopt the less durable capital in economies with weak legal enforcement. To derive this result we must keep in mind that the policy and value functions differ in the two types of economies. However, taking the investment Euler equations for type  $j$  capital (12),  $j \in J$ , in type  $i$  economies and combining them for a firm that is indifferent between the two types of capital yields

$$\frac{Af_k(k) + (1 - \theta_i)q_d(1 - \delta_d)}{\varphi_d(\theta_i)} = \frac{Af_k(k) + (1 - \theta_i)q_{nd}(1 - \delta_{nd})}{\varphi_{nd}(\theta_i)}, \quad (16)$$

which determines the level of investment  $\underline{k}(\theta_i)$  at which the firm is indifferent at the margin. This level depends on legal enforcement and, as shown in the proof in Appendix C, is

higher in economies with weak legal enforcement, so  $\underline{k}(\theta_L) > \underline{k}(\theta_H)$ . For a firm to be able to invest  $\underline{k}(\theta_i)$  it requires net worth of at least  $\underline{w}_{nd}(\theta_i) = \wp_{nd}(\theta_i)\underline{k}(\theta_i)$  and thus  $\underline{w}_{nd}(\theta_L) = \wp_{nd}(\theta_L)\underline{k}(\theta_L) > \wp_{nd}(\theta_H)\underline{k}(\theta_H) = \underline{w}_{nd}(\theta_H)$ ; firms start using more durable capital at a higher level of net worth in weak legal enforcement economies. Similarly, for a firm to be able to stop using less durable capital entirely while investing  $\underline{k}(\theta_i)$  it requires net worth of at least  $\underline{w}_d(\theta_i) = \wp_d(\theta_i)\underline{k}(\theta_i)$  and thus  $\underline{w}_d(\theta_L) = \wp_d(\theta_L)\underline{k}(\theta_L) > \wp_d(\theta_H)\underline{k}(\theta_H) = \underline{w}_d(\theta_H)$ ; firms also stop using less durable capital entirely at a higher level of net worth in weak legal enforcement economies. Finally, firms start to pay dividends once they reach net worth  $\bar{w}(\theta_i) = \wp_d(\theta_i)\bar{k}$ , where  $\bar{k}$  is the first best level of capital, and thus firms delay paying dividends longer in weak legal enforcement economies, as  $\bar{w}(\theta_L) > \bar{w}(\theta_H)$ .

The effect of legal enforcement on technology adoption in terms of durable and non-durable capital is summarized in the following proposition:

**Proposition 3** (Legal enforcement). *Suppose legal enforcement differs across economies, with  $\theta_H > \theta_L$ , and that both types of capital are used in equilibrium in weak legal enforcement economies. If  $\theta_H$  is sufficiently close to 1, then less durable capital is dominated in strong legal enforcement economies and only adopted in weak legal enforcement economies. If the less durable type of capital is used in both types of economies, then a larger fraction of firms in weak legal enforcement economies invest in less durable capital. Firms in such economies substitute to more durable capital at higher levels of net worth. Firms also start to pay dividends at a higher level of net worth in weak legal enforcement economies.*

The parts of the proof not discussed in the text are in Appendix C. Our theory predicts that in economies with weak legal enforcement, more firms invest in less durable types of capital and invest in such capital for longer, that is, until they are older and better capitalized. Indeed, firms in such economies may invest in less durable types of capital that are not adopted in strong legal enforcement economies at all because they are dominated. Thus, there may be new investment in dominated technologies in weak legal enforcement economies. The basic mechanism is that durability raises the requirements for internal funds and weak legal enforcement compounds these effects. Firms in weak legal enforcement economies adopt less durable capital to reduce their financing needs.

## 2.6 Effect of durability on composition of investment

When different types of capital are not perfect substitutes, as assumed so far, but instead imperfect substitutes, say structures and equipment, then all firms use all types of capital and durability affects the composition of investment. Consider the following aggregator

for capital with constant elasticity of substitution

$$k \equiv \left( \sum_{j \in J} \sigma_j k_j^\gamma \right)^{1/\gamma}, \quad (17)$$

where  $k_j$  is type  $j$  capital, the substitution coefficient  $\gamma$  satisfies  $-\infty < \gamma < 1$ , the factor shares  $\sigma_j > 0$ , for all  $j \in J$ , and  $\sum_{j \in J} \sigma_j = 1$ . The elasticity of substitution is  $1/(1 - \gamma)$ . For ease of exposition, we assume that there are two types of capital,  $j \in J \equiv \{d, nd\}$ , as before, but the results obtain more generally.<sup>14</sup> The firm's problem is to maximize (1) subject to (2) through (4), where  $k$  is now as defined in (17). The economy is otherwise unchanged.

Using the definition of the user cost for a financially constrained firm (14) or (15), the first-order condition for type  $j$  capital can be written as  $u_j(w) = \beta \frac{\mu'}{\mu} A f_k(k) \frac{\partial k}{\partial k_j}$ . Dividing the first-order condition for less durable capital by the one for more durable capital we obtain

$$\frac{u_{nd}(w)}{u_d(w)} = \left( \frac{k_d}{k_{nd}} \right)^{1-\gamma} \frac{\sigma_{nd}}{\sigma_d}. \quad (18)$$

Recall that for unconstrained firms, the user cost is  $u_j(w) = u_j$ , that is, equals the frictionless user cost. Therefore, for such firms, the ratio of more durable to less durable capital is determined by the ratio of the frictionless user costs (as well as the factor shares). In contrast, for severely constrained firms, that is, as  $w$  goes to 0, the user cost  $u_j(w) \rightarrow \varphi_j$ , implying that the ratio of more durable to less durable capital is determined by the ratio of the down payments for such firms.

The composition of investment of financially constrained firms is distorted away from more durable toward less durable capital. The reason is that  $\frac{\varphi_{nd}}{\varphi_d} < \frac{u_{nd}}{u_d}$ , that is, the ratio of the down payments of less durable to more durable capital is lower than the ratio of the frictionless user costs. To see this, note that we can equivalently write  $\frac{\varphi_{nd}}{u_{nd}} < \frac{\varphi_d}{u_d}$ , and using the expressions for the down payment (10) and the frictionless user cost (9), we have  $\frac{\varphi_j}{u_j} = 1 + (1 - \theta) \frac{1 - \delta_j}{r + \delta_j}$ , which is lower when  $\delta_j$  is higher. To understand the economic intuition, rewrite the inequality once more as  $\frac{\varphi_d - u_d}{\varphi_d} > \frac{\varphi_{nd} - u_{nd}}{\varphi_{nd}}$ , where  $\frac{\varphi_j - u_j}{\varphi_j}$  is the residual value that cannot be pledged as a fraction of the down payment, and note that  $\frac{\varphi_j - u_j}{\varphi_j} = \left( 1 + \left( (1 - \theta) \frac{1 - \delta_j}{r + \delta_j} \right)^{-1} \right)^{-1}$ ; for more durable capital, the residual value that cannot be pledged comprises a larger fraction of the down payment. More constrained firms hence respond by decreasing the amount of more durable capital and increasing the amount of less durable capital they deploy.

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<sup>14</sup>The case of perfect substitutes considered so far is a special case of this aggregator with  $\gamma = 1$  and  $\sigma_d = \sigma_{nd}$ . Moreover, if  $\gamma = 0$ , we have a Cobb-Douglas aggregator and  $k = k_d^{\sigma_d} k_{nd}^{\sigma_{nd}}$ . Finally, the limit as  $\gamma$  goes to  $-\infty$  is the Leontief aggregator.

Note that, unlike in the case of perfect substitutes,  $\wp_d > \wp_{nd}$  per se is not necessary with imperfect substitutes for our implications for the effect of durability on the composition of investment to obtain (nor is  $q_d > q_{nd}$  necessary), since it is the ratio of the down payment to the user cost,  $\frac{\wp_j}{u_j}$ , that is critical in this case, not the level of the down payments or prices. With imperfect substitutes, the key determinant of investment composition is the fraction of the down payment that is comprised by the residual value that cannot be pledged, which is higher for more durable assets.

We have established the following result:

**Proposition 4** (Composition of investment). *Consider a production function with CES aggregator (17) for more durable and less durable capital. The ratio of more durable to less durable assets  $k_d/k_{nd}$  is determined by the ratio of the frictionless user costs  $u_{nd}/u_d$  for unconstrained firms and by the ratio of the down payments  $\wp_{nd}/\wp_d$  for severely constrained firms. Since  $\frac{\wp_{nd}}{\wp_d} < \frac{u_{nd}}{u_d}$ , the composition of investment of financially constrained firms is distorted away from more durable toward less durable capital.*

Consider the effect of financial development in terms of an increase in legal enforcement, that is, pledgeability  $\theta$ . Financial development does not affect the composition of investment of financially unconstrained firms. In contrast, financial development increases the ratio of more durable to less durable investment for severely constrained firms, as an increase in pledgeability increases the ratio of down payments  $\wp_{nd}/\wp_d$ ; an increase in pledgeability reduces the down payment on more durable assets by more.

For types of capital that are imperfect substitutes our theory predicts that in the cross section of firms, more constrained firms substitute away from more durable assets towards less durable assets and that the relative down payments determine the composition of investment for severely constrained firms. Financial development reduces the distortion away from durable investment for constrained and especially severely constrained firms.

## 2.7 Relation between durability and pledgeability

Throughout we assume that the pledgeability does not vary with the type of capital. This is plausible when the types of capital are only distinguished by their durability and are otherwise the same capital asset, such as different types of trucks, structures, or machines, as perfect substitutability suggests. When different types of capital are imperfect substitutes, however, it is possible that pledgeability also varies with the type of capital. If the more durable capital is also less pledgeable, the effects we emphasize are reinforced. For example, in developed economies, cars may be more easily repossessed than homes making them more pledgeable. That said, in some economies it is likely the

case that structures are more pledgeable than equipment. As long as the more durable capital is not too much more collateralizable than the less durable capital, that is, as long as  $\frac{1-\theta_d}{1-\theta_{nd}} \geq \frac{r+\delta_d}{r+\delta_{nd}} \frac{1-\delta_{nd}}{1-\delta_d}$ , our results obtain. If this inequality were not satisfied, the sign of the distortion would be reversed, but this would be due to the higher pledgeability of the more durable type of capital, not its durability. So the insight that durability and pledgeability are distinct and have opposing effects remains valid.

### 3 Durable assets with finite useful life

We now consider the effect of durability in an economy as in Section 2 except that capital goods last for two periods, such that new assets are durable whereas used assets are non-durable as they have only one period of useful life left. Thus, firms have a choice between new, durable assets and used, non-durable assets. In this economy with one-horse shay depreciation, the price of used, non-durable assets is determined in equilibrium.<sup>15</sup> We show that the purchases of durable assets require more internal funds, despite the fact that their collateral value allows firms to borrow against them, because durable assets are more expensive and hence have a larger financing need. As a consequence, constrained firms buy non-durable assets, that is, used assets, whereas well-capitalized firms buy new assets which are more durable. We also consider the pricing of used assets in equilibrium and the effect of legal enforcement on trade in used capital across countries.

#### 3.1 Environment, entrepreneur's problem, and equilibrium

The environment is as in Section 2.1, that is, the economy has stochastic overlapping generations of entrepreneurs and limited enforcement without exclusion, and we consider a stationary competitive general equilibrium with production. The only difference is how capital depreciates. New assets are durable and last for two periods. We denote the amount of new, durable assets an entrepreneur purchases by  $k_d$ . New assets can be produced from output goods using a linear technology and we normalize the cost of producing new assets to 1, that is,  $q_d = 1$ . Since assets last for two periods, there are also used assets in the economy which have only one period of useful life left and are therefore non-durable. Denote the amount of used, non-durable assets an entrepreneur purchases by  $k_{nd}$ . The price of used assets  $q_{nd}$  is determined in equilibrium. We assume that new, durable assets and used, non-durable assets are perfect substitutes in production and

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<sup>15</sup>This depreciation pattern is known as one-horse shay depreciation (see, for example, Hulten and Wykoff (1981a, 1981b) and Fraumeni (1997)) following Oliver Wendell Holmes, Sr.'s 1858 poem *The Deacon's Masterpiece or, the Wonderful 'One-hoss Shay': A Logical Story*.

assets of  $k_d$  and  $k_{nd}$  this period generate output  $Af(k_d + k_{nd})$  next period where  $A$  is the total factor productivity and  $f$  satisfies the same conditions as before.<sup>16</sup> Our main insight regarding the effect of durability does however extend to the case in which new, durable assets and used, non-durable assets are imperfect substitutes.<sup>17</sup> We study a stationary equilibrium, which we define formally below, in which the price of used capital  $q_{nd}$  and the interest rate on one-period loans  $R$  are constant. To reiterate, except for the nature of depreciation, the economy is as before.

The entrepreneur's problem with new, durable and used, non-durable assets is to choose  $\{d, k_d, k_{nd}, b, w'\}$  given  $w$  to solve

$$v(w) \equiv \max_{\{d, k_d, k_{nd}, b, w'\} \in \mathbb{R}_+^3 \times \mathbb{R}^2} d + \beta v(w') \quad (19)$$

subject to the budget constraints for the current and next period

$$w + b \geq d + k_d + q_{nd}k_{nd}, \quad (20)$$

$$Af(k) + q_{nd}k_d \geq Rb + w', \quad (21)$$

and the collateral constraint

$$\theta q_{nd}k_d \geq Rb, \quad (22)$$

where  $k \equiv k_d + k_{nd}$ . The endogenous state variable, net worth, is defined as output plus the resale value of durable assets minus the loan repayments, that is,  $w' \equiv Af(k_d + k_{nd}) + q_{nd}k_d - Rb$ . The budget constraint in the current period ensures that current net worth plus borrowing covers the current dividend plus the cost of investment in new, durable assets and used, non-durable assets. The budget constraint next period implies that output plus the resale value of durable assets purchased this period can be spent on loan repayments or carried over as net worth for the next period. The collateral constraint states that the firm can borrow up to fraction  $\theta$  of the resale value of the new, durable assets purchased in the current period which will be used assets next period, but cannot borrow against used, non-durable assets purchased this period.

A *stationary equilibrium* is a policy function  $x(w) = [d(w), k_d(w), k_{nd}(w), b(w), w'(w)]$ , an interest rate  $R$ , a price of used, non-durable assets  $q_{nd}$ , and a stationary distribution  $p(w)$  of net worth, such that (i) the policy function  $x(w)$  solves the entrepreneurs' problem in equations (19) to (22) given  $R$  and  $q_{nd}$ ; (ii) the loan market clears, that is,

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<sup>16</sup>The assumption that new and used units are perfect substitutes and that age or durability per se do not play a role dates back to Wicksell (1934).

<sup>17</sup>Appendix D shows that in an economy with two-period assets in which new and used assets are imperfect substitutes analogous results obtain to the ones in the economy with geometric depreciation in Section 2.6.

$\sum_w p(w)b(w) = 0$ ; (iii) the used asset market clears, that is, the supply of used assets equals the demand

$$(1 - \rho) \sum_w p(w)k_d(w) = \sum_w p(w)k_{nd}(w);$$

(iv) the goods market clears, that is, total output of the surviving entrepreneurs plus the endowments of new entrepreneurs equals total dividends plus investment in new capital

$$(1 - \rho) \sum_w p(w)Af(k(w)) + \rho w_0 = \sum_w p(w)d(w) + \sum_w p(w)k_d(w),$$

where  $k(w) \equiv k_d(w) + k_{nd}(w)$ ; and (v) the stationary net worth distribution  $p(w)$  is induced by the entrepreneurs' policy function  $x(w)$ .

Two aspects are worth emphasizing: first, the price of used capital  $q_{nd}$  is now determined by the market clearing condition in the used capital market and hence endogenous; second, the goods market clearing condition does not involve investment in used capital as aggregate net investment in used capital is of course zero. In terms of equilibrium properties, well-capitalized entrepreneurs provide financing and hence the equilibrium interest rate on one-period loans is  $R = \beta^{-1}$ . We determine the equilibrium price of used capital  $q_{nd}$  in Section 3.3 below.

We observe that the problem in (19) to (22) defines a well-behaved dynamic program as before. Denote the multipliers on (20) and (21) by  $\mu$  and  $\beta\mu'$  and on (22) by  $\beta\lambda'$  and let  $\nu$ ,  $\nu_d$ , and  $\nu_{nd}$  be the multipliers on the non-negativity constraints for  $d$ ,  $k_d$ , and  $k_{nd}$ , respectively. The first-order conditions are (5), (7), (8) and

$$\mu = \beta\mu'[Af_k(k) + q_{nd}] + \beta\lambda'\theta q_{nd} + \nu_d, \quad (23)$$

$$\mu q_{nd} = \beta\mu' Af_k(k) + \nu_{nd}, \quad (24)$$

and the envelope condition  $v_w(w) = \mu$ .

### 3.2 User cost and down payment in equilibrium

We start by defining the user costs for new, durable and used, non-durable assets for an unconstrained firm as well as the down payments for the two types of assets in this economy. The user cost of new, durable assets for an unconstrained firm is  $u_d \equiv 1 - R^{-1}q_{nd}$ , as a unit of durable assets costs 1 to buy but can be resold for  $q_{nd}$  as used assets next period which is discounted at the rate  $R^{-1}$  by an unconstrained firm. We refer to this as the user cost of durable assets for an unconstrained firm rather than the frictionless user cost as it involves the equilibrium price of used capital  $q_{nd}$  which may or may not equal the price that would prevail in a frictionless economy, as we discuss below.

Alternatively, we could write this as  $u_d = R^{-1}(r + (1 - q_{nd}))$  in analogy to the user cost with geometric depreciation (9); the term  $1 - q_{nd}$  captures the depreciation in this case. The user cost of used, non-durable assets is  $u_{nd} \equiv q_{nd}$ , as a unit of used, non-durable assets costs  $q_{nd}$  and is worthless next period. In analogy to (9), we could equivalently write  $u_{nd} = R^{-1}q_{nd}(r + 1)$  where the term 1 again captures the depreciation which is full in this case.

The down payment is the minimal amount of internal funds required to deploy a unit of capital. A unit of new, durable assets costs 1 and the firm can borrow  $R^{-1}\theta q_{nd}$  against it, since the unit will turn into a used unit next period with price  $q_{nd}$ , and thus the down payment per unit of new assets is

$$\wp_d \equiv 1 - R^{-1}\theta q_{nd}. \quad (25)$$

Used, non-durable assets have no resale value at the end of the period and hence do not support any borrowing. Thus, the firm has to pay the full price  $q_{nd}$  up front, that is, for non-durable assets the down payment equals the price,  $\wp_{nd} = q_{nd}$ . The down payment on used, non-durable assets thus equals the user cost for an unconstrained firm as  $\wp_{nd} = q_{nd} = u_{nd}$ . In contrast, the down payment on new, durable assets exceeds the user cost for an unconstrained firm, that is,  $\wp_d > u_d$ , which can be seen by rewriting (25) as

$$\wp_d = u_d + R^{-1}(1 - \theta)q_{nd}, \quad (26)$$

because the firm also has to finance out of internal funds the fraction of the resale value it cannot pledge.

Using the down payments defined above, we can rewrite equations (23) and (24) as investment Euler equations as follows:

$$1 = \beta \frac{\mu' Af_k(k) + (1 - \theta)q_{nd}}{\mu \wp_d} + \frac{\nu_d}{\mu \wp_d}, \quad (27)$$

$$1 = \beta \frac{\mu' Af_k(k)}{\mu \wp_{nd}} + \frac{\nu_{nd}}{\mu \wp_{nd}}. \quad (28)$$

Moreover, using the first-order condition (7) to substitute out  $\mu$  in (23) and (24) we obtain  $[1 - R^{-1}q_{nd}] + \frac{\lambda'}{\mu'}[1 - R^{-1}\theta q_{nd}] \geq \beta Af_k(k)$  and  $q_{nd} + \frac{\lambda'}{\mu'}q_{nd} \geq \beta Af_k(k)$ , respectively, or, using the definitions of the user costs for an unconstrained firm and down payments above

$$u_j + \frac{\lambda'}{\mu'}\wp_j \geq \beta Af_k(k), \quad (29)$$

for  $j \in J \equiv \{d, nd\}$ , which is the analogous result to equation (13) in the economy with geometric depreciation. We can use this equation to derive key equilibrium properties.

First, in equilibrium, the user cost of new, durable assets for an unconstrained firm has to be less than or equal to the user cost of used, non-durable assets, that is,  $u_d \leq u_{nd}$ . If  $u_d$  were strictly larger than  $u_{nd}$ , then durable assets would be strictly dominated as  $\wp_d > u_d > u_{nd} = \wp_{nd}$  in this case. But then there would thus be no new investment which is not an equilibrium. Second, in equilibrium, the down payment on new, durable assets has to strictly exceed the down payment on used, non-durable assets, that is,  $\wp_d > \wp_{nd}$ . If  $\wp_d$  were weakly less than  $\wp_{nd}$ , then used assets would be strictly dominated as  $u_{nd} = \wp_{nd} \geq \wp_d > u_d$ . But then there would be no investment in used capital, which is not an equilibrium either. We have proved the following:

**Proposition 5** (Equilibrium in used capital market). *In an equilibrium, new, durable assets have a weakly lower user cost  $u_d \leq u_{nd}$  from the vantage point of an unconstrained firm, but a strictly higher down payment  $\wp_d > \wp_{nd}$  than used, non-durable assets.*

In equilibrium, the down payment required per unit of new assets exceeds the price of used assets (which equals the down payment on used assets), that is,  $\wp_d > q_{nd} = \wp_{nd}$ , as otherwise new, durable assets would dominate purchasing used, that is, non-durable assets. Intuitively, if the down payment on new assets were less than the price of used assets, buying a new unit instead of a used one would yield a positive payoff in the current period and the firm would get an additional positive payoff in the amount of  $(1 - \theta)$  times the resale value of the new unit  $q_{nd}$ , that is,  $(1 - \theta)q_{nd}$ , in the next period, an arbitrage. Table 1 summarizes the financing need of new, durable and used, non-durable assets. To deploy durable assets the firm has to come up with not just the one-period user cost but also the fraction of the residual value that cannot be pledged.<sup>18</sup>

At first blush, it seems surprising that durable assets require more internal funds ( $\wp_d > \wp_{nd}$ ) despite the fact that new assets support more borrowing as the firm can borrow  $R^{-1}\theta q_{nd} > 0$  against them whereas the firm cannot borrow against used assets and has to finance the entire purchase internally. But crucially durable assets are more expensive ( $1 > q_{nd}$ ) and hence have a larger financing need precisely because they are more durable since the assets can be used for two periods. The key insight is that the higher financing need must dominate the higher collateral value in equilibrium. We emphasize that the fact that the collateral value cannot be pledged fully, that is,  $\theta < 1$ , is critical

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<sup>18</sup>These expressions for the down payments can be interpreted as special cases of the down payment with geometric depreciation in equation (11) providing a connection to the results in Section 2: first, if capital fully depreciates every period, that is,  $\delta_j = 1$ , the down payment in (11) is  $\wp_j = u_j$ , which is the down payment for used, non-durable assets here; and second, if the depreciation rate  $\delta_j = 0$ , so capital does not depreciate, then (11) simplifies to  $\wp_j = u_j + R^{-1}(1 - \theta)q_j$ , which is analogous to the expression for the down payment on new, durable assets in equation (26) here.

**Table 1: Durability and Requirements of Internal Funds**

Time	$t$	$t + 1$	$t + 2$
Used, non-durable capital			
Value		$q_{nd}$	0
Collateral value			0
Borrowing		0	
Internal funds required ( $\wp_{nd}$ )		$q_{nd}$	
New, durable capital			
Value	1	$q_{nd}$	0
Collateral value		$\theta q_{nd}$	
Borrowing	$R^{-1}\theta q_{nd}$		
Internal funds required ( $\wp_d$ )	$1 - R^{-1}\theta q_{nd}$		

for the result as otherwise  $\wp_d = u_d$  and in equilibrium  $u_d = u_{nd}$  and all firms would be indifferent between new, durable and used, non-durable assets.

### 3.3 Equilibrium price of used assets

If the economy were frictionless, the rental rate or frictionless user cost of capital  $u^*$  would be  $u^* = \frac{R}{1+R}$  since the purchase cost of one unit of capital has to equal the discounted value of the rental rate over the useful life of the asset, that is,  $1 = u^* + R^{-1}u^*$ . Moreover, the price of used capital in the frictionless economy would be  $q_{nd}^* = u^* = \frac{R}{1+R}$  (and therefore  $u^* = R^{-1}(r + (1 - q_{nd}^*)) = R^{-1}q_{nd}^*(r + 1)$ ).

In our economy with collateral constraints, Proposition 5 shows that  $1 - R^{-1}q_{nd} = u_d \leq u_{nd} = q_{nd}$  which implies that the price and user cost of used, non-durable assets weakly exceeds the price and rental rate in the frictionless economy, that is,  $u_{nd} = q_{nd} \geq q_{nd}^* = u^*$ . This, together with the fact that  $\wp_d > q_{nd}$  in equilibrium, implies that the price of used capital satisfies the following condition in equilibrium

$$\frac{R}{\theta + R} > q_{nd} \geq q_{nd}^* = \frac{R}{1 + R}. \quad (30)$$

In the limit as  $\theta$  goes to 1, equation (30) implies that the price of used capital goes to the frictionless price when the resale value of capital becomes fully pledgeable.

To determine the price of used assets in equilibrium, consider the marginal investor in used assets who is indifferent between investing in new, durable assets and used, non-durable assets and hence  $\nu_d = \nu_{nd} = 0$ . The investment Euler equations (27) and (28)

then imply that

$$(1 - R^{-1}\theta q_{nd}) - q_{nd} = \beta \frac{\mu'}{\mu} (1 - \theta) q_{nd},$$

so the incremental investment required for durable assets,  $\wp_d - q_{nd}$ , equals the discounted resale value. If the marginal investor is unconstrained, then  $\beta\mu'/\mu = R^{-1}$  and the above equation implies that the market clearing price of used assets is  $q_{nd} = \frac{R}{1+R}$ , which equals the frictionless price  $q_{nd}^*$ . If constrained entrepreneurs are the marginal investors and price used assets, then  $\beta\frac{\mu'}{\mu} < R^{-1}$  and hence used assets trade at a premium, that is,  $q_{nd} > \frac{R}{1+R} = q_{nd}^*$  which in turn implies that  $u_d < u_{nd}$ .

If the price of used assets equals the frictionless price  $q_{nd}^*$ , then  $u_d = u^* = u_{nd}$ , and unconstrained entrepreneurs are indifferent between purchasing new, durable and used, non-durable assets. Importantly, however, sufficiently constrained entrepreneurs turn out not to be indifferent even in this case, as we show below. If the price of used assets  $q_{nd} > q_{nd}^*$  instead, then  $u_d < u_{nd}$ , that is, the user cost of durable assets is strictly lower than the user cost of non-durable assets from the vantage point of unconstrained entrepreneurs as new assets can be resold at a premium when they are used. We characterize the choice between new, durable and used, non-durable assets explicitly in the remainder of this section.

We emphasize that the properties that  $u_d \leq u_{nd}$  and  $\wp_d > \wp_{nd}$  arise endogenously here, that is, are equilibrium properties when capital has a finite useful life. In the economy with two types of capital with different geometric depreciation rates in Section 2 whether or not these properties are satisfied is determined by the properties of the linear technologies of producing the two types of capital, although the properties have to be satisfied if both types of capital are used in equilibrium (see Proposition 1).

Finally, to be explicit about the sense in which the new assets with a two-period life are more durable, let us define the depreciation rates for new and used assets: new assets depreciate at rate  $\delta_d \equiv \frac{1 - q_{nd}}{1} \leq 1 - \frac{R}{1+R} = \frac{1}{1+R} < 50\%$  in the first period while in second period (when they are used) they depreciate at rate  $\delta_{nd} \equiv \frac{q_{nd} - 0}{q_{nd}} = 100\%$ . Clearly,  $\delta_d < \delta_{nd}$  and new assets are relatively durable whereas used assets are non-durable.

### 3.4 Choice between new, durable and used, non-durable assets

Consider now the composition of investment for an unconstrained firm. Rewriting the first-order conditions for durable and non-durable investment (23) and (24) using the fact that  $\mu = \mu' = 1$  and  $\lambda' = 0$ , we have  $u_j = \beta A f_k(k) + \nu_j$ , for all  $j \in J$ , that is, unconstrained firms simply compare the user costs. If  $q_{nd} > q_{nd}^*$ , and hence  $u_d < u_{nd}$ ,  $\nu_{nd} > 0$ , that is, an unconstrained firm purchases only new, durable assets and sells assets

once they are used, that is, non-durable. Moreover, the capital stock of an unconstrained firm  $\bar{k}_d$  solves  $1 = \beta[Af_k(k_d) + q_{nd}]$  and the firm is unconstrained once net worth reaches  $\bar{w} = \wp_d \bar{k}_d$ . If  $q_{nd} = q_{nd}^*$ , then  $u_d = u_{nd}$  and hence  $\nu_d = \nu_{nd} = 0$ ; the unconstrained firm is indifferent between investing in new, durable and used, non-durable assets.

We turn to the composition of investment for severely constrained firms next. The budget constraint (20) together with the collateral constraint (22) imply that  $w \geq \wp_d k_d + q_{nd} k_{nd}$ . Thus, as the firm's net worth  $w$  goes to zero, so do  $k_d$  and  $k_{nd}$  and therefore  $k = k_d + k_{nd}$ , and thus  $f_k \rightarrow +\infty$ . The investment Euler equations in turn imply that  $\beta\mu'/\mu$  goes to zero. Combining the investment Euler equations (27) and (28) we have

$$\wp_d - q_{nd} = \beta \frac{\mu'}{\mu} (1 - \theta) q_{nd} + \frac{\nu_d}{\mu} - \frac{\nu_{nd}}{\mu}.$$

The left-hand side is the incremental investment of purchasing new, durable assets instead of used, non-durable assets. The first term on the right-hand side is the additional resale value of the durable assets (net of debt) which is evaluated using the firm's discount factor  $\beta\mu'/\mu$ . This value goes to zero for severely constrained firms and hence these firms purchase only used capital as  $\nu_d > 0$ . Severely constrained firms simply compare down payments.<sup>19</sup>

Our conclusions regarding the composition of investment are summarized as follows:

**Proposition 6** (Choice between new and used assets). **(i)** *If  $q_{nd} > q_{nd}^*$ , unconstrained firms purchase only new, durable assets and sell assets once they are used, that is, non-durable. If  $q_{nd} = q_{nd}^*$ , unconstrained firms are indifferent between new, durable assets and used, non-durable assets.* **(ii)** *Severely constrained firms strictly prefer to purchase only used, non-durable assets, that is, as  $w \rightarrow 0$ ,  $\nu_d > 0$ .*

We emphasize that severely constrained firms strictly prefer to purchase only used, non-durable assets even if used assets do not trade at a premium, because the down payment for durable assets exceeds the price of non-durable assets as long as  $\theta < 1$ .

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<sup>19</sup>To see the intuition for the determinants of the choice between durable and non-durable assets in another way, define the user cost of new, durable assets, which depends on the firm's discount factor, as  $u_d(w) \equiv \wp_d - \beta \frac{\mu'}{\mu} (1 - \theta) q_{nd} = u_d + \beta \frac{\lambda'}{\mu} (1 - \theta) q_{nd}$ , analogously to the expressions for the user cost (14) and (15) in the model with geometric depreciation; to deploy one unit of new, durable assets the firm has to make the down payment  $\wp_d$  in the current period and recovers  $(1 - \theta) q_{nd}$  next period, which the firm evaluates using its discount factor, as the first expression shows. The user cost can also be written as the user cost of durable assets to an unconstrained firm plus the scaled multiplier on the collateral constraint times the amount recovered next period. An unconstrained firm therefore simply compares the frictionless user costs as the multiplier on the collateral constraint is zero. In contrast, a severely constrained firm discounts the amount recovered next period completely and hence the user cost equals the down payment and such firms evaluate the choice by comparing the down payment on new, durable assets to the price of used, non-durable assets.

### 3.5 Trade in used capital

To analyze the implications of our model for trade in used assets, suppose, as in the discussion of the effect of legal enforcement in Section 2, that the world economy consists of two types of economies, economies with strong legal enforcement and economies with weak legal enforcement, that is,  $\theta_H > \theta_L$ , with  $\theta_i \in [0, 1)$ , for  $i \in \{H, L\}$ . Moreover, assume that the world loan and used asset markets are integrated. We focus on the most interesting case in which the world market price for used assets is  $q_{nd} > q_{nd}^*$ , that is, used assets trade at a premium. We show that weak legal enforcement economies are net importers of used assets and, remarkably, used assets may be dominated in strong legal enforcement economies.<sup>20</sup>

Since the world price of used assets  $q_{nd}$  is the same in both types of economies, the user cost of used capital as well as the down payment on used capital is the same everywhere:  $u_{nd} = \wp_{nd} = q_{nd}$ ; indeed, the user cost of new, durable capital for an unconstrained firm is the same everywhere, too, as  $u_d = 1 - R^{-1}q_{nd}$  does not depend on  $\theta_i$ . The down payments on new, durable capital do however depend on legal enforcement as  $\wp_d(\theta_i) \equiv 1 - R^{-1}\theta_i q_{nd}$  and  $\wp_d(\theta_H) < \wp_d(\theta_L)$ , that is, the down payment on new assets is lower in the strong legal enforcement economy. Proposition 5 implies that in equilibrium  $u_d \leq u_{nd}$  and that  $\wp_d(\theta_L) > \wp_{nd}$  as otherwise used capital would be dominated in both economies. But remarkably it is possible that  $\wp_d(\theta_H) \leq \wp_{nd}$ , that is, used capital may be dominated in strong legal enforcement economies if  $\theta_H$  is sufficiently close to 1. To see this, note that if used capital trades at a premium,  $u_d < u_{nd}$ , and that as  $\theta_H$  goes to 1,  $\wp_d(\theta_H) \rightarrow u_d < \wp_{nd}$ . If this is the case, all used capital is shipped to weak legal enforcement economies.

If  $\wp_d(\theta_H) > \wp_{nd}$ , used capital is used in both types of economies. The entrepreneurs' problems again differ due to the difference in legal enforcement and hence so do the policy functions and value functions in the two types of economies. First, consider the problem of an unconstrained firm and recall that the investment of an unconstrained firm solves  $1 = \beta[Af(\bar{k}_d) + q_{nd}]$  and thus the investment of dividend-paying firms in both types of economies is identical and independent of legal enforcement. That said, the net worth threshold at which firms start to pay dividends satisfies  $\bar{w}(\theta_i) = \wp_d(\theta_i)\bar{k}_d$  and since  $\wp_d(\theta_L) > \wp_d(\theta_H)$  this threshold is higher in weak legal enforcement economies than in strong legal enforcement economies. Second, consider a constrained firm that is

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<sup>20</sup>Alternatively, one could assume that economies with weak legal enforcement are also less developed ("poor") compared to economies with strong legal enforcement which may be more developed ("rich") and that entrepreneurs thus also differ in terms of their initial net worth, specifically,  $w_0^L < w_0^H$ ; this would reinforce our conclusions for trade in used assets: less-developed, "poor" economies are net used capital buyers as a larger fraction of entrepreneurs are highly constrained.

indifferent between new and used assets; combining the investment Euler equations (27) and (28) we obtain

$$\frac{Af_k(\underline{k}(\theta_i)) + (1 - \theta_i)q_{nd}}{1 - R^{-1}\theta_i q_{nd}} = \frac{Af_k(\underline{k}(\theta_i))}{q_{nd}}, \quad (31)$$

which determines the level of investment  $\underline{k}(\theta_i)$  at which the firm is indifferent between the two types of assets at the margin. This level depends on legal enforcement and firms in economies with weak legal enforcement are indifferent at a higher level of investment, that is,  $\underline{k}(\theta_L) > \underline{k}(\theta_H)$ , as the proof in Appendix C shows. The lowest level of net worth at which the firm is able to invest  $\underline{k}(\theta_i)$  is  $\underline{w}_{nd}(\theta_i) = q_{nd}\underline{k}(\theta_i)$ , in which case the firm uses only non-durable assets. Clearly, this level is higher with weak legal enforcement, that is,  $\underline{w}_{nd}(\theta_L) > \underline{w}_{nd}(\theta_H)$ . Similarly, the highest level of net worth at which the firm invests  $\underline{k}(\theta_i)$  is  $\underline{w}_d(\theta_i) = \varphi_d(\theta_i)\underline{k}(\theta_i)$ , in which case the firm only invests in durable assets. Since  $\varphi_d(\theta_L) > \varphi_d(\theta_H)$  and  $\underline{k}(\theta_L) > \underline{k}(\theta_H)$ , we conclude that  $\underline{w}_d(\theta_L) > \underline{w}_d(\theta_H)$ . Moreover, since weak legal enforcement does not allow firms to lever as much, the net worth of firms in such economies grows more slowly. Therefore, firms in weak legal enforcement economies use non-durable, used assets for longer or in other words, a larger fraction of firms in weak legal enforcement economies invest in non-durable, used assets.

The effect of legal enforcement on durable and non-durable investment and trade in used assets is summarized as follows:

**Proposition 7** (Trade in used capital). *Suppose legal enforcement differs across economies, with  $\theta_L < \theta_H$ , and world loan and used asset markets are integrated with  $q_{nd} > q_{nd}^*$ . Then weak legal enforcement economies are net importers of used assets. Indeed, if  $\theta_H$  is sufficiently close to 1, used capital is dominated in strong legal enforcement economies and all used capital is shipped to weak legal enforcement economies. Otherwise, firms in weak legal enforcement economies substitute to durable assets at higher levels of net worth, that is,  $\underline{w}_{nd}(\theta_L) > \underline{w}_{nd}(\theta_H)$  and  $\underline{w}_d(\theta_L) > \underline{w}_d(\theta_H)$ . Firms also start to pay dividends at a higher level of net worth in weak legal enforcement economies, that is,  $\bar{w}(\theta_L) > \bar{w}(\theta_H)$ .*

Some details of the proof are in Appendix C. The predictions of our model for trade in used assets are consistent with the empirical evidence provided in the literature.<sup>21</sup>

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<sup>21</sup>We assume that assets provide the same service flow in both periods. If new assets provided a larger service flow in the first period, then the user cost of new assets for unconstrained firms could be higher than that of used assets, although it would still be lower on a per service flow unit basis. Moreover, if new capital were more productive due to technological progress every period, that is, if the economy had vintage capital, the unconstrained firms would adopt the new, durable vintages which are more productive, while more constrained firms would operate using the old, non-durable, and less productive, vintages. Finally, we could nest the model with vintage capital and with geometric depreciation, by

## 4 Renting durable assets

Renting is a significant way in which firms (and households), especially financially constrained ones, avail themselves of durable assets. A key aspect of renting is the associated ease of repossession as argued by Eisefeldt and Rampini (2009) and Rampini and Viswanathan (2013). Renting (or leasing, which we use synonymously here) allows the lessee to deploy assets while the lessor retains ownership which facilitates the repossession of these assets in case of default. In the context of an economy with limited enforcement, we model rented capital as capital the firm cannot abscond with, that is, rented capital can be collateralized fully. The cost of renting is that the lessor has to monitor its use to prevent abuse, for example. But how does durability affect the decision to rent assets instead of buying them (and borrowing against them)?

Consider an economy with capital with geometric depreciation as in Section 2. Suppose there are two types of capital, as before, one which is more durable and one which is less durable, denoted by subscripts  $d$  and  $nd$ , respectively. However, to focus on the rent vs. buy decision, we assume that the two types of assets are required in fixed proportions. Specifically, there are two types of capital which differ in their durability, that is, for  $j \in J \equiv \{d, nd\}$ , type  $j$  capital depreciates at rate  $\delta_j$ , where  $\delta_d < \delta_{nd}$ , and has price  $q_j$ . No assumptions on prices of the two types of capital are required here as the prices affect the cost of owning and renting a particular type of capital in the same way. Denote the type  $j$  capital the firm owns by  $k_j$  and the type  $j$  capital the firm rents (or leases) by  $k_j^l$ . Owned and rented capital of a particular type are assumed to be perfect substitutes. The two types of capital are deployed in fixed proportions, that is, aggregate capital  $k$  is determined by a Leontief aggregator  $k \equiv \min \left\{ \frac{k_d + k_d^l}{\sigma_d}, \frac{k_{nd} + k_{nd}^l}{\sigma_{nd}} \right\}$  where the factor shares  $\sigma_j > 0$  and  $\sum_{j \in J} \sigma_j = 1$ , which implies that  $k_j + k_j^l = \sigma_j k$ .

The rental rate for type  $j$  capital is  $u_j^l \equiv R^{-1} q_j (r + \delta_j + m)$  where  $m$  is the monitoring cost per unit of capital. Limited enforcement implies that this rental rate has to be charged up front. If the monitoring cost were zero, the rental rate would equal the frictionless user cost of capital, albeit payable in advance. A stationary competitive equilibrium can be defined analogously with the main adjustment being that the goods

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assuming that new, durable capital is produced at (exogenous) cost  $q_d$  and depreciates at a geometric rate  $\delta_d$  while at the same time a fraction  $\eta$  of the capital becomes used, non-durable capital. Used, non-durable capital trades at an equilibrium price  $q_{nd}$  and depreciates at rate  $\delta_{nd} > \delta_d$ . This vintage version of the model nests the model with new and used capital analyzed in this section by setting  $\delta_d = 0$ ,  $\delta_{nd} = 1$ , and  $\eta = 1$  (with  $q_d = 1$ ), that is, by assuming that new capital does not depreciate but turns into used capital the next period and that used capital fully depreciates each period. See the working paper version, Rampini (2017), for details.

market clearing condition needs to account for the monitoring costs. In equilibrium, well-capitalized entrepreneurs provide financing at rate  $R = \beta^{-1}$  and are the lessors charging the equilibrium rental rate  $u_j^l$  defined above. Since lessors operate with constant returns to scale and earn an equilibrium return  $R$  (net of depreciation and monitoring cost), for simplicity we do not explicitly account for leasing out capital below as it is equivalent to lending at interest rate  $R$ , but of course aggregate leased capital equals aggregate capital leased out in equilibrium.

The entrepreneur's problem with a rental decision is to choose  $\{d, k, \{k_j, k_j^l\}_{j \in J}, b, w'\}$ , given  $w$ , to solve

$$v(w) \equiv \max_{\{d, k, \{k_j, k_j^l\}_{j \in J}, b, w'\} \in \mathbb{R}_+^6 \times \mathbb{R}^2} d + \beta v(w') \quad (32)$$

subject to (3), (4), and the budget and technological constraints

$$w + b \geq d + \sum_{j \in J} q_j k_j + \sum_{j \in J} u_j^l k_j^l, \quad (33)$$

$$k_j + k_j^l \geq \sigma_j k, \quad j \in J. \quad (34)$$

Only assets that the firm owns can serve as collateral. Introducing aggregate capital  $k$  as a choice variable and equation (34) for  $j \in J$  (which hold with equality at an optimum) are a simple way to impose the Leontief technology.

The first-order conditions with respect to  $k$ ,  $k_j$ , and  $k_j^l$  using the same multipliers as before and multiplier  $\beta \mu' \eta_j$  on equation (34) are

$$\begin{aligned} \sum_{j \in J} \sigma_j \eta_j &= A f_k(k), \\ \mu q_j &= \beta \mu' \eta_j + \beta \mu' q_j (1 - \delta_j) + \beta \lambda' \theta q_j (1 - \delta_j) + \nu_j, \\ \mu u_j^l &= \beta \mu' \eta_j + \nu_j^l. \end{aligned}$$

The investment Euler equation for purchased type  $j$  capital is

$$1 = \beta \frac{\mu' \eta_j + (1 - \theta) q_j (1 - \delta_j)}{\mu \wp_j} + \frac{\nu_j}{\mu \wp_j}.$$

Assume that the down payment for purchasing a type of capital exceeds the rental rate, that is,  $\wp_j > u_j^l$ , as otherwise renting would be dominated. As shown in the previous work, more constrained firms rent capital. To see how durability affects the rent vs. buy decision, suppose a firm is indifferent between renting and owning type  $j$  capital, so  $\nu_j = \nu_j^l = 0$ . Combining first-order conditions we obtain

$$\frac{\lambda'}{\mu} (1 - \theta) (1 - \delta_j) = m.$$

The multiplier on the collateral constraint (scaled by the multiplier on the budget constraint)  $\frac{\lambda}{\mu}$  at which the firm is indifferent between buying and renting depends on the depreciation rate. More durable assets are rented by less constrained firms, whereas less durable assets are rented only if the firm is sufficiently constrained. Suppose the firm needs both structures, say a plant, and equipment, say machines, for production and structures are more durable than machines, which is arguably the case. Severely constrained firms lease both the plant and the machines whereas less constrained firms lease only the plant and unconstrained firms do not lease either. Similarly, if firms require different types of equipment, for example, aircraft, which are very durable, and ground support equipment, which is less durable, our theory predicts that many airlines lease their aircraft, but only severely constrained airlines lease their ground support equipment.<sup>22</sup> Note also that the model implies that there is no renting of assets which depreciate fully each period, that is, with  $\delta_j = 1$ , and no renting when collateral is fully pledgeable.<sup>23</sup>

The key insight is that firms are more inclined to rent more durable assets as such assets are associated with a larger financing need exactly because they are more durable and hence require more internal funds which makes renting them more beneficial as it allows additional financing. Less durable assets are rented only by severely constrained firms, whereas more durable assets are rented even by less constrained firms.

## 5 Related literature

In this section we discuss the related literature on durable goods and used capital markets. Hart and Moore (1994) also consider the effect of durability on financing, among other things, and conclude that “[i]f the assets become more durable . . . [t]he project is more likely to be undertaken” (page 860). Their definition of durability is as follows (page 859): “We say that the assets become longer lived, or more durable, if [the liquidation value]  $L(t)$  rises for all  $0 \leq t \leq T$ .” By interpreting a higher liquidation value as higher durability, they do not distinguish between pledgeability and durability. We argue that the liquidation value should be interpreted as pledgeability, which facilitates financing

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<sup>22</sup>Analogously, for households this implies that severely constrained households rent both their house and their cars whereas less constrained household buy their cars and rent their house, and unconstrained households own both their cars and their house. This prediction seems empirically quite plausible and is consistent with life-cycle patterns in consumer durables purchases, namely the fact that households typically do not own their houses until later in life but own their cars earlier on.

<sup>23</sup>If capital lasts for two periods, as in Section 3, then used, non-durable assets are never rented while new, durable assets might be rented, depending on parameters. This may explain why in practice it is often new assets that are leased, not used ones.

in our model as well, rather than as durability, which impedes financing. Durability of assets in our model is defined in terms of their depreciation, which affects the useful life of the assets and hence both the value in use and the collateral value. In contrast, in their definition durability only affects the collateral value and not the value in use, which is arguably a more appropriate definition for pledgeability than durability.

More specifically, the liquidation value (per unit of capital) in their model can be interpreted as  $L_j k_j \equiv \theta q_j k_j (1 - \delta_j)$ ; the collateral constraint (4) can then be written as  $\sum_{j \in J} L_j k_j \geq Rb$ . With this interpretation,  $L_j = \theta q_j (1 - \delta_j)$ ; thus, the liquidation value involves both pledgeability  $\theta$  and durability  $1 - \delta_j$ , as well as the price of the asset. They consider the effect of the liquidation value  $L_j$  on financing, keeping the price of the asset and the value in use constant; then clearly  $\frac{\partial \varphi_j}{\partial \delta_j} = \frac{\partial}{\partial \delta_j} (q_j - R^{-1} L_j) = R^{-1} \theta q_j > 0$ , leading them to conclude that more durable capital requires a lower down payment. The intuition is that more durable capital has a higher collateral value supporting more debt finance. But this argument is misleading or at least incomplete because it keeps the price of capital  $q_j$  fixed and we show that if two types of capital which differ in their durability are both used in equilibrium, the more durable type of capital must have a higher down payment (and a higher price). Thus, in our view the effect they emphasize should be interpreted as the effect of pledgeability  $\theta$  rather than durability.<sup>24</sup>

Our model abstracts from several features of durable asset markets that have been considered in the literature including adverse selection,<sup>25</sup> illiquidity,<sup>26</sup> and heterogeneity across firms other than that induced by financial constraints.<sup>27</sup> This allows us to focus

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<sup>24</sup>To see this another way, suppose one were to keep the frictionless user cost of capital  $u_j$  constant (instead of the price) and adjust the price accordingly, so that the price of capital is the present value of the future rental payments  $q_j(\delta_j) = \sum_{t=0}^{\infty} \frac{u_j(1-\delta_j)^t}{(1+r)^t} = \frac{R u_j}{r+\delta_j}$ , then  $\frac{\partial}{\partial \delta_j} \varphi_j(\delta_j) = \frac{\partial}{\partial \delta_j} (q_j(\delta_j) - R^{-1} \theta q_j(\delta_j)(1 - \delta_j)) = -q_j(\delta_j) \frac{1-\theta}{r+\delta_j} < 0$ , which would lead one to conclude that more durable capital requires larger down payments, reversing the basic conclusion regarding the effect of durability on financing.

<sup>25</sup>Following Akerlof's (1970) seminal study of the market for used cars, several authors have studied adverse selection in the market for used durables. Bond (1982) finds no evidence that trucks that were purchased used require more maintenance. Hendel and Lizzeri (1999a) consider trade in a durable goods market with adverse selection and heterogeneous consumers and show that trade never breaks down. Hendel and Lizzeri (2002) argue that leasing ameliorates adverse selection in durable goods markets and attribute the increase in car leasing over the last few decades to the increase in the durability of cars which they argue aggravates adverse selection. House and Leahy (2004) show that in the market for durable goods the  $sS$  bounds due to adverse selection contract as heterogeneity increases, implying that as durables age, adverse selection decreases and trade increases.

<sup>26</sup>Gavazza (2011b) shows that aircraft that trade in thinner markets are less liquid and Gavazza (2016) studies the effect of trading frictions and the role of intermediaries in the market for used aircraft. Relatedly, Gavazza (2011a) shows that leased aircraft trade more frequently than owned aircraft and are owned by carriers with more volatile capacity needs.

<sup>27</sup>Bond (1983) studies trade in used equipment in a model with heterogeneous firms which differ in

squarely on the effect of financial constraints in inducing a preference across assets which differ in terms of their durability.

Optimal durability is also analyzed in the literature on durable goods. Much of this literature focuses on a monopolist's choice of durability and argues that a monopolist has incentives to produce less durable goods than a competitive producer would, a phenomenon at times referred to as planned obsolescence (see Waldman (2003) for a comprehensive survey).<sup>28</sup> In a seminal paper Coase (1972) argues that a durable-goods monopolist faces a time inconsistency problem resulting in a complete loss of market power and a competitive outcome.<sup>29</sup> Coase (1972), as well as Bulow (1982) and Stokey (1981), show that leasing or renting the durable goods avoids the time inconsistency problem and allows the monopolist to retain market power. Bulow (1982, 1986) moreover shows that, in the absence of rental markets, a monopolist may choose to produce less durable goods than a competitive producer again in order to avoid Coasian dynamics.<sup>30</sup> In these theories, market power is therefore the *raison d'être* for durable goods of low durability and rental markets for durables. In contrast, in our theory assets of low durability and rental markets are an optimal response to financial constraints in a competitive model.

The choice between new and used capital is also studied by Eisfeldt and Rampini (2007) who focus on the role of maintenance costs. They argue that used capital is cheaper up front but requires maintenance costs which they assume can be paid ex-post, making used capital attractive for constrained firms. The durability of capital per se plays no role in their analysis; in fact, they assume that all capital depreciates at the same geometric rate, that is, is equally durable. While the assumption that used capital

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terms of factor prices and utilization rates. Stolyarov (2002) and Gavazza, Lizzeri, and Roketskiy (2014) study trade in used cars in a model with transaction costs and consumers with heterogeneous utility from the service flow of durables.

<sup>28</sup>An early literature making this case (including Martin (1962), Kleiman and Ophir (1966), Levhari and Srinivasan (1969), and Schmalensee (1970)) is shown by Swan (1970, 1971, 1972) and Sieper and Swan (1973) to be incorrect as the monopolist has the same incentives to minimize the cost of the provision of a given service flow as a competitive producer. Barro (1972) comes to a similar conclusion although he also shows that if consumers are less patient than the monopolist, the monopolist may choose lower durability. Schmalensee (1974) finds that with endogenous maintenance the monopolist does distort durability and Rust (1986) shows that with endogenous scrappage the monopolist distorts durability and derives conditions under which the monopolist produces goods of zero durability. Thus, these authors resuscitate the conventional wisdom to some extent.

<sup>29</sup>Kahn (1986) shows that with increasing marginal costs the monopolist asymptotically produces the competitive amount but produces more slowly.

<sup>30</sup>More recently, Waldman (1996) and Hendel and Lizzeri (1999b) find that monopolists may choose lower durability as a way to price discriminate between consumers in models with consumers with heterogeneous preferences for different vintages that are not perfect substitutes.

requires higher maintenance cost may be plausible, this is a different mechanism from the novel and more fundamental mechanism we propose here, namely that durability itself renders assets harder to finance for constrained borrowers. Moreover, our main results are more general as they obtain for an economy with different types of capital with geometric depreciation which vary in durability as well.<sup>31</sup>

A growing literature studies used capital markets empirically. Ramey and Shapiro (2001) document that used capital sells at substantial discounts in data from aerospace plant closings. Eisfeldt and Rampini (2007) show that smaller and more constrained firms purchase substantially more used capital in U.S. census data on new and used capital expenditures,<sup>32</sup> consistent with the predictions of our model.

Several authors consider international trade in used capital. Sen (1962) considers differences in the relative price of labor as an explanation for the fact that less developed economies are net importers of used capital. Navaretti, Soloaga, and Takacs (2000) show empirically that in less developed economies the share of used equipment imported is higher and Benmelech and Bergman (2011) find that weak legal enforcement is associated with both older aircraft and older technologies, again consistent with our predictions.

Finally, our model is related to the literature on technology adoption. Chari and Hopenhayn (1991) show that new technologies are adopted slowly in an economy with vintage-specific human capital and that economies continue to invest in older vintages. In our model firms may choose to invest in less durable capital that would otherwise be dominated since the lower durability makes this type of capital attractive due to financial constraints even though different types of capital are perfect substitutes in production.

## 6 Conclusion

Durable assets are harder to finance because they require larger down payments. While durability does increase the resale value and hence the collateral value allowing more borrowing, it increases the price of assets and thus the financing need overall by more.

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<sup>31</sup>For consumer durables, Bils and Klenow (1998) find that more durable assets are more likely to be luxuries. This is consistent with an induced preference along the lines of the theory put forth here. Aghion, Angeletos, Banerjee, and Manova (2010) study a business cycle model with two technologies, a one-period investment technology and a two-period investment technology. Their analysis focuses on the implications of the difference in this time-to-build type feature rather than durability.

<sup>32</sup>Eisfeldt and Rampini (2006) show that trade in used capital, which is part of capital reallocation which they define more broadly, is procyclical and provide a calibrated model with countercyclical reallocation frictions to match this basic fact. Lanteri (2018) shows that the relative price of used capital is procyclical and proposes a model in which new and used capital are imperfect substitutes consistent with this property.

Since the effect on the financing need exceeds the increase in collateral value, more durable assets require larger down payments as long as the resale value cannot be fully pledged.

Financial constraints are therefore especially salient for investment in durable assets. For firms, these durable assets include residential and non-residential structures, infrastructure, equipment including aircraft, ships, and trucks. For households, durable assets include consumer durables, especially housing, motor vehicles, and household durables. While we emphasize the effects of durability on the financing of tangible assets, the same results apply for investment in intangible capital, for example, organization capital, which can be collateralized only to a very limited extent or not at all, as  $\theta = 0$  is a special case of our model.

Constrained firms may use less durable assets even if these were otherwise dominated. Constrained firms also buy used capital, which is less durable, instead of new capital, consistent with the data, and rent durable assets to reduce the demands on internal funds. The model thus yields predictions on how the composition of investment, technology adoption, the vintage of capital purchased, and rental choices vary with financial constraints.

Durability exacerbates the effect of legal enforcement on investment. In economies with weak legal enforcement, investment in infrastructure, buildings, and cars and trucks, for example, can be distorted toward lower qualities that are less durable. Moreover, firms and household use less durable qualities and used capital to a greater extent in such economies. And weak legal enforcement countries are net importers of used real assets, consistent with international trade flows. Indeed, firms and households in weak legal enforcement economies may use less durable types of capital and used assets that are dominated in strong legal enforcement economies to reduce their financing needs.

Our model thus provides a novel theory of optimal durability based on financial constraints and competitive markets: Less durable types of capital are used because they alleviate financial constraints.

We emphasize that it is critical to distinguish the durability of assets from their pledgeability both in empirical work and in theory. Pledgeability affects only the collateral value and unambiguously facilitates financing. In contrast, durability affects not only the collateral value but also the cost of the assets and hence the overall financing need, with the net effect of impeding financing.

# Appendix

## Appendix A: Households' choice of consumer durables

This appendix considers households' choice between durable and non-durable goods, which provides a particularly simple version of the main result of the paper. Consider a discrete-time, infinite horizon economy. There is a continuum of households of measure one alive at each date; households survive to the next period with probability  $1 - \rho$  and measure  $\rho \in (0, 1)$  of new households are born every period with net worth  $w_0 > 0$  (to be defined below). Households are risk averse and we formulate the households' problem recursively. Households have preferences  $u(c) + \beta v(w')$  over consumption services  $c$  in the current period and net worth  $w'$  next period, where  $\beta \in (0, 1)$  is the effective rate of time preference (with  $\hat{\beta}$  the rate of time preference and  $\beta \equiv \hat{\beta}(1 - \rho)$ ),  $v(\cdot)$  is the value function, and  $u(\cdot)$  is the utility function which is strictly increasing, strictly concave, continuously differentiable, and satisfies  $\lim_{c \rightarrow 0} u_c(c) = +\infty$ .

Households have access to two types of goods, durable goods and non-durable goods. Denote the amount of durable goods a household purchases in the current period by  $k_d$  and the amount of non-durable goods by  $k_{nd}$ . Durable and non-durable goods provide consumption services and are perfect substitutes in terms of their services, that is,  $c = k_d + k_{nd}$ . The differences between durable goods and non-durable goods are their price and durability: durable goods cost 1 to produce and last for two periods, that is, provide consumption services both in the period in which they are purchased and in the next period; in contrast, non-durable goods cost  $q_{nd}$  where, in equilibrium,  $1 > q_{nd} \geq 1/(1 + \beta)$  but provide consumption services only in the period in which they are purchased. Non-durable goods purchased this period are hence worthless next period, whereas a unit of durable goods purchased this period is worth  $q_{nd}$  next period since it is equivalent to a unit of non-durable goods next period. Households also have a deterministic income of  $y > 0$  each period. Households can buy any amount of durable and/or non-durable goods and can resell durable goods after one period, but do not have access to borrowing or lending otherwise, that is, we set  $\theta = 0$  in terms of the model in the text.

We consider a stationary equilibrium of the economy in which all aggregate quantities and the used capital price  $q_{nd}$  are constant. For simplicity, we focus on the case in which  $q_{nd} > 1/(1 + \beta)$  (as the economic intuition in the case with  $q_{nd} = 1/(1 + \beta)$  is similar).

Given current net worth  $w$ , the household's problem can be written as

$$v(w) \equiv \max_{\{k_d, k_{nd}, w'\} \in \mathbb{R}_+^2 \times \mathbb{R}} u(k_d + k_{nd}) + \beta v(w')$$

subject to the budget constraints for the current and the next period

$$\begin{aligned} w &\geq k_d + q_{nd}k_{nd}, \\ y + q_{nd}k_d &\geq w'. \end{aligned}$$

Notice that consumption services  $c$  have been substituted out and that net worth is defined as income plus the resale value of consumption durables.

Since the constraint set is convex and the operator defined by the Bellman equation satisfies Blackwell's sufficient conditions, there exists a unique value function that solves the Bellman equation which is strictly increasing and strictly concave. Using the multipliers  $\mu$  and  $\beta\mu'$  for the budget constraints in the current and next period, and the multipliers  $\nu_d$  and  $\nu_{nd}$  for the non-negativity constraints, the first-order conditions for durable and non-durable goods and for net worth next period are

$$\begin{aligned}\mu &= u_c(c) + \beta\mu'q_{nd} + \nu_d, \\ \mu q_{nd} &= u_c(c) + \nu_{nd}, \\ \mu' &= v_w(w'),\end{aligned}$$

and the envelope condition is  $v_w(w) = \mu$ . It is not possible that both  $\nu_d > 0$  and  $\nu_{nd} > 0$  at the same time, as otherwise the budget constraint in the current period would be slack.

Over time, an individual household reaches a steady state (conditional on survival) in which the household's net worth and purchases of durable and non-durable goods are constant. In such a steady state, the household only purchases durable goods. To see this, denote the marginal value of net worth in the steady state by  $\bar{\mu}$  and using the first-order conditions we have  $\bar{\mu}(q_{nd} - (1 - \beta q_{nd})) = \nu_{nd} - \nu_d$ . Since the right-hand side is strictly positive, we conclude that  $\nu_{nd} > 0$ ; the household does not buy non-durable goods. (If  $q_{nd} = 1/(1 + \beta)$ , the right-hand side is 0 and hence  $\nu_{nd} = \nu_d = 0$  and the household is indifferent between durable and non-durable goods.) Indeed, using the budget constraints for the current and next periods, we can compute the steady state level of net worth  $\bar{w} = y/(1 - q_{nd})$ , which equals the purchases of durable goods  $\bar{k}_d$  and steady state consumption  $\bar{c}$  (while purchases of non-durable goods are  $\bar{k}_{nd} = 0$ ).

Severely constrained households, that is, households with sufficiently low net worth, buy only non-durable goods. To see this notice that the budget constraint implies that as  $w$  goes to 0,  $c$  does too, and  $\mu \geq u_c(c) \rightarrow +\infty$ . Combining the first-order conditions we have  $\mu(1 - q_{nd}) = \beta\mu'q_{nd} + \nu_d - \nu_{nd}$  and, since  $w' \geq y$ ,  $\mu'$  is bounded above, implying that  $\nu_d > 0$  as  $w$  goes to 0. Indeed, households do not buy any durable goods on a lower interval of net worth levels. For suppose, by contradiction that for  $w_- < w$ ,  $\nu_d > 0$  (that is,  $k_d = 0$ ) at  $w$  while  $k_d^- > 0$  at  $w_-$ . By strict concavity of the value function,  $\mu_- > \mu$ , whereas  $\mu'_- < \mu'$  since  $w'_- = y + q_{nd}k_d^- > y + q_{nd}k_d = w'$ . Using the first-order conditions at  $w$  and  $w_-$  respectively we get  $0 < (\mu_- - \mu)(1 - q_{nd}) = \beta(\mu'_- - \mu')q_{nd} - \nu_{nd}^- - \nu_d < -\nu_{nd}^- - \nu_d$ , a contradiction. Moreover, above this interval, durable goods purchases  $k_d$  are strictly increasing in  $w$ . This is obvious if the household only buys durable goods, so suppose the household buys both durable and non-durable goods. In that case,  $\mu(1 - q_{nd}) = \beta\mu'q_{nd}$ , and by strict concavity  $\mu$  is strictly decreasing in  $w$  and hence so is  $\mu' = v_w(w')$ , implying that  $w'$  and hence  $k_d$  are strictly increasing in  $w$ .

Intuitively, durable goods force households to save, making households with low net worth reluctant to buy durable goods. Such households buy "low quality" non-durable goods because these are cheaper to them as these goods have a smaller up-front cost, whereas households with high net worth buy "high quality" durable goods because these are cheaper to them since they are less constrained which means that the opportunity cost of the additional funds required to purchase durable goods is lower.

## Appendix B: Technology adoption with stochastic productivity

This appendix considers the economy with two types of capital  $j \in J \equiv \{d, nd\}$  with different geometric depreciation rates  $\delta_d < \delta_{nd}$  such that  $u_d < u_{nd}$  and  $\wp_d > \wp_{nd}$  as in Proposition 1, but assumes that firms' productivity  $A'$  is stochastic and independent and identically distributed across firms and over time on a finite set of states  $s' \in S$  with probability  $\pi(s')$ . Following Rampini and Viswanathan (2013) the optimal contract can be implemented with one-period ahead Arrow securities  $b' \equiv b(s')$  subject to state-by-state collateral constraints. In equilibrium, unconstrained entrepreneurs provide state-contingent loans at an expected return  $R = \beta^{-1}$ .

The entrepreneur's problem is to choose  $\{d, k_d, k_{nd}, b', w'\}$ , given  $w$ , to solve

$$v(w) \equiv \max_{\{d, k_d, k_{nd}, b', w'\} \in \mathbb{R}_+^3 \times \mathbb{R}^{2S}} d + E[\beta v(w')] \quad (35)$$

subject to the budget constraints and the state-by-state collateral constraints

$$w + E[b'] \geq d + \sum_{j \in J} q_j k_j, \quad (36)$$

$$A' f(k) + \sum_{j \in J} q_j k_j (1 - \delta_j) \geq R b' + w', \quad \forall s' \in S, \quad (37)$$

$$\theta \sum_{j \in J} q_j k_j (1 - \delta_j) \geq R b', \quad \forall s' \in S, \quad (38)$$

where  $k \equiv \sum_{j \in J} k_j$ . Using the multipliers  $\pi(s')\beta\mu'$  and  $\pi(s')\beta\lambda'$  on (37) and (38) and defining the other multipliers as before, the investment Euler equations for type  $j$  capital,  $j \in J$ , can be written as

$$1 = E \left[ \beta \frac{\mu'}{\mu} \frac{A f_k(k) + (1 - \theta) q_j (1 - \delta_j)}{\wp_j} \right] + \frac{\nu_j}{\mu \wp_j},$$

Define the user cost of type  $j$  capital (for a possibly constrained firm) analogously to (14) and (15) as

$$u_j(w) \equiv \wp_j - E \left[ \beta \frac{\mu'}{\mu} \right] (1 - \theta) q_j (1 - \delta_j) = u_j + E \left[ \beta \frac{\lambda'}{\mu} \right] (1 - \theta) q_j (1 - \delta_j).$$

A firm that pays dividends is unconstrained against all states next period, since  $\mu = 1 = \mu'$  and hence  $\lambda' = 0$ ,  $\forall s' \in S$ . Using the second expression for the user cost we conclude that  $u_j(w) = u_j$  and hence unconstrained firms invest only in the more durable type of capital since  $u_d < u_{nd}$ . Severely constrained firms in contrast invest only in the less durable type of capital since as  $w$  goes to 0, so must  $k_j$ ,  $j \in J$ , and hence  $k = \sum_{j \in J} k_j$ ; using the investment Euler equation and the Inada condition, we conclude that  $\beta \frac{\mu'}{\mu}$  goes to 0,  $\forall s' \in S$ . Using the first expression for the user cost we conclude that  $u_j(w) \rightarrow \wp_j$  and since  $\wp_d > \wp_{nd}$ , the result obtains. Thus our conclusion regarding the effect of durability on investment extends to a stochastic environment.

## Appendix C: Proofs

**Proof of Proposition 3.** Total differentiation of equation (16) yields

$$\frac{d\underline{k}(\theta_i)}{d\theta_i} = \frac{(u_{nd} - u_d)}{(\wp_d(\theta_i) - \wp_{nd}(\theta_i))^2} \frac{q_d q_{nd} (\delta_{nd} - \delta_d)}{Af_{kk}(\underline{k}(\theta_i))} < 0.$$

The critical levels of net worth are  $\underline{w}_{nd}(\theta_i) = \wp_{nd}(\theta_i) \underline{k}(\theta_i)$  (no use of more durable capital below  $\underline{w}_{nd}(\theta_i)$ ),  $\underline{w}_d(\theta_i) = \wp_d(\theta_i) \underline{k}(\theta_i)$  (no use of less durable capital above  $\underline{w}_d(\theta_i)$ ), and  $\bar{w}(\theta_i) = \wp_d(\theta_i) \bar{k}$  (dividend threshold) where  $\bar{k}$  solves  $u_d = \beta Af_k(\bar{k})$ . Since  $\wp_{nd}(\theta_i)$  and  $\wp_d(\theta_i)$  are decreasing in  $\theta_i$  the ordering is immediate. Moreover, since lower  $\theta_i$  allows firms to lever less, the net worth of firms in weak legal enforcement economies grows more slowly. The rest of the proof is in the main text.  $\square$

**Proof of Proposition 7.** Total differentiation of equation (31) yields

$$\frac{d\underline{k}(\theta_i)}{d\theta_i} = \frac{(q_{nd} + R^{-1}q_{nd} - 1)}{(\wp_d(\theta_i) - q_{nd})^2} \frac{q_{nd}^2}{Af_{kk}(\underline{k}(\theta_i))} < 0.$$

The rest of the proof is in the main text.  $\square$

## Appendix D: New and used assets as imperfect substitutes

This appendix considers the economy with assets that last for two periods as in Section 3, but assumes that new, durable and used, non-durable assets are imperfect substitutes with an aggregator for capital  $k$  with constant elasticity of substitution as in (17). The firm's problem is as in (19) to (22) except that the production function is  $f(k)$ . Using the definitions of the user cost of used, non-durable assets  $u_{nd} = q_{nd}$  and of new, durable assets  $u_d(w)$  as in Footnote 19, the first-order conditions imply

$$\frac{u_{nd}}{u_d(w)} = \left( \frac{k_d}{k_{nd}} \right)^{1-\gamma} \frac{\sigma_{nd}}{\sigma_d}$$

which is equivalent to (18) in the economy with capital with geometric depreciation and imperfect substitutes. For unconstrained firms,  $u_d(w) = u_d$  and the ratio of the frictionless user costs  $u_{nd}/u_d$  determines the composition of investment. For severely constrained firms,  $\lim_{w \rightarrow \infty} u_d(w) = \wp_d$  and the ratio of the down payments  $\wp_{nd}/\wp_d$  determines the composition of investment. Since  $\frac{\wp_{nd}}{u_{nd}} = \frac{q_{nd}}{q_{nd}} = 1$  and  $\frac{\wp_d}{u_d} = \frac{1-R^{-1}\theta q_{nd}}{1-R^{-1}q_{nd}} > 1$ ,  $\frac{u_{nd}}{u_d} > \frac{\wp_{nd}}{\wp_d}$  and severely constrained firms substitute away from new assets. Intuitively, the fraction of the down payment comprised by the residual value that cannot be pledged is higher for new, durable assets than for used, non-durable assets, that is,  $\frac{\wp_d - u_d}{\wp_d} = \frac{R^{-1}(1-\theta)q_{nd}}{1-R^{-1}\theta q_{nd}} > 0 = \frac{\wp_{nd} - u_{nd}}{\wp_{nd}}$ . Financial development decreases the distortion of investment away from new, durable assets for severely constrained firms. Note that, unlike in the case of perfect substitutes,  $\wp_d > \wp_{nd}$  is no longer a foregone conclusion with imperfect substitutes and is not necessary for our implications for the effect of durability on the composition of investment to obtain. The key determinant of investment composition is the fraction of the down payment that is comprised by the residual value that cannot be pledged, which is higher for more durable assets.

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