A rationale for imperfect reporting standards*

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We examine the interplay between a regulator's design of a public reporting system and a firm's decisions regarding the gathering and disclosure of private information. The context is one in which the firm seeks to meet a threshold on its market price possibly to avoid delisting on an exchange, exclusion from a market index, downgrading by a credit rating agency, or violation of debt covenants. The regulator, who might be a body that determines accounting standards, seeks to maximize the total information provided by both public reports governed by those standards and voluntary disclosures of information gathered by the firm. Notably, we identify conditions under which the regulator prefers less than fully informative reports as a means of inducing the firm to gather and potentially disclose more information. We obtain further results in cases where the firm's choice with respect to gathering information is or is not observable by investors, the firm's information acquisition decision is made after (rather than before) financial reports are released, the regulator chooses the threshold rather than the properties of the reporting system, and the regulator's choice is guided by the firm's preferences.

Keywords: information acquisition, reporting system, mandatory disclosure, discretionary disclosure

1 Introduction

In this paper, we are interested in the interactions between a firm's regulated financial reporting and its discretionary disclosures. Firms provide information to capital market participants through both these channels. Financial reports are prepared according to accounting standards set by regulators. Discretionary disclosures are voluntary and based on private information that the firm may have acquired. In this paper we study how the properties of reporting systems set by regulators influence and are influenced by the firm's incentives to gather disclosable information. We show that a regulator who seeks to maximize the amount of information available to financial statement users may prefer for financial reports to be imperfectly-informative, even when there are no direct costs associated with compliance or enforcement.

According to the Financial Accounting Standards Board's (FASB) Conceptual Framework, the objective of general purpose financial reporting is to provide financial information about the reporting entity that is useful to existing and potential investors, lenders, and other creditors in making decisions about providing resources to the entity. However, financial reports do not capture all information germane to valuing the firm. Similar to Feltham and Ohlson (1996), we envision firm value as composed of two parts, the value of assets in place and the value of future investment or growth opportunities. While financial reports may reflect the value of assets in place reasonably well, information pertaining to future growth opportunities is less amenable to inclusion in those reports, and may be provided to investors via the firm's discretionary disclosures. The information structure in our model distinguishes between information contained in financial reports the content of which is subject to regulation and additional information the firm may acquire and, if acquired, may disclose. Firms' ability to adjust their information acquisition strategies in response to financial reporting standards provides an interesting channel through which the FASB's standards can have an additional effect on the amount of financial information available to investors. This channel can provide the standard setter with incentives to choose less informative financial reports even when more informative financial reporting would otherwise be feasible and desirable.

We explore the interdependency of regulator's and firm's information decisions in a parsimonious model featuring a firm, a regulator, and (passive) investors. A firm provides information to investors through its regulated financial report and a voluntary disclosure of privately acquired information, and realizes a benefit if investors' posterior beliefs about the firm's value are sufficiently high, i.e., above a threshold. For instance, the firm can maintain a public listing, be included in an index, achieve an investment-grade credit rating, or obtain a waiver of a debt covenant violation if equity investors, credit-raters, or lenders, respectively, have sufficiently high beliefs about the firm.[1]

Investors value the firm for its assets-in-place and its growth opportunities. Information on assets-in-place is provided by a financial report the properties of which are set by a regulator (e.g., the FASB). Information about growth opportunities is outside the purview of the regulator. At its discretion, the firm can choose to gather information about growth opportunities and subsequently disclose this information. The firm's information gathering decision is portrayed by a probability of becoming informed as in Dye (1985) and Jung and Kwon

[1] Beyer and Dye (2018), Dye and Hughes (2017), and Friedman, Hughes, and Michaeli (2017) also consider voluntary disclosure in a settings with nonlinear objective functions due, respectively, to leverage, risk aversion, and thresholds.

(1988). In our analysis, we assume that the firm gathers information after the reporting system is in place but before mandated reports are realized and distributed. Consistent with the Conceptual Framework, the regulator in our model seeks to maximize the value relevant information provided to market participants. This includes setting standards for financial reporting with anticipation of the effects of such standards on the firm's incentives to acquire and disclose additional information.

We begin by examining how the firm's information gathering strategy (probability of becoming informed) is affected by properties of the reporting regime in place. To abstract from unnecessary frictions, we assume the firm's information-gathering is costless. That is, the firm bears no extra (direct) cost from being privately informed more or less often. The firm only seeks to maximize its expected utility, which is equivalent to maximizing the probability of posterior expectations about firm value exceeding the threshold. With this objective, the firm may choose to always, never, or sometimes be informed about the second component of its value.

Broadly speaking, the firm gains from information acquisition because a positive disclosure can cause investors' posterior beliefs to exceed the threshold. By engaging in greater information acquisition, the firm increases the likelihood that it will observe positive private information on which to base a positive disclosure. However, increased information acquisition also makes non-disclosure worse news. The implications of disclosure or non-disclosure on the firm's ability to induce beliefs that exceed the threshold depend on the properties of the financial reporting system and the realized reports from that system. Overall, the firm chooses to be perfectly informed when it faces only gains from the potential for high messages to allow it to meet the threshold, and chooses to be completely uninformed when there is no such benefit. An interior level of information acquisition is chosen when high messages can undo the negative effects of low reports and the threshold may still be met in the case of a good report and non-disclosure of private information. In this last case, the firm balances these two effects to maximize its probability of meeting or exceeding the threshold.

Our second result relates to the regulator's choice of the reporting system. We model the regulator as seeking to maximize the total amount of information available to investors, as measured by the posterior precision of their beliefs about firm value, or, equivalently, the ex ante variance of their posterior expectation of firm value.[2] Now, if firm value were composed only of the first component the regulator would set a financial reporting system that is fully informative of the first component (i.e., a "perfectly-informative reporting system"). Similarly, if the firm's information gathering was independent of the properties of the reporting system, then the regulator would also set a perfectly-informative reporting system. However, when the firm's information gathering is endogenous and depends on the properties of the reporting system, the regulator must also consider how the informativeness (and bias) of the reporting system influence the firm's incentives to gather private disclosable information. When this is taken into account, we find that the regulator always wants to provide incentives to the firm to gather private information as frequently as possible. Sometimes maintaining these incentives is compatible with a perfectly-informative reporting system. However, there are scenarios (i.e., parameter constellations) in which setting a perfectly-informative reporting system would cause the firm to gather private

[2]More information causes greater revision of beliefs and a higher variance of the posterior expectation.

information less frequently. In equilibrium, under these circumstances, the regulator optimally sets an imperfectly-informative reporting system. We emphasize that this occurs even though there are no direct costs to perfectly-informative reporting systems and the regulator's objective is to maximize the amount of information available to investors in expectation. Crucially, the regulator reduces the informativeness of the reporting system to maintain incentives for the firm to acquire (and subsequently disclose) private information.

We consider four model variations. First, we show that the equilibrium is sensitive to the observability of the firm's information acquisition strategy. In the main analysis we assume the strategy is observable. When the strategy is unobservable, the firm's equilibrium strategy is to always acquire information, and this strategy is insensitive to the properties of the reporting system, allowing the regulator to require perfectly-informative financial reports. This creates an incentive for the firm to communicate, i.e., make observable, the probability with which it acquires information. Second, we consider an alternative timeline in which firms decide how frequently to acquire private information after observing the financial reports. We show that our results qualitatively hold: to induce private information acquisition and disclosure, the regulator sets an imperfectly informative reporting system. Third, we allow the regulator to set the threshold the firm faces rather than the properties of the reporting system. We show that the regulator can induce information acquisition for a wide range of threshold values. This leaves the regulator with degrees of freedom to address the specific choice of the threshold to other frictions we do not address. Finally, we show that if the firm could choose the properties of the reporting system, it would choose a fully-informative reporting system only in the knife-edge case of the exogenous threshold being at its highest possible value. This result suggests that regulators whose objective function aligns more closely with firms than with investors may prefer less-informative reporting systems.

We add to a growing stream of literature on the intersection of mandatory financial reporting and discretionary disclosure including Friedman et al. (2017), who examine a firm's choices regarding both its financial reporting system and voluntary disclosure strategy for an exogenous probability of becoming informed. Other recent studies have begun to provide additional conditions under which firms' voluntary disclosure strategies are *not* independent of public signals, as is the case in the model of Acharya, DeMarzo, and Kremer (2011). Cianciaruso, Lee-Lo, and Sridhar (2018) examine interactions between public signals and voluntary disclosures for levered firms and find that disclosure strategies respond to the properties and realizations of public signals. In these models, the public reports or signals pertain to the same underlying state variable as the discretionary disclosures. In our model, in contrast, they pertain to distinct components of firm value.

Other papers studying how mandatorily disclosed information can affect discretionary disclosure choices include Bagnoli and Watts (2007), Einhorn (2005), Gigler and Hemmer (1998, 2001), and Hughes and Pae (2004). We differ from these related studies in that we focus on the firm's endogenous information-gathering choice. We see this choice as a relevant one to study as it is plausibly endogenous and can be changed at a high frequency relative to the properties of the regulated reports.[3] Our paper is also broadly related to the literature on

[3]Empirical studies on interactions between properties of financial reports and discretionary disclosures have provided somewhat mixed results. Gong, Li, and Xie (2009), Lennox and Park (2006), and Francis, Nanda, and Olsson (2008) find that earnings quality proxies are positively associated with features of voluntary disclosure such as frequency. Guay, Samuels,

interactions between private and public information, e.g., Arya and Mittendorf (2007), Fischer and Stocken (2010), Hue and Zheng (2018).

Our study is also related to literature on endogenous information-gathering. Shavell (1994) studies the acquisition and disclosure of information by buyers and sellers, focusing on whether the costly information is acquired too frequently or infrequently. Michaeli (2017) studies how rules limiting disclosure to selective audiences (e.g., Reg FD) influence managers' incentives to gather information and thereby reduce the overall available information. Friedman (2014, 2016), Liang, Rajan, and Ray (2008), Indjejikian and Matejka (2009), Liang and Nan (2014), and Ziv (2000) examine efforts to improve information when it is used for contracting. In these studies, information-gathering typically is valuable because it allows the principal to reduce the amount of risk imposed on risk-averse agents by imperfectly-informative performance measures. Similarly, numerous studies have examined investors' incentives to gather private information. In a rational expectations model of privately informed trading, McNichols and Trueman (1994) and Demski and Feltham (1994) establish conditions under which a forthcoming public report creates and incentive for speculators to gather correlated private information by enabling them to unwind their positions when the report is released. On another dimension, Pae (1999) shows how the option to disclose or not disclose private information can lead to over-investment in costly information acquisition.

Our paper contributes in a number of ways. First, we address interesting

and Taylor (2017) find that greater annual report complexity is associated with voluntary disclosure properties. They interpret their results as suggesting a negative relation between exogenous reporting system quality and discretionary disclosure, in contrast to the prior three studies. He, Plumlee and Wen (2018) similarly find that voluntary disclosures are decreasing in the quality of the periodic mandatory reports provided by a firm's accounting system.

questions involving how the properties of a reporting system set by a regulator interact with a firm's incentives to gather private information that it can but does not have to disclose. Second, we advance a growing literature on how mandatory and voluntary disclosures interact. Third, our results suggest that a regulator interested in maximizing the amount of information available to investors may optimally set an imperfectly-informative reporting system, even in the absence of direct costs related to gathering and processing information or other forces (e.g., "real effects" or coordination problems amongst information users).

The next section introduces the setting. Section 3 analyzes the firm's information acquisition and disclosure choices. Section 4 focuses on the regulator's choice of the properties of the reporting system. Section 5 explores variants of the model. Finally, Section 6 concludes.

2 The economic setting

The main players in our setting are a regulator and a firm. The regulator chooses the properties of reports that the firm must provide to investors. The firm, additionally, may acquire information that it can choose whether to disclose to investors. The firm's goal is, broadly, to convince or persuade investors that the firm's value is sufficiently high. The regulator would like investors to have as much information as possible, in expectation, due potentially to an unmodeled decision investors make conditional on their information.

Formally, firm value is a random state of nature ω. Firm value is given by $\omega = \omega_1 + \omega_2$, where $\omega_1 \in \{G, B\}$ and $\omega_2 \in \{H, L\}$. A natural interpretation is that ω_1 represents the value of assets in place, and ω_2 represents the value of growth opportunities. The common prior belief is that $\Pr(\omega_1 = G) = \alpha$ and $\Pr(\omega_2 = H) = \beta$, with $\alpha, \beta \in (0,1)$. We normalize the state variables to $G = H = 1$ and $B = L = 0$. Hence the prior expectation is $E[\omega] = \alpha + \beta \in (0,2)$.

The firm's reporting system generates a public report $r \in \{r_G, r_B\}$ about ω_1. The properties of the reporting system are given by $\boldsymbol{\rho} = (\rho_G, \rho_B)$, where $\rho_G \equiv \Pr(r = r_G | \omega_1 = G)$ and $\rho_B \equiv \Pr(r = r_G | \omega_1 = B)$. The properties of the reporting system are chosen by the regulator. We focus on reporting systems with $1 \geq \rho_G \geq \rho_B \geq 0$. Let $\boldsymbol{\rho}^F \equiv \boldsymbol{\rho} : \{\rho_G = 1, \rho_B = 0\}$ and $\boldsymbol{\rho}^U \equiv \boldsymbol{\rho} : \{\rho_G = \rho_B\}$ denote reporting systems that are fully-informative and uninformative, respectively. We denote an imperfectly informative reporting system as $\boldsymbol{\rho}^I \equiv \boldsymbol{\rho} \notin \{\boldsymbol{\rho}^U, \boldsymbol{\rho}^F\}$ Furthermore, let $\widehat{\alpha}^G(\boldsymbol{\rho}) \equiv E[\omega_1 | r = r_G]$ and $\widehat{\alpha}^B(\boldsymbol{\rho}) \equiv E[\omega_1 | r = r_B]$. In the remainder, we write $\widehat{\alpha}^G$ and $\widehat{\alpha}^B$ with the understanding that these depend on $\boldsymbol{\rho}$. It is straightforward that $0 \leq \widehat{\alpha}^B \leq \alpha \leq \widehat{\alpha}^G \leq 1$.

In addition to the mandated report, with probability $q \in [0,1]$ the firm privately observes a private signal s that perfectly reveals ω_2. With probability $1-q$ the firm does not observe the signal, i.e.,

$$\begin{aligned}
\Pr(s = \varnothing) &= 1 - q, \\
\Pr(s = \omega_2) &= q.
\end{aligned}$$

Aware of the properties of the reporting system set by the regulator, the firm commits to information gathering through its choice of a probability q of becoming informed prior to the realization of the report. We assume that whenever indifferent between a range of probabilities the firm chooses the lowest feasible q (due to some unmodeled infinitesimal cost of information acquisition). For the main part of the analysis we assume that q is publicly observable. Additionally, we assume that the firm either observes ω_2 perfectly or not at all. In Section 5 we relax the observability of q assumption and discuss the implications of the firm observing instead a noisy signal of ω_2.

We maintain that q is not contractible even if it is observable. This implies that the regulator is unable to control or force information gathering by the firm, thereby capturing the essential conceptual difference between regulated and discretionary disclosure. If q were contractible, the regulator could force the firm to set $q = 1$ and disclose its (verifiable) information. Clearly, this would remove the distinction between regulated and voluntary disclosure.

If the firm observes the signal it can either truthfully communicate it by sending a message $m = s$ or withhold it and send an empty or null message, i.e., $m = \varnothing$. If the firm does not observe a signal it has nothing to communicate, i.e., the firm has no choice but to send $m = \varnothing$. The firm cannot credibly communicate that it did not observe a signal.

We assume that the firm's payoff is given by

$$\Pi = \pi(r, m) - C\mathbb{1}_{\pi(r,m)\leq k}$$

where $\pi(r, m) = E[\omega|r, m] = E[\omega_1|r] + E[\omega_2|m]$ is the market price and $C \geq 0$ is the cost that the firm incurs if investor beliefs fall below a threshold $k \in [0, 2]$. There are several interpretations of the threshold. First, it could represent a listing requirement. Firms whose prices fall below the threshold ($1/share for the NYSE and NASDAQ) are delisted. Second, it could represent a credit rating threshold between investment and non-investment grades for the firm's debt.

Third, it could represent index inclusion based on, for instance, market cap. Evidence for costs related to these institutional thresholds are provided by Macey et al. (2008) for delisting, Kisgen (2006) for ratings downgrades, and Klas et al. (2016) for index inclusion.

With credit ratings, our model can be further reinterpreted. Bonsall et al. (2018) and Kraft (2015) provide evidence of credit ratings agencies basing their ratings on financial information provided in annual reports (i.e., r) and subjective adjustments based on additional information (i.e., m). These provide an additional avenue in which to interpret our additively separable form for ω beyond assets-in-place and growth opportunities.

Observation 1 *$\pi(r_G, m) \geq \pi(r_B, m)$, for any $m \in \{H, L, \varnothing\}$. In addition, $\pi(r, m = H) \geq \pi(r, m = \varnothing) \geq \pi(r, m = L)$, for any $r \in \{r_G, r_B\}$.*

In the main part of the analysis we assume that k is exogenously determined. In Section 5 we relax this assumption. The firm's expected payoff at date 1 is:

$$E[\Pi] \quad = \quad E[\omega] - C(1 - \Pr(\pi(r, m) \geq k))$$

and therefore its objective is to maximize the probability that the price, $\pi(r, m)$, exceeds k. The regulator's objective is to maximize the total available information. Formally, he maximizes the variance of posterior expectations $\vartheta(q, \boldsymbol{\rho}) \equiv Var(E[\omega | r, m, \boldsymbol{\rho}, q])$. By the law of total variance, the variance of posterior expectations is the difference between the prior variance and the expected posterior variance. It is therefore a straightforward measure of the amount of uncertainty resolved, with a higher variance of posterior expectations indicating higher overall informativeness. Equivalent measures of information content have been used in prior studies (e.g., Crawford and Sobel, 1982; Michaeli, 2014; Friedman et al., 2016; Faria-e-Castro et al., 2016).

The timeline of events in the main part of the paper is as follows:

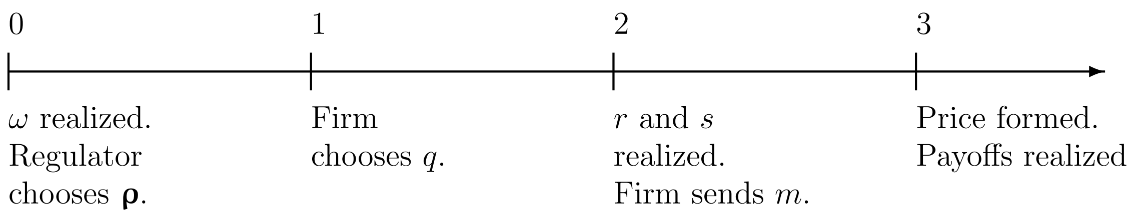

Figure 1: Timeline of events

At date 0 the regulator chooses the properties of the public reprts $\boldsymbol{\rho}$. At date 1, after observing $\boldsymbol{\rho}$, the firm chooses the probability q with which it acquires private information. At date 2 the public report is realized and the firm may receive private information. The firm then decides whether to disclose its information. At date 3 the market prices the firm and payoffs are realized. In what follows, we solve the model via backward induction.

3 Information acquisition and disclosure

Disclosing a high signal increases the posterior beliefs, whereas disclosing a low signal reduces them. Hence, at date 2, the firm discloses $s = H$ and withholds $s = L$. Formally:

Lemma 1 *For any $q \leq 1$, we have $m(s = H) = 1$ and $m(s = L) = m(s = \varnothing) = \varnothing$. For $q = 1$, the firm is indifferent between disclosing and withholding $m(s = L)$.*

By Lemma 1 it is straightforward to see that the probability of disclosure is βq. The binary structure of the firm's private information if obtained implies an invariant disclosure strategy, allowing us to better focus on the firm's exercise of discretion with respect to gathering information.

Corollary 1 *The probability of disclosure is increasing in q, the probability of receipt of private information, and β, the probability of $\omega_2 = H$.*

At date 1, after observing regulator's choice of $\boldsymbol{\rho}$, the firm chooses q to maximize the probability with which it meets the threshold:

$$\check{q}(\boldsymbol{\rho}) \in \arg\max_{q\in[0,1]} \Pr(\pi(r, m) \geq k)$$

Proposition 1 *At date 1, for a given $\boldsymbol{\rho}$, the firm's private information acquisition strategy is:*

$$\check{q}(\boldsymbol{\rho}) = \begin{cases} 0, & \text{if} \quad k \in K_{q=0}(\boldsymbol{\rho}) \\ \widehat{q} \in (0,1), & \text{if} \quad k \in K_{q=\widehat{q}}(\boldsymbol{\rho}) \\ 1, & \text{if} \quad k \in K_{q=1}(\boldsymbol{\rho}) \end{cases}$$

where $\widehat{q} \equiv \frac{\widehat{\alpha}^G+\beta-k}{\beta(\widehat{\alpha}^G+1-k)}$, *and*

(i) $K_{q=0}(\boldsymbol{\rho}) \equiv [0, \widehat{\alpha}^B + \beta] \cup [\widehat{\alpha}^B + 1, \widehat{\alpha}^G + \beta] \cup [\widehat{\alpha}^G + 1, 2]$,

(ii) $K_{q=\widehat{q}}(\boldsymbol{\rho}) \equiv [\widehat{\alpha}^G, \min\{\widehat{\alpha}^G + \beta, \widehat{\alpha}^B + 1, \widehat{\alpha}^G + \frac{\rho_G \alpha}{\widehat{\alpha}^G}\}]$, *and*

(iii) $K_{q=1}(\boldsymbol{\rho}) \equiv [\widehat{\alpha}^B + \beta, \min\{\widehat{\alpha}^B + 1, \alpha + \beta\}] \cup [\alpha + \beta, \widehat{\alpha}^G] \cup [\widehat{\alpha}^G + \frac{\rho_G \alpha}{\widehat{\alpha}^G}, \widehat{\alpha}^B + 1] \cup [\widehat{\alpha}^G + \beta, \widehat{\alpha}^G + 1]$.

In the two cases where the firm chooses $q = 0$, either the threshold is met for sure (e.g., $k = 0$), the threshold can never be met (e.g., $k = 2 > \widehat{\alpha}^G + 1$), or getting a high message will not help overcome the threshold. The firm will always acquire information in cases where the threshold can only be met with a high message. The firm chooses to always gather private information ($q = 1$): (1) if the threshold can be met by a high message following a low report or is always met following a good report irrespective of the message; (2) the threshold can only be met following a good report and a high message; and (3) in some cases where the threshold can be met after both a good report and a null message and a bad report and a high message. Finally, the firm chooses an intermediate probability of observing its private information when it can meet the threshold following a good report and a low message or following a bad report and a high message, and the threshold is not too high. In this region, the firm trades off the higher probability of observing a high message with increasing q against the lower belief following a null message given higher q.

Corollary 2

(i) The firm's optimal probability of observing private information, $\check{q}(\boldsymbol{\rho})$, is non-monotonic in the threshold, k, and the parameters governing the reporting system, $\boldsymbol{\rho}$.

(ii) With $k \in K_{q=\widehat{q}}(\boldsymbol{\rho})$, the firm's optimal probability of observing private information, $\check{q}(\boldsymbol{\rho}) = \widehat{q}$, is increasing in ρ_G and decreasing in ρ_B and k;.

Corollary 3 *Suppose $\boldsymbol{\rho} = \boldsymbol{\rho}^F$. Then, $\lim_{\boldsymbol{\rho}\to\boldsymbol{\rho}^F} K_{q=0}(\boldsymbol{\rho}) = [0, \beta] \cup [1, \alpha + \beta]$, $\lim_{\boldsymbol{\rho}\to\boldsymbol{\rho}^F} K_{q=\widehat{q}}(\boldsymbol{\rho}) = \varnothing$ and $\lim_{\boldsymbol{\rho}\to\boldsymbol{\rho}^F} K_{q=1}(\boldsymbol{\rho}) = [\beta, \min\{1, \alpha + \beta\}] \cup [\alpha + \beta, 1] \cup$*

$[1+\beta, 2]$. *Therefore:*

$$\check{q}(\boldsymbol{\rho}) = \begin{cases} 0, & \textit{if} \quad k \in [0,\beta] \cup [1, \alpha+\beta] \\ 1, & \textit{if} \quad k \in [\beta, \min\{1, \alpha+\beta\}] \cup [\alpha+\beta, 1] \cup [1+\beta, 2] \end{cases}$$

When the public reports are fully-informative, $\widehat{\alpha}^G \to 1$ and $\widehat{\alpha}^B \to 0$ as $\boldsymbol{\rho} \to \boldsymbol{\rho}^F$. This shrinks the region where $\check{q}(\boldsymbol{\rho}) = \widehat{q}$ to nothing, leaving only regions where a high message can help push posterior beliefs over the threshold, implying $\check{q}(\boldsymbol{\rho}) = 1$, and a region where messages fail to help the firm achieve posterior beliefs exceeding the threshold, implying $\check{q}(\boldsymbol{\rho}) = 0$. Consequently, the firm chooses to always acquire private information for high values of k and to never acquire private information for low k.

Corollary 4 *For an uninformative reporting system, we have* $\lim_{\boldsymbol{\rho}\to\boldsymbol{\rho}^U} K_{q=0}(\boldsymbol{\rho}) = [0, \alpha+\beta] \cup [1+\alpha, 2]$, $\lim_{\boldsymbol{\rho}\to\boldsymbol{\rho}^U} K_{q=\widehat{q}}(\boldsymbol{\rho}) = \varnothing$ *and* $\lim_{\boldsymbol{\rho}\to\boldsymbol{\rho}^U} K_{q=1}(\boldsymbol{\rho}) = [\alpha+\beta, 1+\alpha]$. *Therefore:*

$$\check{q}(\boldsymbol{\rho}) = \begin{cases} 0, & \textit{if} \quad k \in [0, \alpha+\beta] \cup [1+\alpha, 2] \\ 1, & \textit{if} \quad k \in [\alpha+\beta, 1+\alpha] \end{cases}$$

As $\boldsymbol{\rho} \to \boldsymbol{\rho}^U$, $\widehat{\alpha}^G \to \alpha$ and $\widehat{\alpha}^B \to \alpha$. As in the case of fully-informative reports, this shrinks the region where $\check{q}(\boldsymbol{\rho}) = \widehat{q}$ to nothing, leaving only regions where a high message can help push posterior beliefs over the threshold, implying $\check{q}(\boldsymbol{\rho}) = 1$, and a region where messages fail to help the firm achieve posterior beliefs exceeding the threshold, implying $\check{q}(\boldsymbol{\rho}) = 0$. As a result, the firm chooses to never acquire and disclose private information for low and high values of k and to always acquire and disclose for intermediate values.

4 Informativeness of public reports

Recall that when choosing the informativeness of the public reports the regulator maximizes the amount of total information as captured by the variance of posterior expectations $\vartheta(q, \boldsymbol{\rho})$.

Lemma 2 *The variance of posterior expectations, $\vartheta(q, \boldsymbol{\rho})$, is increasing in q. For a given q, $\vartheta(q, \boldsymbol{\rho})$ is increasing in ρ_G and decreasing in ρ_B.*

Lemma 2 takes the firm's information acquisition as exogenous. However, requiring more informative reports increases the information about ω_1 and (by Corollary 2) consequently affects the firm's acquisition and disclosure of private information about ω_2. The optimal reporting system therefore trades off information provision about ω_1 with that about ω_2. For simplicity and clarity, we assume ω_1 and ω_2 are equally important for firm value ω.

At date 0 the regulator anticipates the firm's choice of $\check{q}(\boldsymbol{\rho})$ and solves:

$$\begin{aligned}
\max_{\boldsymbol{\rho}} \quad & \vartheta(q, \boldsymbol{\rho}) \\
\text{s.t.} \quad & 1 \geq \rho_G \geq \rho_B \geq 0 \\
& q \in \arg\max_{q} \Pr(\pi(r, m) \geq k)
\end{aligned}$$

Taking into account the firm's choice (Proposition 1), we can restate the regulator's problem as one in which he compares the values of three programs below.

$\mathcal{P}_A^{(R)}$: *Dissuade information acquisition:*

$$\begin{aligned}\max_{\boldsymbol{\rho}} \quad & \vartheta(q=0,\boldsymbol{\rho}) \\ \text{s.t.} \quad & 1 \geq \rho_G \geq \rho_B \geq 0 \\ & k \in K_{q=0}(\boldsymbol{\rho})\end{aligned}$$

$\mathcal{P}_B^{(R)}$: *Induce probabilistic information acquisition:*

$$\begin{aligned}\max_{\boldsymbol{\rho}} \quad & \vartheta(q=\widehat{q},\boldsymbol{\rho}) \\ \text{s.t.} \quad & 1 \geq \rho_G \geq \rho_B \geq 0 \\ & k \in K_{q=\widehat{q}}(\boldsymbol{\rho})\end{aligned}$$

$\mathcal{P}_C^{(R)}$: *Induce information acquisition:*

$$\begin{aligned}\max_{\boldsymbol{\rho}} \quad & \vartheta(q=1,\boldsymbol{\rho}) \\ \text{s.t.} \quad & 1 \geq \rho_G \geq \rho_B \geq 0 \\ & k \in K_{q=1}(\boldsymbol{\rho})\end{aligned}$$

In the Appendix, we decompose each of these further into sub-programs, solve them, and compare expected values to determine the regulator's optimal reporting system given the firm's subsequent choice of information acquisition.

Proposition 2

(i) If $k \in [0,\beta]$, the regulator chooses fully-informative reports ($\boldsymbol{\rho} = \boldsymbol{\rho}^F$) and the firm never acquires information ($q = 0$).

(ii) *If* $k \in [\beta, \max\{\alpha+\beta, 1\}] \cup [1+\beta, 2]$ *the regulator chooses fully-informative reports* ($\boldsymbol{\rho} = \boldsymbol{\rho}^F$) *and the firm always acquires private information* ($q = 1$).

(iii) *If* $k \in [\max\{\alpha+\beta, 1\}, 1+\beta]$, *the regulator chooses imperfectly-informative reports* ($\boldsymbol{\rho} \in \boldsymbol{\rho}^I$) *and the firm always acquires private information* ($q = 1$).

For intermediate and high values of k the regulator's choice of $\boldsymbol{\rho}$ always induces the firm to gather information with probability 1. For high values of k this is possible with a fully-informative reporting system. As a result, perfect information about ω_1 and ω_2 is conveyed implying no informational loss. For intermediate k values, however, inducing $q = 1$ comes at the expense of requiring less than fully-informative public reports about ω_1. This is because with intermediate values of k and a fully-informative reporting system, the firm would choose to gather no private information, as shown in Corollary 3. Reducing the informativeness of the reporting system provides the firm with a benefit from disclosing high messages, thereby encouraging private information acquisition. Hence, for intermediate values of k perfect information about ω_2 is conveyed but there is an information loss regarding ω_1. For extremely low k ($k < \beta$) the regulator cannot induce private information acquisition for any $\boldsymbol{\rho}$. Hence the firm ends up not acquiring and disclosing information about ω_2 but the public reports are fully-informative about ω_1.

5 Model variations

In this section we explore alternative modeling choices. In Section 5.1, we relax our assumption about the observability of the firm's choice of information acquisition, q. In Section 5.2 we consider a setting in which q is chosen after the realization of r. In Section 5.3, we allow the regulator to set k instead of $\boldsymbol{\rho}$. In Section 5.4, we derive the firm's preferred reporting system. Section 5.5 discusses the potential effects of noise in the firm's private signal s.

5.1 Unobservable information acquisition

If the firm's choice of private information acquisition is unobserved by investors then they conjecture the firm's strategy $\check{q}(\boldsymbol{\rho})$. For a given conjecture, investors' interpretation of a non-disclosure ($m = \varnothing$) is fixed. However, the firm would benefit from setting a higher q because this would increase the potential to send a high message ($m = H$). For a given conjecture, then, the firm would optimally set $q = 1$. Therefore, the only rational conjecture for investors to make is $q = 1$. With this as the firm's information acquisition strategy, the regulator no longer needs to worry about the effects of the reporting system on $\check{q}$, and can set a perfectly informative reporting system.

Proposition 3 *When q is unobservable, investors rationally conjecture that the firm chooses $q = 1$. The regulator chooses $\boldsymbol{\rho} = \boldsymbol{\rho}^F$, a fully informative reporting system.*

Proposition 3 highlights the importance of the observability of q. In most prior studies of discretionary disclosure games featuring uncertain receipt of information (e.g., Dye (1985) and Jung and Kwon (1988)), q is given exogenously and assumed to be known by investors. Thus, certain types of firms might be associated with higher or lower q, but this is a firm-level immutable characteristic. In our model, q is a choice variable, which we model as costless to the firm (beyond an infinitesimal cost to motivate $q = 0$ when the firm is indifferent). Empirically, we must look for observable indicators of the firm's choice of q. Such indicators might include the firm's choice of executives or directors – those with a background consistent with an advisory role might increase the firm's likelihood of receiving private information. For example, a biotech firm that places a former FDA executive on its board might be more likely to acquire private information about FDA approval than a firm with a former Secretary of State on the board.[4]

An additional implication of Proposition 3 is that the firm would prefer to be able to make its choice of q observable in many cases. Without observable q, the firm is essentially forced to set $q = 1$, which, by preferences revealed in Proposition 1, is not optimal for the firm with observable q.[5]

To conclude this subsection, we note that the equilibrium in Proposition 3 is sensitive to the assumption that there is no marginal cost to increasing q. Such a cost could push the firm's optimal q down from 1 and would be an equilibrium choice because investors would understand the firm's incentives.[6] With some cost functions (e.g., those with continuous first derivatives), the firm's choice of q would in equilibrium be sensitive to the regulator's choice of $\boldsymbol{\rho}$. This would recover the types of effects on which we focused in the earlier sections and the tenor of the results of Proposition 2.

[4]Theranos, an infamous biotech startup, had two former Secretaries of State on its board.

[5]In July 2013, the SEC required Urban Outfitters to disclose the effect of direct-to-customer sales on the net retail segment sales. In response, Urban Outfitters declared that starting the first quarter of 2014 the company will no longer collect this information.

[6]As a stark example, consider a setting with no cost of setting $q \in [0, 0.7]$, and an infinite cost of setting $q > 0.7$. Here, $q = 0.7$ would be an equilibrium choice.

5.2 Acquisition of private information after the realization of public reports

In this section we consider an alternative timeline in which the firm chooses to acquire information after the public financial report is released. Following a similar logic to the one employed in the proof of Corollary 4, after observing r_i, $i \in \{B, G\}$, the firm's strategy is

$$\check{q}(r_i, \boldsymbol{\rho}) = \begin{cases} 0, & \text{if} \quad k \in [0, \widehat{\alpha}^i + \beta] \cup [1 + \widehat{\alpha}^i, 2] \\ 1, & \text{if} \quad k \in [\widehat{\alpha}^i + \beta, 1 + \widehat{\alpha}^i] \end{cases} \qquad i \in \{B, G\}.$$

Combining the strategy for $i = B$ with that for $i = G$, it is immediate that if $k < \widehat{\alpha}^B + \beta$ or $k > 1 + \widehat{\alpha}^G$ the firm never acquires information for any r. If $k \in [\widehat{\alpha}^G + \beta, 1 + \widehat{\alpha}^B]$ the firm acquires information for any r. Furthermore, if $k \in [\widehat{\alpha}^B + \beta, \widehat{\alpha}^G + \beta]$ the firm acquires information only after r_B, whereas if $k \in [\widehat{\alpha}^B + 1, \widehat{\alpha}^G + 1]$ it does so only after r_G. Depending on the properties of the reporting system, some of these regions may or may not exist. For example, if the reporting system is fully informative, we have $[\widehat{\alpha}^G + \beta, 1 + \widehat{\alpha}^B] = [1 + \alpha, 1]$ and so the firm will never acquire information for any report. Therefore, when choosing the informativeness of the financial reports about ω_1 the regulator trades off reporting information about ω_1 with encouraging acquision and disclosure of information about about ω_2.

Proposition 4 *If $k \leq \beta$ the regulator chooses $\boldsymbol{\rho} = \boldsymbol{\rho}^F$. Otherwise, the regulator chooses $\boldsymbol{\rho} \in \boldsymbol{\rho}^I$. The firm:*

(i) never gathers information if $k \leq \beta$;

(ii) gathers information only after $r = r_B$ *if* $k \in [\beta, \alpha + \beta]$*;*

(iii) always gathers information if $k \in [\alpha + \beta, \max\{1 + \beta, 1 + \alpha\}]$*;*

(iv) gathers information only after $r = r_G$ *if* $k > \max\{1 + \beta, 1 + \alpha\}$*.*

If the threshold is very low (below β) the firm meets the threshold with the prior on ω_2 (i.e., for any r and $\boldsymbol{\rho}$) and therefore no reporting system can induce $q > 0$. Hence, the regulator simply maximizes the information about ω_1 by choosing fully informative reports. For slightly larger k the regulator still cannot induce the firm to gather information after r_G, as a good report is sufficient to meet the threshold. However, in order to have a chance of meeting the threshold after r_B, the firm has to gather information. To maximize the total available information the regulator sets a reporting system that raises the probability of $r = r_B$ thereby making a good report less informative (i.e., by setting $\rho_G < 1$ and $\rho_B = 0$). As k increases further, the firm can no longer meet the threshold with a good report alone – it needs to disclose a high signal as well. Hence, it always acquires information for any r. To make this feasible the regulator chooses $\boldsymbol{\rho} \in \boldsymbol{\rho}^I$.[7] For sufficiently high k disclosure of high signals is no longer sufficient to meet the threshold following $r = r_B$. Therefore, the firm acquires information only after r_G. The regulator sets the reporting system to produce r_G more frequently by setting $\rho_B > 0$ and $\rho_G = 1$. With higher k, good reports need to be more informative (i.e., higher $\hat{\alpha}^G$) for the firm to continue to meet the threshold following r_G and a high message. As $k \to 2$ the threshold is only met with a fully informative good report and a high message.

Intuitively, when irrespective of the report realization the threshold can be

[7]Recall from the preceding discussion that if $\boldsymbol{\rho} = \boldsymbol{\rho}^F$ this is not feasible.

met without the firm acquiring information, or when a good report from a fully informative system and a high message are jointly necessary to meet the threshold, there is no advantage to the regulator in deviating from a fully informative reporting system. Otherwise, the regulator chooses a less informative system in order to induce the firm to acquire information as often as possible. This may be accomplished by making reports less informative such that reports that induce information acquisition are generated more frequently.

5.3 Endogenous choice of threshold

In this section we consider a scenario in which the regulator chooses k instead of $\boldsymbol{\rho}$. This could represent a credit rating agency's choice of ratings cutoffs, assuming that the rating agency benefits from investors having as much information as possible about the underlying firm. The regulator's problem now is

$$\begin{aligned} \max_{k} \quad & \vartheta(\boldsymbol{\rho}, q) \\ \text{s.t.} \quad & 2 \geq k \geq \alpha + \beta \\ & q \in \arg\max_{q} \Pr(\pi(r, m) \geq k) \end{aligned}$$

Since the objective function is independent of k but increasing in q, the result below is immediate.

Proposition 5 *Suppose $\boldsymbol{\rho}$ is given and the regulator chooses k. Then, the regulator chooses any $k \in K_{q=1}(\boldsymbol{\rho})$ as defined in Proposition 1 and the firm always acquires information ($q = 1$).*

In this case inducing the firm to acquire and disclose private information about ω_2 does not come at the expense of information about ω_1. We show that inducing information acquisition requires choosing an interior threshold when $\boldsymbol{\rho} \neq \boldsymbol{\rho}^F$. By doing so, i.e., imposing a delisting or ratings-change threat to firms, the regulator is able to increase the amount of information available in the public domain. Alternatively, if we label the regulator as a credit ratings agency choosing the threshold for an investment-grade rating, this result implies that the ratings agency has leeway in terms of choices of thresholds that would induce the firm to acquire and provide information on which to base discretionary or subjective ratings adjustments (as in Bonsall et al. (2018) and Kraft (2015)).

5.4 Firm's optimal reporting system

In this subsection we consider the reporting system that would be optimal from the firm's perspective. This variant of the model is informative about how a "captured" regulator, i.e., a regulator whose objectives align with the regulated parties, would set the public reporting system.

Proposition 6 *Suppose the firm could choose both q and $\boldsymbol{\rho}$. Let $\widehat{k} \equiv \min\{\frac{\alpha}{\beta} + \beta, 1+\beta, 1+\alpha\}$.*

(i) If $k \in [0, \alpha+\beta]$ the firm chooses $\boldsymbol{\rho} \in \boldsymbol{\rho}^U$ and $q = 0$.

(ii) If $k \in [\min\{1, \alpha+\beta\}, 1)$ the firm chooses $\boldsymbol{\rho} \in \boldsymbol{\rho}^I$ and $q = 1$.

(iii) If $k \in [\max\{1, \alpha+\beta\}, \widehat{k})$ the firm chooses $\boldsymbol{\rho} \in \boldsymbol{\rho}^I$ and $q = 0$.

(iv) If $k \in [\max\{1, \widehat{k}\}, 1+\alpha)$ the firm chooses $\boldsymbol{\rho} \in \boldsymbol{\rho}^U$ and $q = 1$.

(v) If $k \in [1+\alpha, 2)$ the firm chooses $\boldsymbol{\rho} \in \boldsymbol{\rho}^I$ and $q = 1$.

(vi) If $k = 2$ the firm chooses $\boldsymbol{\rho} = \boldsymbol{\rho}^F$ and $q = 1$.

When the threshold is very low ($k \leq \alpha+\beta$) the firm can meet it without disseminating any information. In fact, disseminating information can only make the firm worse off because a bad report and/or acquisition of negative private information can lower the posterior beliefs below k. Hence the firm rationally chooses an uninformative reporting system and not to acquire private information. As k increases beyond the prior, the firm can no longer meet the threshold without disseminating information. The firm now finds it optimal to always acquire private information and provide imperfectly-informative reports that allow it to meet the threshold with either a good report or a high message. As k increases even further, meeting the threshold becomes possible only with a good report (for any message). Consequently, to maximize the probability of meeting k, the firm chooses to never acquire information but provide imperfectly-informative reports. At some high level of k it is no longer feasible to meet the threshold with good reports. If meeting the threshold is still feasible (depending on the magnitude of α and β) with high messages the firm switches to $q = 1$ and chooses uninformative public reports. For very high levels of k, meeting the threshold is only feasible with both a good report and a high message. Hence the firm always acquires information and provides imperfectly-informative reports. As long as $k < 2$, these reports are never fully-informative.

As shown in Figure 2, the preferences of a (non-captured) regulator and a firm over informativeness of public reports are potentially aligned only for intermediate values of k. In this region, they both prefer imperfectly-informative reports. This aligned preference however is driven by different incentives. The regulator seeks to induce the firm to acquire information with probability 1. The firm in contrast maximizes the probability of meeting the threshold. For low and high values of k the preferences of firms and regulators are misaligned, with the firm typically preferring a less informative reporting system. The following proposition describes the types of errors that may occur when the reporting system is set by the regulator or the firm.

Proposition 7 *The reporting system set by the firm potentially produces good reports in bad states and never produces bad reports in good states, i.e.,* $\Pr[r_G|\omega_1 = 0] > 0 = \Pr[r_B|\omega_1 = 1]$. *The reporting system set by the regulator is more informative, but may produce errors in either state, i.e.,* $\Pr[r_G|\omega_1 = 0], \Pr[r_B|\omega_1 = 1] > 0$.

Although the reporting system set by the regulator is more informative (by revealed preference given the regulator's objective function), the regulator may prefer to set a reporting system that produces errors in both underlying states of the world ($\omega_2 \in \{G, B\}$). This leads to incorrect good and bad reports. In contrast, the firm's objective is to meet the threshold as often as possible, giving it an incentive to set a reporting system that allows good reports to be produced when the state is bad. This makes good reports less informative, but causes them to be generated more often. The regulator introduces errors into the reporting system instead to provide incentives for the firm to acquire information about ω_2, i.e., growth opportunities. The regulator wants to provide as much information as possible, and in many cases providing the firm with incentives to gather additional information can come more cheaply from making the reporting system more conservative (lowering ρ_G) than more liberal (increasing ρ_B).

It is instructive to compare the findings in this section with those in Friedman et al. (2017), who find that the firm always chooses imperfectly informative reports and prefers to not receive private information. In contrast, the firm

	Preference of the regulator	Preference of the firm
Low threshold	***Fully-informative*** public reports	***Uninformative*** public reports
Intermediate threshold	***Imperfectly-informative*** public reports	***Imperfectly-informative or uninformative*** public reports
High threshold	***Fully-informative*** public reports	***Imperfectly-informative*** public reports

Figure 2: Comparison of the firm's and the regulator's preferences

here may choose to always acquire private information and set uninformative or imperfectly-informative reports. The reason for the difference in predictions is because public financial reports and messages regarding private information that the firm may have acquired pertain to different components of future payoffs whereas in Friedman et al. (2017) reports and messages pertain to the same component. With two components of firm value, the potential receipt of private information does not create uncertainty in the investor's beliefs about ω_1 when the reporting system is set. Hence, acquiring information about ω_2 may in certain conditions facilitate meeting the threshold without affecting beliefs about ω_1.

5.5 Private signal noise and dishonest messages

Noise in the firm's private signal, s, would tend to make the signal less informative about the underlying state, ω_2. For example, suppose that $\Pr[s = \omega_2] = \delta \in (0.5, 1)$, and $\Pr[s = 1|\omega_2 = 0] = \Pr[s = 0|\omega_2 = 1] = 1 - \delta$. The main analysis takes $\delta = 1$. Setting $\delta < 1$ pushes the conditional expectations away from 0 and 1, as $E[\omega_2|s = 1] = \frac{\delta\beta}{\delta\beta+(1-\delta)(1-\beta)} < 1$ and $E[\omega_2|s = 0] = \frac{(1-\delta)\beta}{(1-\delta)\beta+\delta(1-\beta)} > 0$ for $\delta \in (0.5, 1)$. This has the effect of decreasing $E[\omega_2|m = H]$ and increasing

$E[\omega_2|m = \varnothing]$. While this would change the quantitative features of the results (i.e., the specific equilibrium characterizations as functions of parameters), it would maintain the important features that $\Pr[m = H]$ is increasing in q and $E[\omega_2|m = \varnothing]$ is decreasing in q. Thus, the qualitative features of the equilibrium would be maintained. In words, noise in the firm's private information makes disclosing high messages worse in that they increase beliefs about the firm's growth opportunities less than they would if the firm's private signal were noiseless. Additionally, noise allows investors to have less negative beliefs following non-disclosure. However, firms still benefit from disclosing high messages and bear consequences from non-disclosure.

A further extension to consider involves the prospect for managers to provide dishonest messages. Specifically, there may be some probability that a firm, after observing a low signal, could send a high message. Let $\Pr[m = 1|s = 0] = \lambda$. Like symmetric noise, higher λ would decrease investor's beliefs about ω_2 following $m = 1$. Additionally, because some negative signals are transformed into positive messages, nondisclosures are more likely to result from a lack of information, which means that higher λ would also allow investors to revise their beliefs about ω_2 less negatively. Overall, probabilistic bias as modeled by λ would then have qualitatively similar effects as symmetric noise – changing the functional form but not the nature of the results described in Propositions 1 and 2.

6 Conclusion

We analyze a model in which a regulator sets a reporting system in anticipation of a firm acquiring disclosable information. We show how the properties of the reporting system influence the firm's information acquisition strategy and how the anticipation of information acquisition influences the regulator's design of the public reporting system. The firm's information-acquisition strategy behaves non-monotonically in the properties of the public reporting system, suggesting potential problems in using discretionary disclosure frequency to make inferences about the informational properties of public reporting systems when information acquisition is endogenous. We show that regulators interested in maximizing the total amount of information available to investors may find it optimal to set an imperfectly-informative reporting system, even in the absence of direct costs of doing so. This suggests that maximizing the informativeness of public reports (i.e., requiring fully-informative reports) may lead to less information available to market participants when firms' information acquisition is endogenous.[8]

Our study contributes to the literature on the intersection between regulated reporting and discretionary disclosure. Our focus is on the interplay between the design of a public reporting system by a regulator and a firm's information acquisition strategy. For parsimony, we have abstracted away from several real-world features, such as costs associated with enforcement of reporting standards, compliance costs, costs of information acquisition, and "real effects" of information (e.g., using private information for internal decision-making). Incorporating these, dynamic interactions between the firm and regulator, or multiple firms, could lead to interesting additional insights. We leave this for future work.

[8]From a regulatory perspective, our results are instructive as long as there is a measurable set of firms that face the trade-offs we model in a single-firm setting.

Appendix

Supporting calculations:

$$\begin{aligned}
\Pr(r = r_G) &= \Pr(r = r_G|\omega_1 = G)\Pr(\omega_1 = G) + \Pr(r = r_G|\omega_1 = B)\Pr(\omega_1 = B) \\
&= \rho_G\alpha + \rho_B(1-\alpha) \\
\Pr(r = r_B) &= \Pr(r = r_B|\omega_1 = G)\Pr(\omega_1 = G) + \Pr(r = r_B|\omega_1 = B)\Pr(\omega_1 = B) \\
&= (1-\rho_G)\alpha + (1-\rho_B)(1-\alpha)
\end{aligned}$$

The posterior beliefs about ω_1 are:

$$\begin{aligned}
\widehat{\alpha}^G &\equiv E[\omega_1|r = r_G] \\
&= 1 \times \frac{\Pr(r = r_G|\omega_1 = G)\Pr(\omega_1 = G)}{\Pr(r = r_G)} + 0 \times \frac{\Pr(r = r_G|\omega_1 = B)\Pr(\omega_1 = B)}{\Pr(r = r_G)} \\
&= \frac{\rho_G\alpha}{\rho_G\alpha + \rho_B(1-\alpha)}
\end{aligned}$$

and

$$\begin{aligned}
\widehat{\alpha}^B &\equiv E[\omega_1|r = r_B] \\
&= 1 \times \frac{\Pr(r = r_B|\omega_1 = G)\Pr(\omega_1 = G)}{\Pr(r = r_B)} + 0 \times \frac{\Pr(r = r_B|\omega_1 = B)\Pr(\omega_1 = B)}{\Pr(r = r_B)} \\
&= \frac{(1-\rho_G)\alpha}{(1-\rho_G)\alpha + (1-\rho_B)(1-\alpha)}.
\end{aligned}$$

The probabilities are:

$$\begin{aligned}
\Pr(m = H) &= \Pr(s = \omega_2)\Pr(\omega_2 = H) = q\beta \\
\Pr(m = \varnothing) &= \Pr(m = \varnothing|\omega_2 = H)\Pr(\omega_2 = H) + \Pr(m = \varnothing|\omega_2 = L)\Pr(\omega_2 = L) = 1 - q\beta
\end{aligned}$$

The posterior beliefs about ω_2 are:

$$\begin{aligned}
E[\omega_2|m = H] &= 1 \\
E[\omega_2|m = \varnothing] &= 1 \times \frac{\Pr(m = \varnothing|\omega_2 = H)\Pr(\omega_2 = H)}{\Pr(m = \varnothing)} + 0 \times \frac{\Pr(m = \varnothing|\omega_2 = L)\Pr(\omega_2 = L)}{\Pr(m = \varnothing)} \\
&= \frac{(1-q)\beta}{(1-q)\beta + (1-q)(1-\beta) + q(1-\beta)} \\
&= \frac{(1-q)\beta}{1 - q\beta}
\end{aligned}$$

Hence:

$$\begin{aligned}
\pi(r_G, H) &= E[\omega_1|r = r_G] + E[\omega_2|m = H] = \widehat{\alpha}^G + 1. \\
\pi(r_B, H) &= E[\omega_1|r = r_B] + E[\omega_2|m = H] = \widehat{\alpha}^B + 1.
\end{aligned}$$

$$\begin{aligned}\pi(r_G,\varnothing) &= E[\omega_1|r=r_G]+E[\omega_2|m=\varnothing]=\widehat{\alpha}^G+\frac{(1-q)\beta}{1-q\beta}.\\ \pi(r_B,\varnothing) &= E[\omega_1|r=r_B]+E[\omega_2|m=\varnothing]=\widehat{\alpha}^B+\frac{(1-q)\beta}{1-q\beta}.\end{aligned}$$

Proof of Proposition 1: Case A: High threshold $k \geq \alpha+\beta$

The firm's problem boils down to comparing the values of the following programs:

$\mathcal{P}_1^{(F)}$ *Never meet the threshold:*

$$\begin{aligned}&\max_{q\in[0,1]} 0\\ &\text{s.t.}\quad \widehat{\alpha}^G+1\leq k\end{aligned}$$

$\mathcal{P}_2^{(F)}$ *Meet threshold with ($r=r_G$, $m=H$):*

$$\begin{aligned}&\max_{q\in[0,1]} q\beta(\rho_G\alpha+\rho_B(1-\alpha))\\ &\text{s.t.}\quad \widehat{\alpha}^G+1\geq k\\ &\qquad \widehat{\alpha}^G+\frac{(1-q)\beta}{1-q\beta}\leq k\\ &\qquad \widehat{\alpha}^B+1\leq k\end{aligned}$$

$\mathcal{P}_3^{(F)}$ *Meet threshold with ($r=r_G$, $m=H$) and ($r=r_G$, $m=\varnothing$):*

$$\begin{aligned}&\max_{q\in[0,1]} \rho_G\alpha+\rho_B(1-\alpha)\\ &\text{s.t.}\quad \widehat{\alpha}^G+\frac{(1-q)\beta}{1-q\beta}\geq k\\ &\qquad \widehat{\alpha}^B+1\leq k\end{aligned}$$

$\mathcal{P}_4^{(F)}$ *Meet threshold with ($r=r_G$, $m=H$) and ($r=r_B$, $m=H$):*

$$\begin{aligned}&\max_{q\in[0,1]} q\beta\\ &\text{s.t.}\quad \widehat{\alpha}^G+\frac{(1-q)\beta}{1-q\beta}\leq k\\ &\qquad \widehat{\alpha}^B+1\geq k\end{aligned}$$

$\mathcal{P}_5^{(F)}$ *Meet threshold with ($r=r_G$, $m=H$) and ($r=r_B$, $m=H$) and ($r=r_G$,*

$m = \varnothing$):

$$\max_{q\in[0,1]} \rho_G\alpha + \rho_B(1-\alpha) + q\beta((1-\rho_G)\alpha + (1-\rho_B)(1-\alpha))$$
$$\text{s.t.} \quad \widehat{\alpha}^G + \frac{(1-q)\beta}{1-q\beta} \geq k$$
$$\widehat{\alpha}^B + 1 \geq k$$

First, consider $\mathcal{P}_1^{(F)}$. This program is only feasible when $\widehat{\alpha}^G + 1 - k < 0$ is satisfied. The optimal solution is $q = 0$ and the value of the program is $V = 0$.

Next, consider $\mathcal{P}_2^{(F)}$. The first and third constraints are independent of q and can be restated as $\widehat{\alpha}^B + 1 \leq k \leq \widehat{\alpha}^G + 1$. Program $\mathcal{P}_2^{(F)}$ is only feasible when the condition above holds. The LHS of the second constraint is decreasing in q, i.e., high q relaxes the constraint. Let

$$R_G(q) \equiv \widehat{\alpha}^G + \frac{(1-\widehat{q})\beta}{1-\widehat{q}\beta} - k$$

and $\widehat{q}$ be the probability that satisfies the equality $R_G(\widehat{q}) = 0$, or,

$$\widehat{q} \equiv R_G^{-1}(q) = \frac{\widehat{\alpha}^G + \beta - k}{\beta\,(\widehat{\alpha}^G + 1 - k)}.$$

Then, the second constraint can be restated as $q > \widehat{q}$. Note that $\widehat{q} < 1$ because $R_G'(q) \leq 0$ and $\lim_{q\to 1} R_G(q) = \widehat{\alpha}^G - k \leq 1 - k \leq \widehat{\alpha}^B + 1 - k < 0$ by the third constraint. It follows that $q > \widehat{q}$ is not an empty set. Moreover, $\widehat{q} > 0$ holds if $\lim_{q\to 0} R_G(q) = \widehat{\alpha}^G + \beta - k > 0$. On the other hand, if $\lim_{q\to 0} R_G(q) = \widehat{\alpha}^G + \beta - k < 0$ it follows that $\widehat{q} < 0$ and so the second constraint $q > \widehat{q}$ holds for any $q \in [0,1]$. The objective function is increasing in q. Summarizing, the solution is $q = 1$ and the value is $V = \beta\frac{\rho_G\alpha}{\widehat{\alpha}^G}$.

Now consider $\mathcal{P}_3^{(F)}$. The second constraint is independent of q and sets the condition $\widehat{\alpha}^B + 1 \leq k$ for Program $\mathcal{P}_3^{(F)}$ to be feasible. Using the solution of Program $\mathcal{P}_2^{(F)}$, we can restate the first constraint as $q < \widehat{q}$. As before, $\widehat{q} < 1$ because $R_G'(q) \leq 0$ and $\lim_{q\to 1} R_G(q) = \widehat{\alpha}^G - k \leq 1 - k \leq \widehat{\alpha}^B + 1 - k < 0$ by the second constraint. Moreover, $\widehat{q} > 0$ holds if $\lim_{q\to 0} R_G(q) = \widehat{\alpha}^G + \beta - k > 0$. On the other hand, if $\lim_{q\to 0} R_G(q) = \widehat{\alpha}^G + \beta - k < 0$ it follows that $\widehat{q} < 0$ and so the first constraint $q < \widehat{q}$ cannot be satisfied for any $q \in [0,1]$ and therefore $\mathcal{P}_3^{(F)}$ is not feasible. Lastly, the objective function is independent of q. Summarizing: for $\widehat{\alpha}^G + \beta - k < 0$, the program is not feasible; for $\widehat{\alpha}^G + \beta - k > 0$, $q = 0$ and its value is $V = \frac{\rho_G\alpha}{\widehat{\alpha}^G}$.

Now consider $\mathcal{P}_4^{(F)}$. The second constraint is independent of q and sets the condition $\widehat{\alpha}^B + 1 \geq k$ for Program $\mathcal{P}_4^{(F)}$ to be feasible. Using the solution of Program $\mathcal{P}_2^{(F)}$, we can restate the first constraint as $q > \widehat{q}$. Furthermore, $\widehat{q} < 1$ if $\lim_{q\to 1} R_G(q) = \widehat{\alpha}^G - k < 0$. Otherwise the constraint $q > \widehat{q}$ cannot be satisfied for any $q \in [0,1]$ and therefore $\mathcal{P}_4^{(F)}$ is not feasible. Moreover, $\widehat{q} > 0$ holds if

$\lim_{q\to 0} R_G(q) = \widehat{\alpha}^G + \beta - k > 0$. On the other hand, if $\lim_{q\to 0} R_G(q) = \widehat{\alpha}^G + \beta - k < 0$ it follows that $\widehat{q} < 0$ and so the first constraint $q > \widehat{q}$ is always satisfied. Summarizing: for $\widehat{\alpha}^G - k > 0$, the program is not feasible; for $\widehat{\alpha}^G - k < 0$: $q = 1$ and $V = \beta$.

Now consider $\mathcal{P}_5^{(F)}$. The second constraint is independent of q and sets the condition $\widehat{\alpha}^B + 1 \geq k$ for Program $\mathcal{P}_5^{(F)}$ to be feasible. Using the solution of Program $\mathcal{P}_2^{(F)}$, we can restate the first constraint as $q < \widehat{q}$. Furthermore, $\widehat{q} < 1$ if $\lim_{q\to 1} R_G(q) = \widehat{\alpha}^G - k < 0$. Otherwise, the constraint $q < \widehat{q}$ is always satisfied for any $q \in [0, 1]$. Moreover, $\widehat{q} > 0$ holds if $\lim_{q\to 0} R_G(q) = \widehat{\alpha}^G + \beta - k > 0$. Otherwise, $\mathcal{P}_5^{(F)}$ is not feasible. The objective function is increasing in q. Summarizing: For $\widehat{\alpha}^G - k > 0$: $q = 1$ and $V = \frac{\rho_G\alpha}{\widehat{\alpha}^G} + \beta\left(1 - \frac{\rho_G\alpha}{\widehat{\alpha}^G}\right)$. For $\widehat{\alpha}^G - k < 0$: (i) when $\widehat{\alpha}^G + \beta - k > 0$: $q = \frac{\widehat{\alpha}^G + \beta - k}{\beta(\widehat{\alpha}^G + 1 - k)}$ and $V = \frac{\rho_G\alpha}{\widehat{\alpha}^G} + \left(\frac{\widehat{\alpha}^G + \beta - k}{\widehat{\alpha}^G + 1 - k}\right)\left(1 - \frac{\rho_G\alpha}{\widehat{\alpha}^G}\right)$. (ii) when $\widehat{\alpha}^G + \beta - k < 0$ the program is not feasible.

Comparison results for $k \geq \alpha + \beta$:

A. Case $k > \widehat{\alpha}^G + 1$: Only program $\mathcal{P}_1^{(F)}$ is feasible: The optimal solution is $q = 0$ (and $V = 0$).

B. Case $k < \widehat{\alpha}^G + 1$ and $k > \widehat{\alpha}^B + 1$: Comparing the value of program $\mathcal{P}_2^{(F)}$ with that of $\mathcal{P}_3^{(F)}$:

1. For $\widehat{\alpha}^G + \beta \leq k$: program $\mathcal{P}_3^{(F)}$ is not feasible. Hence the firm chooses $q = 1$ (and so $V = \frac{\rho_G\alpha}{\widehat{\alpha}^G}\beta$).

2. For $\widehat{\alpha}^G + \beta > k$: the solution of $\mathcal{P}_3^{(F)}$ is $q_2 = 0$ while the solution of $\mathcal{P}_2^{(F)}$ is $q_1 = 1$. It is straightforward to see that the difference between their values is $\frac{\rho_G\alpha}{\widehat{\alpha}^G}(1 - \beta) \geq 0$. Hence, the firm chooses $q = 0$ (and $V = \frac{\rho_G\alpha}{\widehat{\alpha}^G}$).

C. Case $k < \widehat{\alpha}^G + 1$ and $k < \widehat{\alpha}^B + 1$: Comparing the value of program $\mathcal{P}_4^{(F)}$ with that of $\mathcal{P}_5^{(F)}$:

1. For $\widehat{\alpha}^G > k$: program $\mathcal{P}_4^{(F)}$ is not feasible. Hence the firm chooses $q = 1$ (and so $V = \frac{\rho_G\alpha}{\widehat{\alpha}^G} + \beta\left(1 - \frac{\rho_G\alpha}{\widehat{\alpha}^G}\right)$).

2. For $\widehat{\alpha}^G \leq k$:

   a. if $\widehat{\alpha}^G + \beta \leq k$: program $\mathcal{P}_5^{(F)}$ is not feasible. Hence the firm chooses $q = 1$ (and so $V = \beta$).

   b. if $\widehat{\alpha}^G + \beta > k$: The differential between the value of $\mathcal{P}_5^{(F)}$ and $\mathcal{P}_4^{(F)}$ is:

$$(1 - \widehat{q}\beta)\frac{\rho_G\alpha}{\widehat{\alpha}^G} - (1 - \widehat{q})\beta \propto \frac{(1 - \widehat{q}\beta)}{(1 - \widehat{q})}\frac{\rho_G\alpha}{\widehat{\alpha}^G} - \beta \equiv \Delta V$$

   Using the fact that $R_G(\widehat{q}) = 0$ and so $\frac{(1-\widehat{q}\beta)}{(1-\widehat{q})} = \frac{\beta}{k - \widehat{\alpha}^G}$, and noting that

$k - \widehat{\alpha}^G \in (0, \beta)$, the value differential is:

$$\Delta V = \frac{(1-\widehat{q}\beta)}{(1-\widehat{q})} \frac{\rho_G \alpha}{\widehat{\alpha}^G} - \beta = \beta \frac{\rho_G \alpha}{\widehat{\alpha}^G (k - \widehat{\alpha}^G)} - \beta \propto \frac{\rho_G \alpha}{\widehat{\alpha}^G} + \widehat{\alpha}^G - k.$$

If $k < \frac{\rho_G \alpha}{\widehat{\alpha}^G} + \widehat{\alpha}^G$ then $q = \frac{\widehat{\alpha}^G + \beta - k}{\beta(\widehat{\alpha}^G + 1 - k)}$ (and so $V = \frac{\rho_G \alpha}{\widehat{\alpha}^G} + \left(\frac{\widehat{\alpha}^G + \beta - k}{\widehat{\alpha}^G + 1 - k}\right)\left(1 - \frac{\rho_G \alpha}{\widehat{\alpha}^G}\right)$).
If $k > \frac{\rho_G \alpha}{\widehat{\alpha}^G} + \widehat{\alpha}^G$ then $q = 1$ (and so $V = \beta$).

Case B: Low threshold $k \leq \alpha + \beta$: To separate the cases we now denote the programs with a superscript (F, L) instead of (F). The main difference is that now program $\mathcal{P}_1^{(F)}$ is substituted by:

$\mathcal{P}_1^{(F,L)}$ *Always meet the threshold:*

$$\max_{q \in [0,1]} 1$$
$$\widehat{\alpha}^B + \frac{(1-q)\beta}{1-q\beta} \geq k$$

The rest of the programs remain the same. First, consider $\mathcal{P}_1^{(F,L)}$. The objective function is independent of q, whereas the RHS of the constraint is decreasing in q and so high q tightens the constraint. As $q \to 0$, we get $\lim_{q \to 0} \widehat{\alpha}^B + \frac{(1-q)\beta}{1-q\beta} = \widehat{\alpha}^B + \beta$. The firm chooses $q = 0$, the value is $V = 1$ and the program is feasible if $k \leq \widehat{\alpha}^B + \beta$.

Next, consider $\mathcal{P}_2^{(F,L)}$. The second and third constraints are independent of q and can be restated as $\widehat{\alpha}^B + 1 \leq k \leq \alpha + \beta$. Program $\mathcal{P}_2^{(F,L)}$ is only feasible when the condition above holds. As before, the second constraint can be restated as $q \geq \widehat{q}$. Note that $\widehat{q} \leq 1$ because $R'_G(q) \leq 0$ and $\lim_{q \to 1} R_G(q) = \widehat{\alpha}^G - k \leq 1 - k \leq \widehat{\alpha}^B + 1 - k \leq 0$ by the second constraint. It follows that $q \geq \widehat{q}$ is not an empty set. Moreover, $\widehat{q} \geq 0$ holds because $\lim_{q \to 0} R_G(q) = \widehat{\alpha}^G + \beta - k \geq \alpha + \beta - k \geq 0$. The objective function is increasing in q. Summarizing, the solution is $q = 1$ and the value is $V = \beta \frac{\rho_G \alpha}{\widehat{\alpha}^G}$. The program is feasible if $k \in [\widehat{\alpha}^B + 1, \alpha + \beta]$.

Now consider $\mathcal{P}_3^{(F,L)}$. The second and third constraints are independent of q and set the condition $k \in [\widehat{\alpha}^B + 1, \alpha + \beta]$ for Program $\mathcal{P}_3^{(F,L)}$ to be feasible. Using the solution of Program $\mathcal{P}_2^{(F,L)}$, we can restate the first constraint as $q \leq \widehat{q}$. As before, $\widehat{q} \leq 1$ because $R'_G(q) \leq 0$ and $\lim_{q \to 1} R_G(q) = \widehat{\alpha}^G - k \leq 1 - k \leq \widehat{\alpha}^B + 1 - k \leq 0$ by the second constraint. Moreover, $\widehat{q} \geq 0$ holds because $\lim_{q \to 0} R_G(q) = \widehat{\alpha}^G + \beta - k \geq \alpha + \beta - k \geq 0$. Lastly, the objective function is independent of q. Hence: $q = 0$, $V = \frac{\rho_G \alpha}{\widehat{\alpha}^G}$ and the program is feasible if $k \in [\widehat{\alpha}^B + 1, \alpha + \beta]$.

Now consider $\mathcal{P}_4^{(F,L)}$. The second and third constraints are independent of q and set the condition $k \leq \min\{\widehat{\alpha}^B + 1, \alpha + \beta\}$ for Program $\mathcal{P}_4^{(F,L)}$ to be feasible. Using the solution of Program $\mathcal{P}_2^{(F,L)}$, we can restate the first constraint as $q \geq \widehat{q}$. Furthermore, $\widehat{q} \leq 1$ if $\lim_{q \to 1} R_G(q) = \widehat{\alpha}^G - k \leq 0$. Otherwise the constraint $q \geq \widehat{q}$ cannot be satisfied for any $q \in [0, 1]$ and therefore $\mathcal{P}_4^{(F,L)}$ is not feasible. Moreover, $\widehat{q} \geq 0$ holds because

$\lim_{q\to 0} R_G(q) = \widehat{\alpha}^G + \beta - k \geq \alpha + \beta \geq 0$.

Noting that the objective function is increasing in q, we can summarize: For $k \leq \min\{\widehat{\alpha}^G, \alpha+\beta\}$, the program is not feasible. For $k \geq \widehat{\alpha}^G$ but $k \leq \min\{\widehat{\alpha}^B + 1, \alpha+\beta\}$: $q = 1$ and $V = \beta$.

Now consider $\mathcal{P}_5^{(F,L)}$. The second and third constraints are independent of q and set the condition $k \leq \min\{\widehat{\alpha}^B + 1, \alpha+\beta\}$ for Program $\mathcal{P}_5^{(F,L)}$ to be feasible. Using the solution of Program $\mathcal{P}_2^{(F,L)}$, we can restate the first constraint as $q \leq \widehat{q}$. Furthermore, $\widehat{q} \leq 1$ if $\lim_{q\to 1} R_G(q) = \widehat{\alpha}^G - k \leq 0$. Otherwise, the constraint $q \leq \widehat{q}$ is always satisfied for any $q \in [0,1]$. Moreover, $\widehat{q} \geq 0$ holds because $\lim_{q\to 0} R_G(q) = \widehat{\alpha}^G + \beta - k \geq \alpha + \beta - k \geq 0$. The objective function is increasing in q. Summarizing: For $k \leq \min\{\widehat{\alpha}^G, \alpha+\beta\}$: $q = 1$ and $V = \frac{\rho_G \alpha}{\widehat{\alpha}^G} + \beta\left(1 - \frac{\rho_G\alpha}{\widehat{\alpha}^G}\right)$; For $k \geq \widehat{\alpha}^G$ but $k \leq \min\{\widehat{\alpha}^B + 1, \alpha+\beta\}$ $q = \frac{\widehat{\alpha}^G + \beta - k}{\beta(\widehat{\alpha}^G + 1 - k)}$ and $V = \frac{\rho_G\alpha}{\widehat{\alpha}^G} + \left(\frac{\widehat{\alpha}^G+\beta-k}{\widehat{\alpha}^G+1-k}\right)\left(1 - \frac{\rho_G\alpha}{\widehat{\alpha}^G}\right)$.

Comparing the programs:

(1) if $k \leq \widehat{\alpha}^B + \beta$: programs $\mathcal{P}_1^{(F,L)}$, $\mathcal{P}_4^{(F,L)}$ and $\mathcal{P}_5^{(F,L)}$ can be satisfied but the value of program $\mathcal{P}_1^{(F,L)}$ is the highest one. Hence, the solution is $q = 0$ and the threshold is always met (for any r).

(2) if $k \geq \widehat{\alpha}^B + \beta$, there are several possible cases (note that $\widehat{\alpha}^G \leq 1 \leq \widehat{\alpha}^B + 1$ for any $\boldsymbol{\rho}$):

Case (a): $\widehat{\alpha}^B + \beta \leq \widehat{\alpha}^G \leq \widehat{\alpha}^B + 1 \leq \alpha + \beta$

Then, for any $k \leq \widehat{\alpha}^G$ only $\mathcal{P}_5^{(F,L)}$ is feasible and the solution is $q = 1$. For any $k \in [\widehat{\alpha}^G, \widehat{\alpha}^B + 1]$ both $\mathcal{P}_5^{(F,L)}$ (with $q = \widehat{q}$) and $\mathcal{P}_4^{(F,L)}$ are feasible. It is easy to see that, by definition of $R(\widehat{q})$, the difference between the value of $\mathcal{P}_4^{(F,L)}$ and that of $\mathcal{P}_5^{(F,L)}$ is $\beta(1-\widehat{q}) + \frac{\rho_G\alpha}{\widehat{\alpha}^G}(1 - \beta\widehat{q}) \propto \frac{\beta(1-\widehat{q})}{1-\beta\widehat{q}} + \frac{\rho_G\alpha}{\widehat{\alpha}^G} = k - \widehat{\alpha}^G + \frac{\rho_G\alpha}{\widehat{\alpha}^G} > 0$. For $k \in [\widehat{\alpha}^B + 1, \alpha+\beta]$ both $\mathcal{P}_2^{(F,L)}$ and $\mathcal{P}_3^{(F,L)}$ are feasible but the value of $\mathcal{P}_3^{(F,L)}$ is higher. In summary, for any $k \in [\widehat{\alpha}^B + \beta, \widehat{\alpha}^B + 1]$ the solution is $q = 1$. For any $k \in [\widehat{\alpha}^B + 1, \alpha + \beta]$ the solution is $q = 0$.

Case (b): $\widehat{\alpha}^B + \beta \leq \widehat{\alpha}^G \leq \alpha + \beta \leq \widehat{\alpha}^B + 1$

Then, for any $k \leq \widehat{\alpha}^G$ only $\mathcal{P}_5^{(F,L)}$ is feasible and the solution is $q = 1$. For any $k \in [\widehat{\alpha}^G, \alpha+\beta]$ both $\mathcal{P}_5^{(F,L)}$ (with $q = \widehat{q}$) and $\mathcal{P}_4^{(F,L)}$ are feasible. It is easy to see that, by definition of $R(\widehat{q})$, the difference in values is $\beta(1-\widehat{q}) + \frac{\rho_G\alpha}{\widehat{\alpha}^G}(1 - \beta\widehat{q}) \propto \frac{\beta(1-\widehat{q})}{1-\beta\widehat{q}} + \frac{\rho_G\alpha}{\widehat{\alpha}^G} = k - \widehat{\alpha}^G + \frac{\rho_G\alpha}{\widehat{\alpha}^G} \geq 0$. In summary, for any $k \in [\widehat{\alpha}^B + \beta, \alpha+\beta]$ the solution is $q = 1$.

Case (c): $\widehat{\alpha}^B + \beta \leq \alpha + \beta \leq \widehat{\alpha}^G \leq \widehat{\alpha}^B + 1$

Then, for any $k \in [\widehat{\alpha}^B + \beta, \alpha+\beta]$ only $\mathcal{P}_5^{(F,L)}$ is feasible and the solution is $q = 1$.

In summary: if $k \leq \widehat{\alpha}^B + \beta$ or $k \in [\widehat{\alpha}^B + 1, \alpha+\beta]$ the firm never gathers information ($\check{q}(\boldsymbol{\rho}) = 0$). Otherwise, the firm always gathers information ($\check{q}(\boldsymbol{\rho}) = 1$).

Proof of Lemma 2: Note that, for given q,

$$
\begin{aligned}
Var(E[\omega_1|r,\boldsymbol{\rho}]) &= \Pr(r=r_G)\times(E[\omega_1|r=r_G])^2+\Pr(r=r_B)\times(E[\omega_1|r=r_B])^2 \\
&\quad -(E[\omega_1])^2 \\
&= \frac{\rho_G^2\alpha^2}{\rho_G\alpha+\rho_B(1-\alpha)}+\frac{(1-\rho_G)^2\alpha^2}{(1-\rho_G)\alpha+(1-\rho_B)(1-\alpha)}-\alpha^2 \\
&= \widehat{\alpha}^G\rho_G\alpha+\widehat{\alpha}^B(1-\rho_G)\alpha-\alpha^2 \\
Var(E[\omega_2|m,q]) &= \Pr(m=H)\times 1^2+\Pr(m=\varnothing)\times(E[\omega_2|m=\varnothing])^2-(E[\omega_2])^2 \\
&= q\beta+\frac{(1-q)^2\beta^2}{1-q\beta}-\beta^2
\end{aligned}
$$

and therefore

$$
\begin{aligned}
\vartheta(\boldsymbol{\rho},q) &= Var(E[\omega|r,m,\boldsymbol{\rho},q]) \\
&= Var(E[\omega_1|r,\boldsymbol{\rho}])+Var(E[\omega_2|m,q]) \\
&= \widehat{\alpha}^G\rho_G\alpha+\widehat{\alpha}^B(1-\rho_G)\alpha-\alpha^2+q\beta+\frac{(1-q)^2\beta^2}{1-q\beta}-\beta^2
\end{aligned}
$$

The comparative statics follows immediately.

Proof of Proposition 2:
Case A: High threshold $k\geq\alpha+\beta$: By the proofs of Proposition 1 and Lemma 2, simplifying and using the fact that $R_G(\widehat{q})=0$ and so $k-\widehat{\alpha}^G=\frac{\beta(1-\widehat{q})}{(1-\widehat{q}\beta)}$, the three main programs can be decomposed into:

$$
\begin{aligned}
\mathcal{P}_{A1}^{(R)}: \quad & \max_{\boldsymbol{\rho}} \quad \widehat{\alpha}^G\rho_G\alpha+\widehat{\alpha}^B(1-\rho_G)\alpha-\alpha^2 \\
& \text{s.t.} \quad 1\geq\rho_G\geq\rho_B\geq 0 \\
& \qquad\quad k\geq\widehat{\alpha}^G+1 \\
\mathcal{P}_{A2}^{(R)}: \quad & \max_{\boldsymbol{\rho}} \quad \widehat{\alpha}^G\rho_G\alpha+\widehat{\alpha}^B(1-\rho_G)\alpha-\alpha^2 \\
& \text{s.t.} \quad 1\geq\rho_G\geq\rho_B\geq 0 \\
& \qquad\quad k\geq\widehat{\alpha}^B+1 \\
& \qquad\quad k\leq\widehat{\alpha}^G+\beta \\
\mathcal{P}_{B}^{(R)}: \quad & \max_{\boldsymbol{\rho}} \quad \widehat{\alpha}^G\rho_G\alpha+\widehat{\alpha}^B(1-\rho_G)\alpha+(1-\beta)(\widehat{\alpha}^G-k)+\beta-\alpha^2-\beta^2 \\
& \text{s.t.} \quad 1\geq\rho_G\geq\rho_B\geq 0 \\
& \qquad\quad k>\widehat{\alpha}^G \\
& \qquad\quad k<\min\{\widehat{\alpha}^G+\beta,\widehat{\alpha}^B+1,\widehat{\alpha}^G+\frac{\rho_G\alpha}{\widehat{\alpha}^G}\}
\end{aligned}
$$

$$\begin{aligned}
\mathcal{P}_{C1}^{(R)}: \quad & \max_{\boldsymbol{\rho}} \quad \widehat{\alpha}^G \rho_G \alpha + \widehat{\alpha}^B (1-\rho_G)\alpha + \beta - \alpha^2 - \beta^2 \\
& \text{s.t.} \quad 1 \geq \rho_G \geq \rho_B \geq 0 \\
& \qquad k < \widehat{\alpha}^G \\
& \qquad k > \alpha + \beta \\
\mathcal{P}_{C2}^{(R)}: \quad & \max_{\boldsymbol{\rho}} \quad \widehat{\alpha}^G \rho_G \alpha + \widehat{\alpha}^B (1-\rho_G)\alpha + \beta - \alpha^2 - \beta^2 \\
& \text{s.t.} \quad 1 \geq \rho_G \geq \rho_B \geq 0 \\
& \qquad k \geq \widehat{\alpha}^G + \beta \\
& \qquad k \leq \widehat{\alpha}^G + 1 \\
& \qquad NOT \left[\widehat{\alpha}^G + \frac{\rho_G \alpha}{\widehat{\alpha}^G} < \widehat{\alpha}^G + \beta < \widehat{\alpha}^B + 1\right] \\
\mathcal{P}_{C3}^{(R)}: \quad & \max_{\boldsymbol{\rho}} \quad \widehat{\alpha}^G \rho_G \alpha + \widehat{\alpha}^B (1-\rho_G)\alpha + \beta - \alpha^2 - \beta^2 \\
& \text{s.t.} \quad 1 \geq \rho_G \geq \rho_B \geq 0 \\
& \qquad k \geq \widehat{\alpha}^G + \frac{\rho_G \alpha}{\widehat{\alpha}^G} \\
& \qquad k \leq \widehat{\alpha}^B + 1 \\
& \qquad \widehat{\alpha}^G + \frac{\rho_G \alpha}{\widehat{\alpha}^G} < \widehat{\alpha}^B + 1 < \widehat{\alpha}^G + \beta \\
\mathcal{P}_{C4}^{(R)}: \quad & \max_{\boldsymbol{\rho}} \quad \widehat{\alpha}^G \rho_G \alpha + \widehat{\alpha}^B (1-\rho_G)\alpha + \beta - \alpha^2 - \beta^2 \\
& \text{s.t.} \quad 1 \geq \rho_G \geq \rho_B \geq 0 \\
& \qquad k \geq \widehat{\alpha}^G + \frac{\rho_G \alpha}{\widehat{\alpha}^G} \\
& \qquad k \leq \widehat{\alpha}^G + 1 \\
& \qquad \widehat{\alpha}^G + \frac{\rho_G \alpha}{\widehat{\alpha}^G} < \widehat{\alpha}^G + \beta < \widehat{\alpha}^B + 1
\end{aligned}$$

First, consider $\mathcal{P}_{A1}^{(R)}$. The RHS of the second constraint is increasing in ρ_G and decreasing in ρ_B, i.e., high ρ_G and low ρ_B tighten the constraint. On the other hand, the objective function is increasing in ρ_G and decreasing in ρ_B. Setting the second constraint binding yields $k-1 = \widehat{\alpha}^G$ or $\rho_B = \frac{\alpha(2-k)\rho_G}{(1-\alpha)(k-1)}$. Substituting into the objective function it continues to be increasing in ρ_G and so the solution is $\rho_G = 1$ and $\rho_B = \frac{\alpha(2-k)}{(1-\alpha)(k-1)}$. Note that the first constraint can be restated as $1 \geq \frac{\alpha(2-k)}{(1-\alpha)(k-1)} \geq 0$. The second inequality holds as long as the second constraint is satisfied. The first inequality holds if $k \geq 1+\alpha$ which is satisfied as long as the second inequality is satisfied. The second constraint is satisfied with equality by construction. The value of the program is then $\alpha(k-1-\alpha)$ and it is feasible if $k \geq 1+\alpha$.

Now consider $\mathcal{P}_{A2}^{(R)}$. The objective function is increasing in ρ_G and decreasing in ρ_B. The RHS of the third constraint is increasing in ρ_G and decreasing in ρ_B, i.e., high ρ_G and low ρ_B relax the constraint. Furthermore, the RHS of the second constraint is decreasing in ρ_G and increasing in ρ_B, i.e., high ρ_G and low ρ_B relax the constraint.

Therefore, the optimal choice is $\rho_G = 1$ and $\rho_B = 0$. Note that they satisfy the first constraint. The second constraint can be restated as $k \geq 1$ and the third as $k \leq 1+\beta$. The value of the program is then $\alpha(1-\alpha)$ and it is feasible if $k \in [1, 1+\beta]$.

Next, consider $\mathcal{P}_B^{(R)}$. The objective function is increasing in ρ_G and decreasing in ρ_B. The RHS of the second, third and fifth constraints are increasing in ρ_G and decreasing in ρ_B. The RHS of the fourth constraint is decreasing in ρ_G and decreasing in ρ_B. Hence, high ρ_G and low ρ_B relax the third and fifth constraint but tighten the second and fourth constraints. Hence, either the second or the fourth constraint will bind. (If one of them binds the other one is automatically satisfied.)

Suppose the second constraint binds. Then, the regulator sets the reporting system to satisfy $k = \widehat{\alpha}^G$, i.e., $\rho_B = \frac{\alpha(1-k)\rho_G}{(1-\alpha)k}$. Substituting into the objective function, we note that it continues to be increasing in ρ_G and hence $\rho_G = 1$ and $\rho_B = \frac{\alpha(1-k)}{(1-\alpha)k}$. The value in such case is $\alpha(k-\alpha) + \beta(1-\beta)$. Note that the first, second, third and fifth constraints are satisfied. The fourth constraint can be restated as $k < 1$.

Suppose now that the fourth constraint binds. Then, the regulator sets the reporting system to satisfy $k = \widehat{\alpha}^B + 1$, i.e., $\rho_B = \frac{1+\alpha-k-\alpha(2-k)\rho_G}{(1-\alpha)(1-k)}$. Substituting into the objective function, we note that it continues to be increasing in ρ_G if either $k < 1$ (which contradicts $k = \widehat{\alpha}^B + 1$) or $k < 2+\alpha-\beta$ and $k < 1+\alpha$ (the last inequalities hold as long as the constraints are satisfied). Hence $\rho_G = 1$ and $\rho_B = 1$. The value in such case is $(1-\beta)(\alpha-k)+\beta(1-\beta)$. This solution however is not feasible because the third constraint can be restated as $k < \alpha+\beta$ which contradicts our assumption. Therefore, the solution to $\mathcal{P}_3^{(R)}$ is $\rho_G = 1$ and $\rho_B = \frac{\alpha(1-k)}{(1-\alpha)k}$. The value is $\alpha(k-\alpha)+\beta(1-\beta)$ and the program is feasible if $k < 1$.

Next, $\mathcal{P}_{C1}^{(R)}$. The objective function is increasing in ρ_G and decreasing in ρ_B. The RHS of the second constraint is increasing in ρ_G and decreasing in ρ_B, i.e., high ρ_G and low ρ_B relax the constraint. Therefore, the optimal choice is $\rho_G = 1$ and $\rho_B = 0$. Note that they satisfy the first constraint. The second constraint is satisfied if $k < 1$. The value of the program is then $\alpha(1-\alpha)+\beta(1-\beta)$ and it is feasible if $k < 1$. Note that $\alpha(1-\alpha)+\beta(1-\beta)$ is the maximum value that the variance of the posterior expectation can obtain, so the solution here will apply for $k < 1$. In the remainder of the proof, we focus on $k > 1$.

Next, $\mathcal{P}_{C2}^{(R)}$. The objective function is increasing in ρ_G and decreasing in ρ_B. The RHS of the second and third constraints are increasing in ρ_G and decreasing in ρ_B. Hence, high ρ_G and low ρ_B relax the third but tighten the second constraint. Setting the second constraint binding, $k = \widehat{\alpha}^G + \beta$, yields $\rho_B = \frac{\alpha(1+\beta-k)\rho_G}{(1-\alpha)(k-\beta)}$. Substituting into the objective function it continues to be increasing in ρ_G and so the solution is $\rho_G = 1$ and $\rho_B = \frac{\alpha(1+\beta-k)}{(1-\alpha)(k-\beta)}$. Note that the first constraint is satisfied if $k < 1+\beta$. The second constraint is satisfied with equality by construction. The third constraint can be restated as $0 \leq 1-\beta$ which is always satisfied. At $\rho_G = 1$ and $\rho_B = \frac{\alpha(1+\beta-k)}{(1-\alpha)(k-\beta)}$, $\widehat{\alpha}^G + \beta < \widehat{\alpha}^B + 1 \Leftrightarrow k < 1$ so the 'NOT' condition is satisfied by $k > 1$. The solution for $k < 1$ could be different, but we know from the solution to $\mathcal{P}_{C1}^{(R)}$: that the global optimum when $k < 1$ is given by $\rho_G = 1$ and $\rho_B = 0$. Therefore, in the relevant region of $k > 1$, the value of the program is $\alpha(k-\beta-\alpha)+\beta(1-\beta)$ and it is feasible if

$k < 1+\beta$. For $k > 1+\beta$, the conditions in $\mathcal{P}_5^{(R)}$ are satisfied by $\rho_G = 1$ and $\rho_B = 0$, and the value of the program is $\alpha(1-\alpha)+\beta(1-\beta)$ which is the global maximum. So, we have established that the regulator's utility is maximized with $\rho_G = 1$ and $\rho_B = 0$ whenever $k \le 1$ or $k > 1+\beta$. We focus on $k \in (1, 1+\beta)$ in the remainder.

Continuing, $\mathcal{P}_{C3}^{(R)}$. The objective function is increasing in ρ_G and decreasing in ρ_B. The RHS of the second constraint is increasing in ρ_G and decreasing in ρ_B. The RHS of the third constraint is decreasing in ρ_G and increasing in ρ_B. It follows that high ρ_G and low ρ_B tighten the constraints. We can ignore the last constraints, as $\widehat{\alpha}^G + \frac{\rho_G \alpha}{\widehat{\alpha}^G} = \widehat{\alpha}^B + 1$ is only feasible if these are also equal to k, in which case constraints two and three will bind as well. Higher ρ_G and lower ρ_B will tend to slacken the $\widehat{\alpha}^B + 1 < \widehat{\alpha}^G + \beta$ constraint, implying that this constraint will not bind.

Suppose the third constraint binds, $k = \widehat{\alpha}^B + 1$, i.e., $\rho_B = \frac{1+\alpha-k-\alpha(2-k)\rho_G}{(1-\alpha)(1-k)}$. Substituting into the objective function, we note that it continues to be increasing in ρ_G if $k < 1$ and decreasing otherwise. Hence either $\rho_G = 1$ (resulting in $\rho_B = 1$) or $\rho_G = 0$ (resulting in $\rho_B = 0$). Either way, the reporting system is uninformative. However, an uninformative reporting system violates $\widehat{\alpha}^B + 1 < \widehat{\alpha}^G + \beta$ in the last constraint so this cannot be the solution.

Suppose instead that the second constraint binds. Note that the third constraint implies $k < \alpha + 1$. Then, solving for ρ_B yields $\rho_B = \frac{k-2\alpha\rho_G \pm \sqrt{k^2-4\alpha\rho\mathrm{G}}}{2-2\alpha}$ which implies $k^2 - 4\alpha\rho_G \ge 0 \Leftrightarrow \frac{k^2}{4\alpha} \ge \rho_G$. Solving for ρ_G, we have $\frac{k-1+2(\alpha-1)\rho_B - \sqrt{(k-1)^2+4\rho_B(1-\alpha)}}{2\alpha}$ and $\frac{k-1+2(\alpha-1)\rho_B + \sqrt{(k-1)^2+4\rho_B(1-\alpha)}}{2\alpha}$. The first is negative.[9] The second is positive and weakly greater than ρ_B as long as $\rho_B \le k-\alpha$. Substituting the ρ_G into the objective function and taking the derivative yields five (5) critical points:

1. $\rho_B = 0$. In this case $\rho_G = \frac{k-1}{\alpha}$ and the value of the objective function is $\frac{(k-1)(1-\alpha)^2}{2-k} + \beta(1-\beta)$. This solution is feasible (i.e., satisfies the other constraints) in the region of $1 \le k < 1-\alpha$ when $\alpha + k^2 + 3 \ge 4k$ and either $\alpha \le \beta$ or $\alpha + \beta k + 1 \le 2\beta + k$. Overall, the feasibility conditions simplify to (i) $1 \le k < 2$, (ii) $-k^2+4k-3 < \alpha < \frac{k^2-3k+1}{k-3}$ and (iii) $\frac{-\alpha+k-1}{k-2} \le \beta \le k-\alpha$.

2. $\rho_B = k-\alpha$. Here, $\rho_G = \rho_B = k-\alpha$ and the value of the objective function is $\beta(1-\beta)$, but the constraint that $\widehat{\alpha}^B + 1 < \widehat{\alpha}^G + \beta$ is violated.

3. FOC.1: $\rho_B = k-\alpha$ is the same as the upper-bound, and is infeasible (see previous item).

4. FOC.2: $\frac{4\alpha+2k\left(\sqrt{8\alpha-8k+9}-5\right)-5\sqrt{8\alpha-8k+9}+15}{8-8\alpha}$. This solution yields $\rho_B \in [0, k-\alpha]$ when $k < \frac{3}{2}$ and $\alpha \le 5+2k^2-6k$. Substituting, we have
$\rho_G = \frac{-4\alpha-2k\left(\sqrt{8\alpha-8k+9}-7\right)+5\sqrt{8\alpha-8k+9}+2\sqrt{8\alpha-10\sqrt{8\alpha-8k+9}+4k\left(\sqrt{8\alpha-8k+9}+k-7\right)+34}-19}{8\alpha}$.
We have $0 \le \rho_B \le \rho_G \le 1$ if the above conditions and $2k^2 < 9\alpha$ hold.

[9] Can show by $(k-1+2(\alpha-1)\rho_B)^2 - ((k-1)^2+4\rho_B(1-\alpha)) = -4(1-\alpha)\rho_B(k-(1-\alpha)\rho_B) < 0$ for $k > 1$.

The expession for the value of the objective fucntion is long: $-\beta^2+\beta$
$$-\frac{4(\alpha-1)^2\left(\frac{\sqrt{4\alpha+2k^2+2k\left(\sqrt{8\alpha-8k+9}-7\right)-5\sqrt{8\alpha-8k+9}+17}}{\sqrt{2}}+\frac{4\alpha+2k\left(\sqrt{8\alpha-8k+9}-5\right)-5\sqrt{8\alpha-8k+9}+15}{4(\alpha-1)}+k-1\right)^2}{\left(\sqrt{8\alpha+4k^2+4k\left(\sqrt{8\alpha-8k+9}-7\right)-10\sqrt{8\alpha-8k+9}+34}+2k-6\right)\left(\sqrt{8\alpha+4k^2+4k\left(\sqrt{8\alpha-8k+9}-7\right)-10\sqrt{8\alpha-8k+9}+34}+2k-2\right)}.$$
Substituting into the conditions, this solution is feasible as long as: (i) $1\le k<\frac{4}{3}$, (ii) $k-\frac{7}{9}\le\alpha\le 2k^2-6k+5$ and (iii) $\frac{\sqrt{9-8(k-\alpha)}-1}{2}\le\beta<1$.

5 FOC.3: $\frac{-4\alpha^2-11\alpha+10(\alpha-1)k+\sqrt{(\alpha-1)^2(5-2k)^2(8\alpha-8k+9)}+15}{8(\alpha-1)^2}$. This solution implies $\rho_B>k-\alpha$, which would lead to an infeasible (imaginary) value for ρ_G.

Notably, the conditions for solution 4 imply the conditions for solution 1, so solution 1 is a possibility whenever solution 4 is. However, under these conditions, it is possible for the regulator to prefer solution 4.

Last, $\mathcal{P}^{(R)}_{C4}$. For this program, we can start by noting that the objective function is increasing in ρ_G and decreasing in ρ_B. These also slacken the third constraint, but tighten the last, $\widehat{\alpha}^G+\beta<\widehat{\alpha}^B+1$. Note that a reporting system with $\rho_G=1$ and $\rho_B=0$ will violate this constraint. So, there are three potential constraints that may bind: A) $k\ge\widehat{\alpha}^G+\frac{\rho_G\alpha}{\widehat{\alpha}^G}$; B) $\widehat{\alpha}^G+\frac{\rho_G\alpha}{\widehat{\alpha}^G}<\widehat{\alpha}^G+\beta$; or C) $\widehat{\alpha}^G+\beta<\widehat{\alpha}^B+1$.

A: $k=\widehat{\alpha}^G+\frac{\rho_G\alpha}{\widehat{\alpha}^G}$. solving for ρ_B yields $\rho_B=\frac{k-2\alpha\rho_G\pm\sqrt{k^2-4\alpha\rho G}}{2-2\alpha}$ which implies $k^2-4\alpha\rho_G\ge 0\Leftrightarrow\frac{k^2}{4\alpha}\ge\rho_G$. Solving for ρ_G, we have $\frac{k-1+2(\alpha-1)\rho_B-\sqrt{(k-1)^2+4\rho_B(1-\alpha)}}{2\alpha}$, and $\frac{k-1+2(\alpha-1)\rho_B+\sqrt{(k-1)^2+4\rho_B(1-\alpha)}}{2\alpha}$. The first is negative.[10] The second is positive and weakly greater than ρ_B as long as $\rho_B\le k-\alpha$. Substituting the ρ_G into the objective function and taking the derivative yields five (5) critical points:

1. $\rho_B=0$. In this case $\rho_G=\frac{k-1}{\alpha}$ and the value of the objective function is $\frac{(k-1)(1-\alpha)^2}{2-k}+\beta(1-\beta)$. This solution is feasible (i.e., satisfies the other constraints) in the region of $1\le k<2$ when $1+\alpha+2\beta\ge k(1-\beta)$ and either $\beta\le 1/2$ or $\beta<\frac{1}{3-k}$. Overall, the feasibility conditions simplify to (i) $1\le k<2$, (ii) $k-1+\beta(2-k)<\alpha\le k-\beta$ and (iii) $k-1<\beta<\frac{1}{3-k}$.

2. $\rho_B=k-\alpha$. Here, $\rho_G=\rho_B=k-\alpha$ and the value of the objective function is $\beta(1-\beta)$. This solution is feasible if $k<1+\alpha$.

3. FOC.1: $\rho_B=k-\alpha$ is the same as the previous.

4. FOC.2: $\frac{4\alpha+2k\left(\sqrt{8\alpha-8k+9}-5\right)-5\sqrt{8\alpha-8k+9}+15}{8-8\alpha}$. This solution yields $\rho_B\in[0,k-\alpha]$ when $k<\frac{7}{4}$ and $\alpha\le 5+2k^2-6k$. Substituting, we have
$$\rho_G=\frac{-4\alpha-2k\left(\sqrt{8\alpha-8k+9}-7\right)+5\sqrt{8\alpha-8k+9}+2\sqrt{8\alpha-10\sqrt{8\alpha-8k+9}+4k\left(\sqrt{8\alpha-8k+9}+k-7\right)+34}-19}{8\alpha}.$$
We have $0\le\rho_B\le\rho_G\le 1$ if the above conditions and $2k^2<9\alpha$ hold.

[10] Can show by $(k-1+2(\alpha-1)\rho_B)^2-((k-1)^2+4\rho_B(1-\alpha))=-4(1-\alpha)\rho_B(k-(1-\alpha)\rho_B)<0$ for $k>1$.

The expression for the value of the objective function is long: $-\beta^2+\beta$
$$-\frac{4(\alpha-1)^2\left(\sqrt{k^2+k\left(\sqrt{8\alpha-8k+9}-7\right)+\frac{1}{2}\left(4\alpha-5\sqrt{8\alpha-8k+9}+17\right)}+\frac{4\alpha+2k\left(\sqrt{8\alpha-8k+9}-5\right)-5\sqrt{8\alpha-8k+9}+15}{4(\alpha-1)}+k-1\right)^2}{\left(\sqrt{8\alpha+4k^2+4k\left(\sqrt{8\alpha-8k+9}-7\right)-10\sqrt{8\alpha-8k+9}+34}+2k-6\right)\left(\sqrt{8\alpha+4k^2+4k\left(\sqrt{8\alpha-8k+9}-7\right)-10\sqrt{8\alpha-8k+9}+34}+2k-2\right)}.$$
Substituting into the conditions, this solution is feasible as long as: (i) $1\le k<\frac{4}{3}$ (ii) $k-\frac{7}{9}<\alpha\le 2k^2-6k+5$ (iii) $\frac{3-\sqrt{9-8(k-\alpha)}}{4}\le\beta\le\frac{\sqrt{9-8(k-\alpha)}-1}{2}$.

5. FOC.3: $\frac{-4\alpha^2-11\alpha+10(\alpha-1)k+\sqrt{(\alpha-1)^2(5-2k)^2(8\alpha-8k+9)}+15}{8(\alpha-1)^2}$. This solution implies $\rho_B>k-\alpha$, which would lead to an infeasible (imaginary) value for ρ_G.

B) $\widehat{\alpha}^G+\frac{\rho_G\alpha}{\widehat{\alpha}^G}=\widehat{\alpha}^G+\beta$. If this constraint binds and the second constraint is slack, then this implies $\widehat{\alpha}^G+\beta\le k$, which implies the threshold cannot be met after a good report and nondisclosure, which violates an assumption of the program.

C) $\widehat{\alpha}^G+\beta=\widehat{\alpha}^B+1$. Note that this constraint implies $\rho_B<\rho_G$ is a strict inequality, otherwise the binding constriant would imply $\beta=1$ while we have assumed $\beta<1$. Solving for ρ_G yields $\frac{\alpha(2(\beta-1)\rho_B-1)-2\beta\rho_B+\beta+2\rho_B-\sqrt{4(\alpha-1)(\beta-1)\rho_B+(\alpha-\beta)^2}}{2\alpha(\beta-1)}$ and $\frac{\alpha(2(\beta-1)\rho_B-1)-2\beta\rho_B+\beta+2\rho_B+\sqrt{4(\alpha-1)(\beta-1)\rho_B+(\alpha-\beta)^2}}{2\alpha(\beta-1)}$. The latter solution is not feasible as it never satisfies $\rho_B<\rho_G$. For the former solution, we can substitute it into the objective function, which is concave (via the 2nd derivative test) for $\rho_B>-\frac{(\alpha-\beta)^2}{4(1-\alpha)(1-\beta)}$. The first-order condition for a maximum is satisfied at $\rho_B=\frac{1-2\alpha+\beta}{4(1-\alpha)}$ which is positive for $\alpha\le\frac{1+\beta}{2}$. For $\alpha>\frac{1+\beta}{2}$, we have $\rho_B=0$. Below, we explore each:

1) $\alpha\le\frac{1+\beta}{2}$ and $\rho_B=\frac{1-2\alpha+\beta}{4(1-\alpha)}$: In this case, $\rho_G=\frac{1+2\alpha-\beta}{4\alpha}$ and the value of the objective function is $\frac{1+2\beta-3\beta^2}{4}$. The conditions of the program are satisfied at the solution with all of the following holding: (i) $2\le 2k-2\alpha+\beta\le 3$, (ii) $\beta\ge\frac{1}{2}$ (which implies $\alpha\le\frac{1+\beta}{2}$), and (iii) $\alpha\le\frac{k^2+\beta-1}{2}$

2) $\alpha>\frac{1+\beta}{2}$ and $\rho_B=0$: In this case, $\rho_G=\frac{\alpha-\beta}{\alpha(1-\beta)}$ and the objective function's value is $\alpha(1-\alpha)+\beta(\alpha-\beta)$. Note that $\alpha>\frac{1+\beta}{2}\Rightarrow\alpha>\beta$. The conditions of the program are satisfied at this solution for $k>\frac{3}{2}$ and: (i) $\beta k+1\ge 3\beta$ and $\alpha\le\beta(2-k)+k-1$, or (ii) $\beta k+1<3\beta$, which implies $\beta>2/3$.

Comparison reveals that:

(i) if $k\in(\alpha+\beta,1]$ the regulator chooses a fully-informative reporting system ($\rho_G=1$ and $\rho_B=0$) and the firm always acquires information ($q=1$). This is from the solution to $\mathcal{P}^{(R)}_{C1}$, which dominates in this region.

(ii) if $k\in[1+\beta,2]$ the regulator chooses a fully-informative reporting system ($\rho_G=1$ and $\rho_B=0$) and the firm always acquires information ($q=1$). This is from the solution to $\mathcal{P}^{(R)}_{C2}$, which dominates in this region.

(iii) if $k\in(1,1+\beta)$, the regulator choses an imperfectly-informative reporting system $(\rho_G,\rho_B)\ne(0,1)$ as given in $\mathcal{P}^{(R)}_{C2}$, $\mathcal{P}^{(R)}_{C3}$, or $\mathcal{P}^{(R)}_{C4}$.

Case B: Low threshold $k \leq \alpha + \beta$: Taking into account the firm's choice when $k \leq \alpha + \beta$:

$$\check{q}(\boldsymbol{\rho}) = \begin{cases} 0, & \text{if} \quad k \leq \widehat{\alpha}^B + \beta \quad \text{or} \quad k \in [\widehat{\alpha}^B + 1, \alpha + \beta] \\ 1 & \text{otherwise.} \end{cases}$$

we can restate the regulator's problem as one in which he compares the values of the programs below:

$$\begin{aligned}
\mathcal{P}_{A1}^{(R,L)}: \qquad & \max_{\boldsymbol{\rho}} \quad \widehat{\alpha}^G \rho_G \alpha + \widehat{\alpha}^B (1-\rho_G)\alpha - \alpha^2 \\
& \text{s.t.} \quad 1 \geq \rho_G \geq \rho_B \geq 0 \\
& \qquad\ k \leq \widehat{\alpha}^B + \beta \\
\mathcal{P}_{A2}^{(R,L)}: \qquad & \max_{\boldsymbol{\rho}} \quad \widehat{\alpha}^G \rho_G \alpha + \widehat{\alpha}^B (1-\rho_G)\alpha - \alpha^2 \\
& \text{s.t.} \quad 1 \geq \rho_G \geq \rho_B \geq 0 \\
& \qquad\ k \geq \widehat{\alpha}^B + 1 \\
& \qquad\ k \leq \alpha + \beta \\
\mathcal{P}_{C1}^{(R,L)}: \qquad & \max_{\boldsymbol{\rho}} \quad \widehat{\alpha}^G \rho_G \alpha + \widehat{\alpha}^B (1-\rho_G)\alpha - \alpha^2 + \beta(1-\beta) \\
& \text{s.t.} \quad 1 \geq \rho_G \geq \rho_B \geq 0 \\
& \qquad\ k \geq \widehat{\alpha}^B + \beta \\
& \qquad\ k \leq \widehat{\alpha}^B + 1 \\
& \qquad\ \widehat{\alpha}^B + 1 \leq \alpha + \beta \\
\mathcal{P}_{C2}^{(R,L)}: \qquad & \max_{\boldsymbol{\rho}} \quad \widehat{\alpha}^G \rho_G \alpha + \widehat{\alpha}^B (1-\rho_G)\alpha - \alpha^2 + \beta(1-\beta) \\
& \text{s.t.} \quad 1 \geq \rho_G \geq \rho_B \geq 0 \\
& \qquad\ k \geq \widehat{\alpha}^B + \beta \\
& \qquad\ k \leq \alpha + \beta \\
& \qquad\ \widehat{\alpha}^B + \beta \leq \widehat{\alpha}^G \leq \alpha + \beta \leq \widehat{\alpha}^B + 1 \\
\mathcal{P}_{C3}^{(R,L)}: \qquad & \max_{\boldsymbol{\rho}} \quad \widehat{\alpha}^G \rho_G \alpha + \widehat{\alpha}^B (1-\rho_G)\alpha - \alpha^2 + \beta(1-\beta) \\
& \text{s.t.} \quad 1 \geq \rho_G \geq \rho_B \geq 0 \\
& \qquad\ k \geq \widehat{\alpha}^B + \beta \\
& \qquad\ k \leq \alpha + \beta \\
& \qquad\ \alpha + \beta \leq \widehat{\alpha}^G \leq \widehat{\alpha}^B + 1
\end{aligned}$$

First, consider $\mathcal{P}_{A1}^{(R,L)}$. The objective function is increasing in ρ_G and decreasing in ρ_B. The RHS of the second constraint is decreasing in ρ_G and increasing in ρ_B. Hence, high ρ_G and low ρ_B tighten the constraint. However, note that if $k \leq \beta$ the second constraint is always satisfied for any $\boldsymbol{\rho}$. Hence, if $k \leq \beta$ the regulator will choose $\rho_G = 1$ and $\rho_B = 0$. The value at this solution is $\alpha(1-\alpha)$.

Now consider the case $k > \beta$. Setting the second constraint binding, $k = \widehat{\alpha}^B + \beta$, yields $\rho_B = \frac{k-\beta-\alpha(1-\rho_G(1-(k-\beta)))}{(1-\alpha)(k-\beta)}$. Substituting in the objective function we note that it is decreasing if $k > \beta$. Then $\rho_G = 0$ and $\rho_B = \frac{\alpha+\beta-k}{(1-\alpha)(\beta-k)} < 0$. Which violates the first constraint. In summary, the program is only feasible when $k \leq \beta$. Then, the solution is $\rho_G = 1$ and $\rho_B = 0$ and the value is $\alpha(1-\alpha)$.

Now consider $\mathcal{P}_{A2}^{(R,L)}$. The objective function is increasing in ρ_G and decreasing in ρ_B. The RHS of the second constraint is decreasing in ρ_G and increasing in ρ_B. Hence, high ρ_G and low ρ_B relax the second constraint. The third constraint is independent of $\boldsymbol{\rho}$. Setting $\rho_G = 1$ and $\rho_B = 0$, the first constraint is satisfied and the second constraint can be restated as $k \geq 1$. The solution of the program is $\rho_G = 1$ and $\rho_B = 0$, its value is $\alpha(1-\alpha)$ and it is feasible when $k \in [1, \alpha+\beta]$.

Now consider $\mathcal{P}_{C1}^{(R,L)}$. The objective function is increasing in ρ_G and decreasing in ρ_B. The RHS of the second, third and fourth constraints are decreasing in ρ_G and increasing in ρ_B. Hence, high ρ_G and low ρ_B relax the second and fourth constraint but tighten the third constraint. However, when $k \leq 1$ the third constraint is always satisfied for any $\boldsymbol{\rho}$. Setting $\rho_G = 1$ and $\rho_B = 0$, the first constraint is satisfied, the second constraint can be restated as $k \geq \beta$, the third as $k \leq 1$ and the fourth as $\alpha + \beta \geq 1$. The value is $\alpha(1-\alpha) + \beta(1-\alpha)$.

Suppose $k > 1$. Setting the third constraint binding, $k = \widehat{\alpha}^B + 1$, yields $\rho_B = \frac{1+\alpha-k-\alpha(2-k)\rho_G}{(1-\alpha)(1-k)}$. Substituting in the objective function we note that it continues to be increasing in ρ_G. Hence, $\rho_G = 1$ and $\rho_B = 1$. The first and third constraints are satisfied. The second constraint can only be satisfied if $k = \alpha + \beta$. The fourth constraint can be restated as $\alpha + 1 \leq \alpha + \beta$ and it can never be satisfied. Hence, the program is not feasible. In summary, the program is only feasible when $k \in [\beta, 1]$ and $\alpha+\beta \geq 1$. Then, the solution is $\rho_G = 1$ and $\rho_B = 0$ and the value is $\alpha(1-\alpha)+\beta(1-\alpha)$.

Now consider $\mathcal{P}_{C2}^{(R,L)}$. The objective function is increasing in ρ_G and decreasing in ρ_B. The RHS of the second constraint is decreasing in ρ_G and increasing in ρ_B. Hence, high ρ_G and low ρ_B relaxes the second constraint. The third constraint is independent of $\boldsymbol{\rho}$. Regarding the fourth constraint: (i) high ρ_G and low ρ_B relax $\widehat{\alpha}^B + \beta \leq \widehat{\alpha}^G$; (ii) tighten $\widehat{\alpha}^G \leq \alpha + \beta$ and $\alpha + \beta \leq \widehat{\alpha}^B + 1$.

Suppose $\widehat{\alpha}^G \leq \alpha + \beta$ binds. Then $\rho_B = \frac{\alpha\rho_G(1-(\alpha+\beta))}{(1-\alpha)(\alpha+\beta)}$. Substituting into the objective function, we note that it continues to be increasing in ρ_G. Hence, $\rho_G = 1$ and $\rho_B = \frac{\alpha(1-(\alpha+\beta))}{(1-\alpha)(\alpha+\beta)}$. The first constraint is satisfied if $1 \geq \alpha+\beta$. The second constraint can be restated as $k \geq \beta$. The fourth constraint can be restated as $\beta \leq \alpha+\beta \leq \alpha+\beta \leq 1$. The value is $\beta(1+\alpha-\beta)$.

Suppose $\alpha+\beta \leq \widehat{\alpha}^B+1$ binds. Then $\rho_B = \frac{1-\beta-\alpha\rho_G(2-(\alpha+\beta))}{(1-\alpha)(1-(\alpha+\beta))}$. Substituting into the objective function, we note that it continues to be increasing in ρ_G as long as $1 \geq \alpha+\beta$. Suppose this is the case. Then, $\rho_G = 1$ and $\rho_B = 1$. The first constraint is satisfied. The fourth constraint can be restated as $\alpha + \beta \leq \alpha \leq \alpha + \beta \leq \alpha + 1$ which can only be satisfied if $\beta = 0$. The value is $R = \beta(1-\beta)$. Now suppose $1 \geq \alpha + \beta$ does not hold. Then, $\rho_G = 0$ and $\rho_B = \frac{1-\beta}{(1-\alpha)(1-(\alpha+\beta))} < 0$ which violates the first constraint.

Noting that the value of the program when $\widehat{\alpha}^G \leq \alpha + \beta$ binds is larger than the value when $\alpha + \beta \leq \widehat{\alpha}^B + 1$ binds, we conclude that the solution of the program is

$\rho_G = 1$ and $\rho_B = \frac{\alpha(1-(\alpha+\beta))}{(1-\alpha)(\alpha+\beta)}$, its value is $\beta(1+\alpha-\beta)$ and it is feasible if $\alpha+\beta \leq 1$ and $k \in [\beta, \alpha+\beta]$.

Lastly, consider $\mathcal{P}_{C3}^{(R,L)}$. The objective function is increasing in ρ_G and decreasing in ρ_B. The RHS of the second constraint is decreasing in ρ_G and increasing in ρ_B. Hence, high ρ_G and low ρ_B relax the second constraint. The third constraint is independent of $\boldsymbol{\rho}$. Regarding the fourth constraint: high ρ_G and low ρ_B relax $\alpha+\beta < \widehat{\alpha}^G$ but tighten $\widehat{\alpha}^G \leq \widehat{\alpha}^B + 1$. Setting this constraint binding, $\widehat{\alpha}^G = \widehat{\alpha}^B + 1$, yields $\rho_B = \frac{1+\alpha-2\alpha\rho_G+\sqrt{(1+\alpha)^2-4\alpha\rho_G}}{2(1-\alpha)}$. Substituting into the objective function, we note that it continues to be increasing in ρ_G. Hence, $\rho_G = 1$ and $\rho_B = \frac{1-\alpha+\sqrt{(1-\alpha)^2}}{2(1-\alpha)} = 1$. The first constraint is satisfied. The fourth constraint can be restated as $\alpha+\beta \leq \alpha \leq \alpha+1$ which can only be satisfied if $\beta = 0$. The value is $\beta(1-\beta)$.

Comparison reveals that:

(i) If $k \leq \beta$, the regulator sets $\rho_G = 1$ and $\rho_B = 0$ and the firm never acquires information.

(ii) If $k > \beta$ and $1 \geq \alpha+\beta$, the regulator sets $\rho_G = 1$ and $\rho_B = \frac{\alpha(1-(\alpha+\beta))}{(1-\alpha)(\alpha+\beta)}$ and the firm always acquires information.

(iii) If $k > \beta$ and $1 \leq \alpha+\beta$, the regulator sets $\rho_G = 1$ and $\rho_B = 0$ and the firm always acquires information.

Proof of Proposition 4: Using the proof of Lemma 2

$$
\begin{aligned}
\vartheta(\boldsymbol{\rho}, q) &= \widehat{\alpha}^G \rho_G \alpha + \widehat{\alpha}^B (1-\rho_G)\alpha - \alpha^2 \\
&\quad + \Pr(q=1) \left(q\beta + \frac{(1-q)^2\beta^2}{1-q\beta} - \beta^2 \right)\Bigg|_{q=1} \\
&\quad + \Pr(q=0) \left(q\beta + \frac{(1-q)^2\beta^2}{1-q\beta} - \beta^2 \right)\Bigg|_{q=0} \\
&= \widehat{\alpha}^G \rho_G \alpha + \widehat{\alpha}^B (1-\rho_G)\alpha - \alpha^2 + Pr(q=1)\beta(1-\beta).
\end{aligned}
$$

The regulator compares the values of the following programs:

$\mathcal{P}_{A1}^{(R,T)}$: *Dissuade information acquisition after any r:*

$$
\begin{aligned}
\max_{\boldsymbol{\rho}} \quad & \widehat{\alpha}^G \rho_G \alpha + \widehat{\alpha}^B (1-\rho_G)\alpha - \alpha^2 \\
\text{s.t.} \quad & 1 \geq \rho_G \geq \rho_B \geq 0 \\
& k \leq \widehat{\alpha}^B + \beta
\end{aligned}
$$

$\mathcal{P}_{A2}^{(R,T)}$: *Dissuade information acquisition after any r:*

$$\begin{aligned} \max_{\boldsymbol{\rho}} \quad & \widehat{\alpha}^G \rho_G \alpha + \widehat{\alpha}^B (1-\rho_G)\alpha - \alpha^2 \\ \text{s.t.} \quad & 1 \geq \rho_G \geq \rho_B \geq 0 \\ & k \geq 1 + \widehat{\alpha}^G \end{aligned}$$

$\mathcal{P}_{B}^{(R,T)}$: *Induce information acquisition after any r:*

$$\begin{aligned} \max_{\boldsymbol{\rho}} \quad & \widehat{\alpha}^G \rho_G \alpha + \widehat{\alpha}^B (1-\rho_G)\alpha - \alpha^2 + \beta(1-\beta) \\ \text{s.t.} \quad & 1 \geq \rho_G \geq \rho_B \geq 0 \\ & k \geq \widehat{\alpha}^G + \beta \\ & k \leq 1 + \widehat{\alpha}^B \end{aligned}$$

$\mathcal{P}_{C}^{(R,T)}$: *Induce information acquisition only after r_B:*

$$\begin{aligned} \max_{\boldsymbol{\rho}} \quad & \widehat{\alpha}^G \rho_G \alpha + \widehat{\alpha}^B (1-\rho_G)\alpha - \alpha^2 + ((1-\rho_G)\alpha + (1-\rho_B)(1-\alpha))\beta\,(1-\beta) \\ \text{s.t.} \quad & 1 \geq \rho_G \geq \rho_B \geq 0 \\ & k \geq \widehat{\alpha}^B + \beta \\ & k \leq \widehat{\alpha}^G + \beta \end{aligned}$$

$\mathcal{P}_{D}^{(R,T)}$: *Induce information acquisition only after r_G:*

$$\begin{aligned} \max_{\boldsymbol{\rho}} \quad & \widehat{\alpha}^G \rho_G \alpha + \widehat{\alpha}^B (1-\rho_G)\alpha - \alpha^2 + (\rho_G\alpha + \rho_B(1-\alpha))\beta\,(1-\beta) \\ \text{s.t.} \quad & 1 \geq \rho_G \geq \rho_B \geq 0 \\ & k \geq \widehat{\alpha}^B + 1 \\ & k \leq \widehat{\alpha}^G + 1 \end{aligned}$$

Consider first $\mathcal{P}_{A1}^{(R,T)}$. The objective function is increasing in ρ_G and decreasing in ρ_B. If $k \leq \beta$ the second constraint is always satisfied for any $\boldsymbol{\rho}$. Hence, if $k \leq \beta$, the solution is $\boldsymbol{\rho} = \boldsymbol{\rho}^F$ and the value of the program is $\alpha(1-\alpha)$. Now consider $k > \beta$. The RHS of the second constraint is decreasing in ρ_G and increasing in ρ_B. Hence, high ρ_G and low ρ_B tightens the second constraint. Setting the constraint binding and solving for ρ_G yields $\rho_G = \frac{(\alpha-1)\rho_B(\beta-k)+\alpha+\beta-k}{\alpha(\beta-k+1)}$. Substituting into the objective function, it continues to be decreasing in ρ_B and so $\rho_B = 0$. Then $\rho_G = \frac{\alpha+\beta-k}{\alpha(1+\beta-k)}$. The first constraint is satisfied if $k < \alpha + \beta$. The second constraint is satisfied with equality by construction. The value of the program is $(1-\alpha)(\alpha+\beta-k)$. To summarize: If $k \leq \beta$, the solution is $\boldsymbol{\rho} = \boldsymbol{\rho}^F$ and the value of the program is $\alpha(1-\alpha)$. If $k \in [\beta, \alpha+\beta]$, the solution is $\rho_B = 0$ and $\rho_G = \frac{\alpha+\beta-k}{\alpha(1+\beta-k)}$. The value of the program is $(1-\alpha)(\alpha+\beta-k)$.

Now consider $\mathcal{P}_{A2}^{(R,T)}$. The objective function is increasing in ρ_G and decreasing in ρ_B. The RHS of the second constraint is increasing in ρ_G and decreasing in ρ_B. Hence, high ρ_G and low ρ_B tightens the second constraint. Setting the constraint binding and solving for ρ_B yields $\rho_B = \frac{\alpha(2-k)\rho_G}{(1-\alpha)(k-1)}$. Substituting into the objective function, it continues to be increasing in ρ_G (if $k > 1$ which is indeed the case if the second constraint is satisfied). Hence, $\rho_G = 1$ and $\rho_B = \frac{\alpha(2-k)}{(1-\alpha)(k-1)}$. The second constraint is satisfied with equality by construction. The first inequality is satisfied if $k > 1+\alpha$ but that must be true if the second constraint is satisfied. To summarize, the solution is $\rho_G = 1$ and $\rho_B = \frac{\alpha(2-k)}{(1-\alpha)(k-1)}$, the value of the program is $\alpha(k-1-\alpha)$ and it is feasible if $k > 1+\alpha$.

Now consider $\mathcal{P}_{B}^{(R,T)}$. The objective function is increasing in ρ_G and decreasing in ρ_B. The RHS of the second constraint is increasing in ρ_G and decreasing in ρ_B. The RHS of the third constraint is decreasing in ρ_G and increasing in ρ_B. Hence, high ρ_G and low ρ_B tightens both the second and the third constraints. Suppose the second constraint binds. Solving for ρ_B yields $\rho_B = \frac{\alpha(1+\beta-k)\rho_G}{(1-\alpha)(k-\beta)}$. Substituting into the objective function, it continues to be increasing in ρ_G if $k > \beta$ (this is indeed the case whenever the second constraint is satisfied). So $\rho_G = 1$. Then $\rho_B = \frac{\alpha(1+\beta-k)}{(1-\alpha)(k-\beta)}$. The first constraint is satisfied if $k \geq \alpha+\beta$ and $k \leq 1+\beta$. The third constraint is satisfied if $k \leq 1$.Combining the conditions yields $k \in [\alpha+\beta, 1]$. The value of the program then is $\alpha(k-\alpha-\beta)+\beta(1-\beta)$. Now suppose that the third constraint binds. Solving for ρ_G yields $\rho_G = \frac{(\alpha-1)(k-1)\rho_B-\alpha+k-1}{\alpha(k-2)}$. Substituting into the objective function, it continues to be decreasing in ρ_B. So, $\rho_B = 0$ and $\rho_G = \frac{1+\alpha-k}{\alpha(2-k)}$. The first constraint is satisfied if $k \in [1, 1+\alpha]$. The second constraint can be restated as $k > 1+\beta$. The third constraint is satisfied with equality by construction. Combining the conditions yields $k \in [1, 1+\alpha]$. The value of the program is $(1-\alpha)(1+\alpha-k)+\beta(1-\beta)$. Comparing the solutions: If $k \in [\alpha+\beta, 1]$ then $\rho_G = 1$ and $\rho_B = \frac{\alpha(1+\beta-k)}{(1-\alpha)(k-\beta)}$. The value of the program then is $\alpha(k-\alpha-\beta)+\beta(1-\beta)$. If $k \in [1+\beta, 1+\alpha]$ then $\rho_B = 0$ and $\rho_G = \frac{1+\alpha-k}{\alpha(2-k)}$. The value of the program is $(1-\alpha)(1+\alpha-k)+\beta(1-\beta)$.

Now consider $\mathcal{P}_{C}^{(R,T)}$. High ρ_G and low ρ_B relax the second and third constraint. The objective function is also decreasing in ρ_B. Hence $\rho_B = 0$. Substituting into the objective function and solving for ρ_G yields $\rho_G = \frac{1}{\alpha}\left(1+\frac{(1-\alpha)}{\sqrt{\beta(1-\beta)}}\right)$ and $\rho_G = \frac{1}{\alpha}\left(1-\frac{(1-\alpha)}{\sqrt{\beta(1-\beta)}}\right)$ but only $\rho_G = \frac{1}{\alpha}\left(1-\frac{(1-\alpha)}{\sqrt{\beta(1-\beta)}}\right)$ can satisfy the first constraint. Specifically, $\rho_G = \frac{1}{\alpha}\left(1-\frac{(1-\alpha)}{\sqrt{\beta(1-\beta)}}\right)$ is bounded away from zero if $\alpha \geq 1-\sqrt{\beta(1-\beta)}$. If this is the case the second constraint can be restated as $k \geq 1+\beta-\sqrt{(1-\beta)\beta}$, the third constraint as $k \leq 1+\beta$, and the value of the program is $(1-\alpha)\left(\alpha+2\sqrt{(1-\beta)\beta}-1\right)$. If this is not the case, i.e., if $\alpha < 1-\sqrt{\beta(1-\beta)}$ the first constraint binds and $\rho_G = \rho_B = 0$ and the second and third constraint can be restated as $k = \alpha+\beta$. This is a knife-edge case and we ignore it. Lastly, consider the case when $k < 1+\beta-\sqrt{\beta(1-\beta)}$. In this case the second constraint has to bind. The solution is $\rho_G = \frac{\alpha+\beta-k}{\alpha(1+\beta-k)}$ and

$\rho_B = 0$. The value of the program then is $\frac{(1-\alpha)\left(-k(\alpha+2\beta+1)+\alpha\beta+\alpha+2\beta+k^2\right)}{1+\beta-k}$ and it is feasible if $k > \beta$. Comparing the solutions: If $k \in [1+\beta-\sqrt{(1-\beta)\beta}, 1+\beta]$ and $\alpha \geq 1-\sqrt{\beta(1-\beta)}$ then $\rho_G = \frac{1}{\alpha}\left(1-\frac{(1-\alpha)}{\sqrt{\beta(1-\beta)}}\right)$ and $\rho_B = 0$. The value of the program then is $(1-\alpha)\left(\alpha+2\sqrt{(1-\beta)\beta}-1\right)$. If $k \in [\beta, 1+\beta-\sqrt{(1-\beta)\beta}]$ then $\rho_G = \frac{\alpha+\beta-k}{\alpha(1+\beta-k)}$ and $\rho_B = 0$ and the value of the program is $\frac{(1-\alpha)\left(-k(\alpha+2\beta+1)+\alpha\beta+\alpha+2\beta+k^2\right)}{1+\beta-k}$.

Lastly consider $\mathcal{P}_D^{(R,T)}$. High ρ_G and low ρ_B relax the second and third constraint. The objective function is also increasing in ρ_G. Hence $\rho_G = 1$. Substituting into the objective function and solving for ρ_B yields $\rho_B = 1-\frac{1}{1-\alpha}\left(1+\frac{\alpha}{\sqrt{\beta(1-\beta)}}\right)$ and $\rho_B = 1-\frac{1}{1-\alpha}\left(1-\frac{\alpha}{\sqrt{\beta(1-\beta)}}\right)$ but only $\rho_B = 1-\frac{1}{1-\alpha}\left(1-\frac{\alpha}{\sqrt{\beta(1-\beta)}}\right)$ can satisfy the first constraint. Specifically, it remains bounded away from 1 if $\alpha \leq \sqrt{\beta(1-\beta)}$. If this is the case the second constraint can be restated as $k \geq 1$. The third constraint can be restated as $k \leq 1+\sqrt{\beta(1-\beta)}$ and the value of the program is $2\alpha\sqrt{\beta(1-\beta)}-\alpha^2$. If this is not the case, i.e., if $\alpha > \sqrt{\beta(1-\beta)}$, then $\rho_G = \rho_B = 1$ and the second and third constraint can be restated as $k = 1+\alpha$. This is a knife edge case and we ignore it. Lastly, consider the case when $k > 1+\sqrt{\beta(1-\beta)}$. In this case the second constraint has to bind. The solution is $\rho_G = 1$ and $\rho_B = \frac{\alpha(2-k)}{(1-\alpha)(k-1)}$. The value of the program then is $\frac{\alpha(\beta(1-\beta)+(k-1)(k-1-\alpha))}{k-1}$. Comparing the solutions: If $\alpha \leq \sqrt{\beta(1-\beta)}$ and $k \in [1, 1+\sqrt{\beta(1-\beta)}]$ then $\rho_G = 1$ and $\rho_B = 1-\frac{1}{1-\alpha}\left(1-\frac{\alpha}{\sqrt{\beta(1-\beta)}}\right)$. The value of the program is $2\alpha\sqrt{\beta(1-\beta)}-\alpha^2$. If $k \in [1+\sqrt{\beta(1-\beta)}, 2]$ then $\rho_G = 1$ and $\rho_B = \frac{\alpha(2-k)}{(1-\alpha)(k-1)}$. The value of the program is $\frac{\alpha(\beta(1-\beta)+(k-1)(k-1-\alpha))}{k-1}$.

Comparison reveals that:

(i) if $k < \beta$ only $\mathcal{P}_{A1}^{(R,T)}$ is feasible. The regulator chooses $\boldsymbol{\rho} = \boldsymbol{\rho}^F$ and the firm chooses $q = 0$ after any report.

(ii) if $k \in [\beta, \min\{1+\beta-\sqrt{\beta(1-\beta)}, \alpha+\beta\}$ both $\mathcal{P}_C^{(R,T)}$ and $\mathcal{P}_{A1}^{(R,T)}$ are feasible but $\mathcal{P}_C^{(R,T)}$ has a higher value. The regulator chooses $\boldsymbol{\rho} = \left(0, \frac{\alpha+\beta-k}{\alpha(1+\beta-k)}\right)$ and the firm chooses $q = 1$ after r_B.

(iii) if $k \in [\min\{1+\beta-\sqrt{\beta(1-\beta)}, \alpha+\beta\}, \alpha+\beta]$ both $\mathcal{P}_{A1}^{(R,T)}$ and $\mathcal{P}_C^{(R,T)}$ are feasible but the value of $\mathcal{P}_C^{(R,T)}$ is larger. The regulator chooses $\boldsymbol{\rho} = \left(0, \frac{1}{\alpha}\left(1-\frac{(1-\alpha)}{\sqrt{\beta(1-\beta)}}\right)\right)$ and the firm chooses $q = 1$ after r_B, otherwise chooses $q = 0$. Note that the condition for feasibility of $\mathcal{P}_C^{(R,T)}$ is a necessary and sufficient condition for $1+\beta-\sqrt{\beta(1-\beta)}, \alpha+\beta\} < \alpha+\beta$. If this is not true then the case $k \in [\min\{1+\beta-\sqrt{\beta(1-\beta)}, \alpha+\beta\}, \alpha+\beta]$ can never exist.

(iv) if $k \in [\alpha+\beta, 1]$ both $\mathcal{P}_B^{(R,T)}$ and $\mathcal{P}_C^{(R,T)}$ may be feasible. The value of $\mathcal{P}_B^{(R,T)}$

is larger. Hence the regulator chooses $\boldsymbol{\rho} = \left(\frac{\alpha(1+\beta-k)}{(1-\alpha)(k-\beta)}, 1\right)$ and the firm always chooses $q = 1$ (for any report).

(iv) if $k \in [1, 1+\beta]$ the programs $\mathcal{P}_B^{(R,T)}$, $\mathcal{P}_D^{(R,T)}$ and $\mathcal{P}_C^{(R,T)}$ may be feasible but the value of $\mathcal{P}_B^{(R,T)}$ is the highest. The regulator chooses $\boldsymbol{\rho} = \left(\rho_B = 0, \rho_G = \frac{1+\alpha-k}{\alpha(2-k)}\right)$ and the firm always chooses $q = 1$ for any report.

(v) if $k \in [1+\beta, 1+\alpha]$ both $\mathcal{P}_B^{(R,T)}$ and $\mathcal{P}_D^{(R,T)}$ are feasible but the value of $\mathcal{P}_B^{(R,T)}$ is larger. The regulator chooses $\boldsymbol{\rho} = \left(0, \frac{1+\alpha-k}{\alpha(2-k)}\right)$ and the firm always chooses $q = 1$ for any report. Note that the condition for feasibility of $\mathcal{P}_B^{(R,T)}$ is a necessary and sufficient condition for $1+\beta < 1+\alpha$. If this is not true then the case $k \in [1+\beta, 1+\alpha]$ can never exist.

(vi) if $k \in [1+\alpha, 1+\sqrt{\beta(1-\beta)}]$ both $\mathcal{P}_{A2}^{(R,T)}$ and $\mathcal{P}_D^{(R,T)}$ are feasible but the value of $\mathcal{P}_D^{(R,T)}$ is larger. The regulator chooses $\boldsymbol{\rho} = \left(1 - \frac{1}{1-\alpha}\left(1 - \frac{\alpha}{\sqrt{\beta(1-\beta)}}\right), 1\right)$ and the firm chooses $q = 1$ only after r_G. Note that the condition for feasibility of $\mathcal{P}_D^{(R,T)}$ is a necessary and sufficient condition for $1+\alpha < 1+\sqrt{\beta(1-\beta)}$. If this is not true then the case $k \in [1+\alpha, 1+\sqrt{\beta(1-\beta)}]$ can never exist.

(vii) if $k > 1+\sqrt{\beta(1-\beta)}$ both $\mathcal{P}_D^{(R,T)}$ and $\mathcal{P}_{A2}^{(R,T)}$ are feasible but the value of $\mathcal{P}_D^{(R,T)}$ is higher. The regulator chooses $\boldsymbol{\rho} = \left(\frac{\alpha(2-k)}{(1-\alpha)(k-1)}, 1\right)$ and the firm chooses $q = 1$ if $r = r_G$.

Summarizing the solution: for any $k \leq \beta$ the regulator chooses $\boldsymbol{\rho} = \boldsymbol{\rho}^F$. Otherwise, chooses $\boldsymbol{\rho} \in \boldsymbol{\rho}^I$. The firm chooses to:

(i) never gather information if $k \leq \beta$;

(ii) gather information only after $r = r_B$ if $k \in [\beta, \alpha+\beta]$;

(iii) always gather information if $k \in [\alpha+\beta, \max\{1+\beta, 1+\alpha\}]$;

(iv) gather information only after $r = r_G$ if $k > \max\{1+\beta, 1+\alpha\}$.

Proof of Proposition 6: The firm solves and compares the values of the following programs:

$\mathcal{P}_1^{(FF)}$ *Never meet the threshold:*

$$\begin{aligned}
&\max_{q,\boldsymbol{\rho}} 0 \\
&\text{s.t.} \quad 0 \leq q \leq 1 \\
&\qquad 0 \leq \rho_B \leq \rho_G \leq 1 \\
&\qquad \widehat{\alpha}^G + 1 \leq k
\end{aligned}$$

$\mathcal{P}_2^{(FF)}$ *Meet threshold with ($r = r_G$, $m = H$):*

$$\begin{aligned}
\max_{q,\boldsymbol{\rho}} \ & q\beta(\rho_G\alpha + \rho_B(1-\alpha)) \\
\text{s.t.} \ & 0 \le q \le 1 \\
& 0 \le \rho_B \le \rho_G \le 1 \\
& \widehat{\alpha}^G + 1 \ge k \\
& \widehat{\alpha}^G + \frac{(1-q)\beta}{1-q\beta} \le k \\
& \widehat{\alpha}^B + 1 \le k
\end{aligned}$$

$\mathcal{P}_3^{(FF)}$ *Meet threshold with ($r = r_G$, $m = H$) and ($r = r_G$, $m = \varnothing$):*

$$\begin{aligned}
\max_{q,\boldsymbol{\rho}} \ & \rho_G\alpha + \rho_B(1-\alpha) \\
\text{s.t.} \ & 0 \le q \le 1 \\
& 0 \le \rho_B \le \rho_G \le 1 \\
& \widehat{\alpha}^G + \frac{(1-q)\beta}{1-q\beta} \ge k \\
& \widehat{\alpha}^B + 1 \le k
\end{aligned}$$

$\mathcal{P}_4^{(FF)}$ *Meet threshold with ($r = r_G$, $m = H$) and ($r = r_B$, $m = H$):*

$$\begin{aligned}
\max_{q,\boldsymbol{\rho}} \ & q\beta \\
\text{s.t.} \ & 0 \le q \le 1 \\
& 0 \le \rho_B \le \rho_G \le 1 \\
& \widehat{\alpha}^G + \frac{(1-q)\beta}{1-q\beta} \le k \\
& \widehat{\alpha}^B + 1 \ge k
\end{aligned}$$

$\mathcal{P}_5^{(FF)}$ *Meet threshold with ($r = r_G$, $m = H$) and ($r = r_B$, $m = H$) and ($r = r_G$, $m = \varnothing$):*

$$\begin{aligned}
\max_{q,\boldsymbol{\rho}} \ & \rho_G\alpha + \rho_B(1-\alpha) + q\beta((1-\rho_G)\alpha + (1-\rho_B)(1-\alpha)) \\
\text{s.t.} \ & 0 \le q \le 1 \\
& 0 \le \rho_B \le \rho_G \le 1 \\
& \widehat{\alpha}^G + \frac{(1-q)\beta}{1-q\beta} \ge k \\
& \widehat{\alpha}^B + 1 \ge k
\end{aligned}$$

First, consider $\mathcal{P}_1^{(FF)}$. The objective function is independent of q and $\boldsymbol{\rho}$. High ρ_G and low ρ_B tighten the third constraint. Hence, the firm prefers $\boldsymbol{\rho} \in \boldsymbol{\rho}^U$. This program is only feasible when $\alpha + 1 \le k$ is satisfied. The optimal solution is $q = 0$ and $\boldsymbol{\rho} \in \boldsymbol{\rho}^U$

and the value of the program is $V = 0$.

Next, consider $\mathcal{P}_2^{(FF)}$. The objective function is increasing in ρ_G, ρ_B and q. The LHS of the fourth constraint is decreasing in q, i.e., high q relaxes it. As before, the fourth constraint can be restated as $q > \widehat{q}$. Note that $\widehat{q} < 1$ because $R'_G(q) \leq 0$ and $\lim_{q\to 1} R_G(q) = \widehat{\alpha}^G - k \leq 1 - k \leq \widehat{\alpha}^B + 1 - k < 0$ by the fifth constraint. It follows that $q > \widehat{q}$ is not an empty set. Hence, $q = 1$. The third and fifth constraints are independent of q but high ρ_G and low ρ_B relax them. Hence $\rho_G = 1$. Then, $\widehat{\alpha}^B = 0$ and the fifth constraint can be restated as $k \geq 1$ (this is a sufficient condition for the fourth constraint at $q = 1$ to be satisfied too). It remains to consider the optimal ρ_B. Given that high ρ_B increases the objective function but tightens the third constraint, the firm will choose it to bind that constraint, i.e., $\widehat{\alpha}^G + 1 = k$, which yields $\rho_B = \frac{\alpha(2-k)}{(1-\alpha)(k-1)}$. The second constraint (and more specifically $\frac{\alpha(2-k)}{(1-\alpha)(k-1)} \leq 1$) is satisfied if $k \geq 1 + \alpha$. Summarizing, the solution is $q = 1$, $\boldsymbol{\rho} = \left(1, \frac{\alpha(2-k)}{(1-\alpha)(k-1)}\right)$ and the value is $V = \beta\alpha\left(1 + \frac{(2-k)}{(k-1)}\right)$. It is feasible if $k \geq 1 + \alpha$.

Now consider $\mathcal{P}_3^{(FF)}$. The objective function is increasing in ρ_G, ρ_B and independent of q. Using the solution of Program $\mathcal{P}_2^{(FF)}$, we can restate the third constraint as $q \leq \widehat{q}$. As before, $\widehat{q} < 1$ because $R'_G(q) \leq 0$ and $\lim_{q\to 1} R_G(q) = \widehat{\alpha}^G - k \leq 1 - k \leq \widehat{\alpha}^B + 1 - k < 0$ by the fourth constraint. Moreover, $\widehat{q} > 0$ holds if $\lim_{q\to 0} R_G(q) = \widehat{\alpha}^G + \beta - k > 0$. On the other hand, if $\lim_{q\to 0} R_G(q) = \widehat{\alpha}^G + \beta - k < 0$ it follows that $\widehat{q} < 0$ and so the third constraint $q \leq \widehat{q}$ cannot be satisfied for any $q \in [0, 1]$. Hence a necessary condition for this program to be feasible is $\widehat{\alpha}^G + \beta - k \geq 0$. Setting $q = 0$, we only need to make sure $\widehat{\alpha}^G + \beta - k \geq 0$ and $\widehat{\alpha}^B + 1 - k \leq 0$ hold. High ρ_G relaxes both of these constraints (as well as increases the objective function), hence $\rho_G = 1$. Then, $\widehat{\alpha}^B + 1 - k = 1 - k$, i.e, the program is feasible if $k \geq 1$. Setting the constraint $\widehat{\alpha}^G + \beta - k \geq 0$ binding and solving for ρ_B yields $\rho_B = \frac{\alpha(1+\beta-k)}{(1-\alpha)(k-\beta)}$. Note that the second constraint and especially $\frac{\alpha(1+\beta-k)}{(1-\alpha)(k-\beta)} \geq 0$ is satisfied if $1 + \beta \geq k$. Summarizing, the solution is $q = 0$, $\boldsymbol{\rho} = \left(1, \frac{\alpha(1+\beta-k)}{(1-\alpha)(k-\beta)}\right)$ and the value is $V = \alpha\left(1 + \frac{(1+\beta-k)}{(k-\beta)}\right)$. It is feasible if $k \in [1, 1 + \beta]$.

Now consider $\mathcal{P}_4^{(FF)}$. The objective function is increasing in q and is independent of $\boldsymbol{\rho}$. Using the solution of Program $\mathcal{P}_2^{(FF)}$, we can restate the third constraint as $q \geq \widehat{q}$. Furthermore, $\widehat{q} < 1$ if $\lim_{q\to 1} R_G(q) = \widehat{\alpha}^G - k < 0$. Otherwise the constraint $q \geq \widehat{q}$ cannot be satisfied for any $q \in [0, 1]$ and therefore $\mathcal{P}_4^{(FF)}$ is not feasible. Hence, $q = 1$ and a necessary condition for this program to be feasible is that $k \geq \widehat{\alpha}^G$. At the same time, $\widehat{\alpha}^B + 1 \geq k$ has to be satisfied. High ρ_G and low ρ_B tighten both constraints and so $\rho_G = \rho_B$. Summarizing, the solution is $q = 1$, $\boldsymbol{\rho} \in \boldsymbol{\rho}^U$ and the value is $V = \beta$. It is feasible if $k \geq \alpha$ (which holds since $k \geq \alpha + \beta$) and $k < 1 + \alpha$.

Lastly, consider $\mathcal{P}_5^{(FF)}$. First, note that $(1 - \rho_G)\alpha + (1 - \rho_B)(1 - \alpha) = 1 - (\rho_G\alpha + \rho_B(1 - \alpha))$. Hence, we can restate the objective function as $(1 - q\beta)(\rho_G\alpha + \rho_B(1 - \alpha)) + q\beta$. It is increasing in q, ρ_G and ρ_B. Using the solution of Program $\mathcal{P}_2^{(FF)}$, we can restate the third constraint as $q \leq \widehat{q}$. Hence, the solution is $q = \widehat{q}$. To ensure that $\widehat{q} \in [0,1]$ we need to verify that $\lim_{q\to 0} R_G(q) = \widehat{\alpha}^G + \beta - k \geq 0$ and $\lim_{q\to 1} R_G(q) = \widehat{\alpha}^G - k \leq 0$. Consider also the fourth constraint: $\widehat{\alpha}^B + 1 \geq k$. Low ρ_B relaxes $\widehat{\alpha}^G + \beta - k \geq 0$ but tightens both $\widehat{\alpha}^G \leq k$ and $\widehat{\alpha}^B + 1 \geq k$. If one of them binds, the other one is satisfied.

Suppose $\widehat{\alpha}^G \leq k$ binds, i.e., $\rho_B = \frac{(1-k)\alpha\rho_G}{k(1-\alpha)}$. Then, $\widehat{\alpha}^B + 1 \geq \widehat{\alpha}^B + \widehat{\alpha}^G \geq \widehat{\alpha}^G = k$ and $\widehat{\alpha}^G + \beta - k = \beta \geq 0$. Substituting in the objective function, it continues to be increasing in ρ_G and hence $\rho_G = 1$ and $\rho_B = \frac{(1-k)\alpha}{k(1-\alpha)}$. The second constraint is satisfied if $k \in [\alpha, 1]$. Taking into account that we are only considering the case $k \geq \alpha + \beta$, this can simplified to $k \leq 1$. The constraint $\widehat{\alpha}^B + 1 \geq k$ can also be simplified to $k \leq 1$. At this $\boldsymbol{\rho}$ we have $\widehat{q} = 1$. The value of the program is $V = \frac{\alpha(1-\beta)}{k} + \beta$.

Suppose $\widehat{\alpha}^B + 1 \geq k$ binds, i.e., $\rho_B = \frac{1+\alpha-k-\alpha(2-k)\rho_G}{(1-\alpha)(1-k)}$. Substituting into the objective function we note that it becomes decreasing in ρ_G, hence we set $\widehat{\alpha}^G + \beta - k$ binding and solve for ρ_G: $\rho_G = \frac{(1+\alpha-k)(k-\beta)}{\alpha(1-\beta)}$. Therefore $\rho_B = \frac{(1+\alpha-k)(1+\beta-k)}{(1-\alpha)(1-\beta)}$. However, whenever $k \geq \alpha + \beta$, the second constraint is not satisfied. To summarize, the solution is $q = 1$, $\boldsymbol{\rho} = \left(1, \frac{(1-k)\alpha}{k(1-\alpha)}\right)$ and the value is $V = \frac{\alpha(1-\beta)}{k} + \beta$. It is feasible if $k \leq 1$.

Let $\hat{k} \equiv \min\{\frac{\alpha}{\beta} + \beta, 1+\beta, 1+\alpha\}$. Summarizing with our observation about the case of $k \leq \alpha + \beta$:

(i) If $k \in [0, \alpha+\beta]$ the solution is $\boldsymbol{\rho} \in \boldsymbol{\rho}^U$ and $q = 0$.

(ii) If $k \in [\min\{1, \alpha+\beta\}, 1)$ then the value of program $\mathcal{P}_5^{(FF)}$ is the highest. The solution is $q = 1$, $\boldsymbol{\rho} = \left(1, \frac{\alpha(1-k)}{(1-\alpha)k}\right)$.

(iii) If $k \in [\max\{1, \alpha+\beta\}, \widehat{k})$ then the value of program $\mathcal{P}_3^{(FF)}$ is the highest. The solution is $q = 0$, $\boldsymbol{\rho} = \left(1, \frac{\alpha(1+\beta-k)}{(1-\alpha)(k-\beta)}\right)$.

(iv) If $k \in [\max\{1, \widehat{k}\}, 1+\alpha)$ then the value of program $\mathcal{P}_4^{(FF)}$ is the highest. The solution is $q = 1$, $\boldsymbol{\rho} \in \boldsymbol{\rho}^U$.

(v) If $k \in [1+\alpha, 2]$ then the value of program $\mathcal{P}_2^{(FF)}$ is the highest. The solution is $q = 1$, $\boldsymbol{\rho} = \left(1, \frac{\alpha(2-k)}{(1-\alpha)(k-1)}\right)$.

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